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A Location Problem with the A-distance in a Competitive Environment

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Abstract— **This paper proposes a new location problem of competitive facilities, e.g. shops. In the most studies of competitive facility location, the distance between the facilities and their customers is represented as the Euclid distance. The proposing location problem introduces the A-distance, proposed by Widmayer etc., for representing the situation that the directions which customers can move are given. For solving the formulated facility location problem efficiently, it is shown that the problem is reformulated as a combinatorial optimization problem, and its solving method based on genetic algorithms is proposed. The efficiency of the solving method is shown by applying to several examples of the competitive facility location problems.**

Keywords: location, competitiveness, linear programming, combinatorial optimization, genetic algorithms

1 Introduction

A competitive facility location problem (CFLP) is one of optimal location problems for commercial facilities, e.g. shops and supermarkets, and an objective for CFLPs is mainly to obtain as many customers as possible. Mathematical studies on CFLPs are originated by Hotelling [6]. He considered CFLPs under the conditions that (i) customers are uniformly distributed on a line segment, (ii) each of decision makers (DMs) can locate and move her/his own facility at any times, and (iii) all customers only use the nearest facility. CFLPs on a plain were studied by Okabe and Suzuki [12], etc. As extension of Hotelling's CFLP, Wendell and McKelvey [17] assumed that there exist customers on a finite number of points, called demand points (DPs), and they considered CFLPs on a network whose nodes are DPs.

Based upon CFLPs by Wendell and McKelvey, Hakimi [5] considered CFLPs under the conditions that the DM locates her/his facilities on a network that competitive facilities were already located. Drezner [3] extended Hakimi's CFLPs to CFLPs on a plane that there are DPs and competitive facilities. As extension of their CFLPs, CFLPs with quality or size of facilities are considered by Uno et al. [15], Fernández et al. [4], Bruno and Improta [1], and Zhang and Rushton, CFLPs with fuzziness are considered by Moreno Pérez et al [11], and CFLPs based on maximal covering are considered by Plastria and Vanhaverbeke [13].

In the studies of CFLPs including the above CFLPs, the distance between the facilities and their customers is an important factor and usually represented as the Euclid distance. However, if the facilities are located in urban areas, CFLPs with the Euclid distance may be often unsuitable because such problems are not considered a limitation of the direction that customers can move in urban areas. For cases that the facilities are located in the city whose streets are set out neatly in a grid, e.g. Manhattan and Kyoto, CFLPs with the rectilinear distance are suitable [18]. By extending the definition of the rectilinear distance, Widmayer et al. [19] proposed "the A-distance", which is a distance for the situation that customers can move at given several directions. Matsutomi and Ishii [10] proposed a location problem of public facilities, e.g. hospitals, fire department, and ambulance service, with the A-distance, and Uno et al. [16] extended the above problem to a multiobjective problem. For the details of the relation between optimal location problems and various types of distance, the readers can refer to the survey of Martini et al. [9].

In this study, we propose a new location problem of competitive facilities by introducing the A-distance. Since the formulated CFLP is a nonlinear programming problem, it is difficult to find a strict optimal solution of the problem directly. Then, we show that the CFLP can be reformulated as a combinatorial optimization problem. In order to solve the combinatorial optimization problem efficiently, we propose an efficient solving method based upon genetic algorithms by utilizing characteristics of the CFLPs. For details of the genetic algorithms, the readers can refer to the studies of Sakawa et al. [14]. We apply the solving method to examples of the CFLPs in order to show its efficiency.

The remaining structure of this article is organized as follows: In Section 2, we describe the definition of the A-distance proposed by Widmayer et al. [19]. In Section 3, we formulate the CFLP with the A-distance into a nonlinear programming problem. Since it is difficult to solve

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the formulated problem directly, we show that one of its optimal solutions can be found by solving a combinatorial optimization problem in Section 4. In Section 5, we propose an efficient solving method based upon genetic algorithms by utilizing characteristics of the CFLPs. We show the efficiency of the solving method by applying to several examples of the CFLPs with the A-distance in Section 6. Finally, in Section 7, concluding comments and future extensions are summarized.

2 A-distance

In this section, we will describe the definition of the A-distance proposed by Widmayer et al. [19] in order to introduce the A-distance into the CFLP in the next sections.

We consider the case that there are β directions that customers can move in any points of the plane $\boldsymbol{R}^2$. Let $\alpha_1, \ldots, \alpha_\beta$ be the directions which are satisfied $0 \leq \alpha_1 < \ldots < \alpha_\beta < \pi$, and let $A \equiv \{\alpha_1, ..., \alpha_\beta\}$ be the set of the directions. The line segment between $\boldsymbol{p}_1$ and $\boldsymbol{p}_2 \in \boldsymbol{R}^2$ is denoted by $\overline{\boldsymbol{p}_1\boldsymbol{p}_2}$. If the direction of $\overline{\boldsymbol{p}_1\boldsymbol{p}_2}$ is in A, the line segment is called "A-oriented."

Definition 1 *For $\boldsymbol{p}_1$ and $\boldsymbol{p}_2 \in \boldsymbol{R}^2$, the A-distance between these two points is defined as follows:*

$$d_A(\boldsymbol{p}_1, \boldsymbol{p}_2) \equiv \begin{cases} ||\boldsymbol{p}_1 - \boldsymbol{p}_2||, \\ \quad \textit{if } \overline{\boldsymbol{p}_1\boldsymbol{p}_2} \textit{ is A-oriented,} \\ \min\limits_{\boldsymbol{p}_3 \in \boldsymbol{R}^2} \{d_A(\boldsymbol{p}_1, \boldsymbol{p}_3) + d_A(\boldsymbol{p}_3, \boldsymbol{p}_2)\}, \\ \quad \textit{otherwise,} \end{cases} \quad (1)$$

where $|| \cdot ||$ is the Euclid norm.

An example of the A-distance is shown in Fig. 1. Widmayer et al. [19] shows that one of the shortest paths by means of the A-distance between any two points is the combination of at most two line segments.

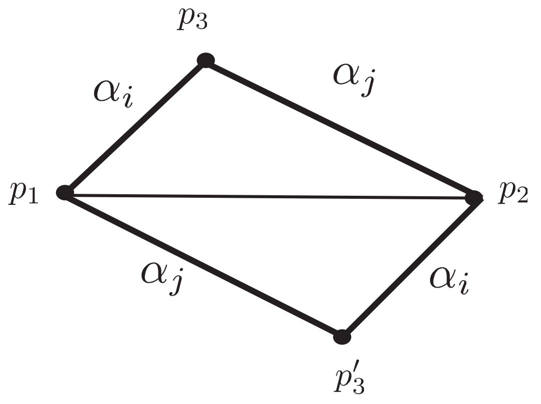

Figure 1: A-distance

The A-distance can be considered a generalization of the rectilinear distance (the Manhattan distance), because the rectilinear distance is represented as the A-distance for cases that $A = \{0, \pi/2\}$. On the other hand, the Euclid distance is represented as the A-distance for cases that $A = [0, \pi)$.

Definition 2 *For a point $\boldsymbol{p} \in \boldsymbol{R}^2$ and $r > 0$, the locus of points whose A-distances to $\boldsymbol{p}$ are r is called "the A-circle" whose center and radius are $\boldsymbol{p}$ and r, respectively.*

An example of the A-circle is shown in Fig. 2.

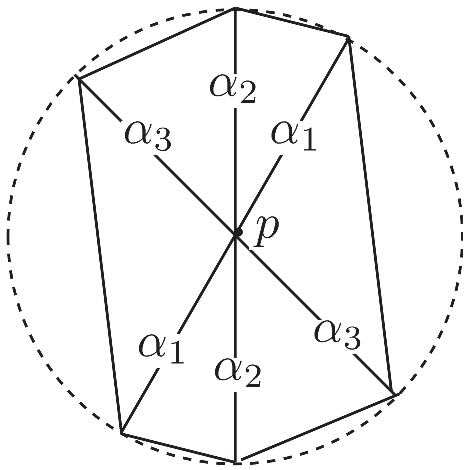

Figure 2: A-circle ($A = \{\pi/3, \pi/2, 3\pi/4\}$)

Let $\boldsymbol{p} = (p_1, p_2)$ and $r > 0$ be the center and radius of an A-circle, respectively. Then, the region in the A-circle not including its boundary is represented as follows:

$$\{(z_1, z_2) \mid a_{l1}z_1 + a_{l2}z_2 < b_l, \ l = 1, \ldots, 2\beta\}, \quad (2)$$

where for each $l = 1, \ldots, \beta$,

$$\left. \begin{array}{l} a_{l1} = \sin\alpha_{l+1} - \sin\alpha_l, \ a_{l2} = \cos\alpha_l - \cos\alpha_{l+1}, \\ b_l = p_1 a_{l1} + p_2 a_{l2} + r\sin(\alpha_{l+1} - \alpha_l), \\ a_{l+\beta,1} = -a_{l1}, \ a_{l+\beta,2} = -a_{l2}, \\ b_{l+\beta} = 2\sin\alpha_l - b_l. \end{array} \right\} \quad (3)$$

3 Formulation of CFLP with the A-distance

In our CFLPs, we assume that all customers only exist on DPs in plain R^2. For convenience sake, by aggregating all customers on the same DP, we regard one DP as one customer.

There are n DPs in R^2, and let $D = \{1, \cdots, n\}$ be a set of indices of the DPs. Let m denote the number of new facilities that the DM locates, and let k denote the number of competitive facilities which have been already located in R^2. The sets of indices of the new facilities and the competitive facilities are denoted by $F = \{1, \ldots, m\}$ and $F_E = \{m+1, \cdots, m+k\}$, respectively.

Let $\boldsymbol{u}_i \in R^2$ be the site of DP $i \in D$, and let $\boldsymbol{x}_j \in R^2$ be the site of facility $j \in F \cup F_E$. Then, the A-distances between DP i to facility j is represented as $d_A(\boldsymbol{u}_i, \boldsymbol{x}_j)$. It is assumed that DP i uses facility j if the following relations are satisfied:

$$d_A(\boldsymbol{u}_i, \boldsymbol{x}_j) < d_A(\boldsymbol{u}_i, \boldsymbol{x}_l),\ \forall l > j, \quad (4)$$

$$d_A(\boldsymbol{u}_i, \boldsymbol{x}_j) \leq d_A(\boldsymbol{u}_i, \boldsymbol{x}_l),\ \forall l < j. \quad (5)$$

Both (4) and (5) represent that all customers use the nearest facility in the sense of the A-distance and they give priority to the competitive facilities if the two or more A-distances are the same.

Let $\boldsymbol{x} = (\boldsymbol{x}_1, \ldots, \boldsymbol{x}_m)$ be the location of the new facilities. Then we use the following 0-1 variable to represent whether or not DP i uses new facility $j \in F$:

$$\varphi_i^j(\boldsymbol{x}) = \begin{cases} 1, & \text{if DP } i \text{ uses the new facility } j, \\ 0, & \text{otherwise.} \end{cases} \quad (6)$$

Let $w_i > 0$ be the buying power (BP) of DP i. New facility $j \in F$ can obtain the BP w_i if DP i uses the facility j. The objective of the DM is maximizing the sum of BP that all the new facilities obtain. Then, the CFLP with the A-distance is formulated as the following optimization problem:

$$\left.\begin{array}{ll} \text{maximize} & \displaystyle\sum_{i=1}^{n}\sum_{j=1}^{m} w_i \cdot \varphi_i^j(\boldsymbol{x}) \\ \text{subject to} & \boldsymbol{x} \in R^{2m} \end{array}\right\} \quad (7)$$

Problem (7) is a nonconvex nonlinear programming problem and we need to find at least one optimal solution of (7). However, for most CFLPs with the Euclid distance [3, 15], it is difficult to find an optimal solution by using general analytic solving methods with derived functions of the objective function, Kuhn-Tucker conditions, etc. The above difficulty is also true of the CFLPs with the A-distance. Moreover, as shown in Section 6, it is also difficult to find an optimal solution by using heuristic solving methods for nonlinear programming problems, e.g. genetic algorithm for numerical optimization for constrained problem (GENOCOP) [8]. In the next section, we show that (7) can be reformulated as a combinatorial optimization problem.

4 Reformulation to a combinatorial optimization problem

For DP $i \in D$, the A-distance to the competitive facilities is denoted as follows:

$$\bar{d}_i^E \equiv \min_{j \in F_E} \{d_A(\boldsymbol{u}_i, \boldsymbol{v}_j)\}, \quad (8)$$

Let $C_i \subseteq \boldsymbol{R}^2$ be the region in the A-circle whose center and radius are $\boldsymbol{u}_i$ and $\bar{d}_i^E$, respectively, which does not include the boundary. Then, the following theorem plays an important role to find an optimal solution of (7).

Theorem 3 *For a set of DPs $\hat{D} \subseteq D$, one of the new facilities can obtain BP from all DPs in $\hat{D}$ by locating in $Cir(\hat{D})$ if the following equation is satisfied:*

$$Cir(\hat{D}) \equiv \bigcap_{i \in \hat{D}} C_i \neq \emptyset \quad (9)$$

PROOF: *For obtaining BP from DP $i \in \hat{D}$, a new facility needs to be located at points whose A-distances from DP i are less than $\bar{d}_i^E$, that is, located in C_i. Therefore, one of the new facilities can obtain BP from all DPs in $\hat{D}$ by locating in $Cir(\hat{D})$ if (9) is satisfied.* □

Let I_C be the family of the set of DPs satisfying (9). Then, from Theorem 3, there exist $\hat{D}_1, \ldots, \hat{D}_m \in I_C$ such that an optimal solution of P is obtained if the DM locates new facilities $1, \ldots, m$ in $Cir(\hat{D}_1), \ldots, Cir(\hat{D}_m)$, respectively. Therefore, (7) can be reformulated as the following combinatorial optimization problem:

$$\left.\begin{array}{ll} \text{maximize} & \displaystyle\sum_{i=1}^{n}\sum_{j=1}^{m} w_i \cdot \varphi_i^j(\boldsymbol{x}) \\ \text{subject to} & \boldsymbol{x}_j \in Cir(\hat{D}_j), \forall j = 1, \ldots, m, \\ & \hat{D}_j \in I_C, \forall j = 1, \ldots, m \end{array}\right\} \quad (10)$$

For DP i, we use (3) in Section 2 for representing the A-circle C_i. Then, we can examine whether $\hat{D}_j$ is included in I_C and then find a point in $Cir(\hat{D}_j)$ if $\hat{D}_j \in I_C$ by using the first phase of the two phase simplex method, that is solving the following linear programming problem with auxiliary variables $\tau_1, \ldots, \tau_{2\beta} \geq 0$:

$$\left.\begin{array}{ll} \text{minimize} & \tau_1 + \cdots + \tau_{2\beta} \\ \text{subject to} & a_{l1} z_1^j + a_{l2} z_2^j + \varepsilon + \tau_l \\ & \quad \leq \min_{i \in \hat{D}} \{b_l^i\}, \forall l = 1, \ldots, 2\beta, \\ & \tau_1, \ldots, \tau_{2\beta} \geq 0, \end{array}\right\} \quad (11)$$

where ε is a sufficiently small positive number. Let $(z_1^{j*}, z_2^{j*}, \tau_1^*, \ldots, \tau_{2\beta}^*)$ be an optimal solution of (11). Then, if the optimal value of (11) is 0, $Cir(\hat{D}_j)$ is a nonempty set and (z_1^{j*}, z_2^{j*}) is a location of facility j that can obtain BP from all DPs in $\hat{D}_j$. Thus we can find an optimal solution of (10) by solving (11) for all subsets of the set of DPs and then choosing m optimal solutions in all given optimal solutions.

Because the number of the all subsets of DPs is $2^n - 1$, the computational complexity of choosing m optimal solutions is ${}_{2^m-1}C_m$. Then, it is NP-hard to find a strict optimal solution of (10). In the next section, we propose an efficient solving method to find an approximate optimal solution of (10).

5 Solving method based on genetic algorithms

Problem (10) is a combinatorial optimization problem and can be regarded as a nonlinear 0-1 programming problem by representing $\hat{D}_1, \ldots, \hat{D}_m$ as a set of 0-1 variables. For nonlinear 0-1 programming problems, a genetic algorithm is one of the efficient solution algorithms. In this section, we propose an efficient solving method based upon the genetic algorithm with double strings [14] by utilizing characteristics of the CFLPs.

5.1 Coding

We use the following 0-1 variable to represent whether DP i is included in $\hat{D}_j$:

$$s_i^j = \begin{cases} 1, & \text{if } i \in \hat{D}_j, \\ 0, & \text{if } i \notin \hat{D}_j. \end{cases} \tag{12}$$

Then, we represent the solution of (10) $\hat{D}_1, \ldots, \hat{D}_m$ as the individual represented as the following status matrix:

$$S = \begin{pmatrix} s_1^1 & \cdots & s_1^m \\ \vdots & \ddots & \vdots \\ s_n^1 & \cdots & s_n^m \end{pmatrix}. \tag{13}$$

5.2 Generation of the initial population

We propose the following method in order to generate initial individuals in the feasible set of (10):

Step 1: Set the location of new facility $\boldsymbol{x}$ randomly.

Step 2: Give an individual as the following status matrix:

$$S = \begin{pmatrix} \varphi_1^1(\boldsymbol{x}) & \ldots & \varphi_1^m(\boldsymbol{x}) \\ \vdots & \ddots & \vdots \\ \varphi_n^1(\boldsymbol{x}) & \ldots & \varphi_n^m(\boldsymbol{x}) \end{pmatrix}. \tag{14}$$

For S given by the above method, it is clearly satisfied that $\hat{D}_1, \ldots, \hat{D}_m \in I_C$.

5.3 Fitness function

In usual genetic algorithms, an objective function is used as a fitness function. We also use the objective function as the fitness function basically. However, for individual $S \in \{0,1\}^{mn}$, there is no suitable location of new facility j if $Cir(\hat{D}_j) = \emptyset$. Such individuals are usually enumerated by either the following two methods:

- The individuals have fatal genes, that is, the fitness value of the individuals is 0.
- Facility j is not located on any points if $Cir(\hat{D}_j) = \emptyset$, and then the fitness value of the individuals is the sum of the BP obtained by the facilities satisfying $Cir(\hat{D}_j) \neq \emptyset$.

There are generally many such individuals in (10) and their fitness values are low by using both methods. This means that these methods make many individuals which do not contribute the search of the optimal solutions of (10). Then, we propose another method such that facility j satisfying $Cir(\hat{D}_j) = \emptyset$ is located on a suitable point.

From the first phase of the two phase simplex method, the objective function of (11) represents the degree of violation for constraints. The optimal solution of (11) minimizes the degree of violation for constraints. Then, if facility j satisfying $Cir(\hat{D}_j) = \emptyset$ is located on the optimal solution of (11), it is expected that the facility can obtain BP from most DPs in $\hat{D}_j$. We thus represent the fitness value of individual as the sum of BP obtained by the new facilities whose locations are given by solving (11) for $\hat{D}_1, \ldots, \hat{D}_m$.

Moreover, for each new facility $j \in F$, the left-side coefficients of constraints of (11) are common to all individuals. Then, we can use the sensitivity analysis [7] to solve (11) efficiently.

5.4 Mutation

In addition to crossover, mutation and inversion in the genetic algorithms with double strings [14], we propose a new mutation utilizing characteristics of CFLPs.

Individual S is often different from the status of use for $\boldsymbol{x}^S$ due to the following two results:

- If $\hat{D}_j \notin I_C$ for at least one new facility $j \in F$, then status matrix S and the status of use if the location of facility j is $\boldsymbol{x}_j^S$ are not the same.
- Even if $\hat{D}_j \in I_C$, DP i that $s_i^j = 1$ may use another new facility whose attractive power is more than that of facility j.

In general genetic algorithms, descendant individuals inheriting characteristics of ancestor generations are generated by genetic operations, e.g. crossover and mutation. However, the individuals mentioned above are far from containing several characteristics, e.g. a status of use, a location of facilities, etc. Then, general crossover, mutation and inversion are often ineffective for such individuals. Accordingly, we propose a mutation that individuals are changed so as to adjust their status matrices to the statuses of use if the new facilities are located on the sites found for the individuals. This mutation is represented as follows:

Step 1. For individual S, find the location of new facilities $\boldsymbol{x}^S$ by solving (11) for $\hat{D}_1, \ldots, \hat{D}_m$.

Step 2. Replace S with the following status matrix:

$$S' = \begin{pmatrix} \varphi_1^1(\boldsymbol{x}^S) & \dots & \varphi_1^m(\boldsymbol{x}^S) \\ \vdots & \ddots & \vdots \\ \varphi_n^1(\boldsymbol{x}^S) & \dots & \varphi_n^m(\boldsymbol{x}^S) \end{pmatrix}. \quad (15)$$

Step 3. For individual S', find the location of new facilities $\boldsymbol{x}^{S'}$ and its fitness value by solving (11) for $\hat{D}_1, \dots, \hat{D}_m$.

6 Numerical experiments

In this section, we show the efficiency of the solution algorithm in the previous sections by applying to several examples of the CFLPs. In these examples, we give three directions such that $A = \{0, \pi/4, \pi/2\}$. The sites of DPs $\boldsymbol{u}_1, \dots, \boldsymbol{u}_n$ are given in $[0, 10] \times [0, 10]$ randomly, and their BP $w_1, \dots, w_n$ are given in $\{1, \dots, 10\}$ randomly. We give three competitive facilities, that is $k = 3$, and for competitive facility $j \in F_E$, its site $\boldsymbol{x}_j$ is given in $[0, 10] \times [0, 10]$ randomly.

Next, we give parameters about our solving method; for details of parameters of genetic algorithms, the reader can read reference of Sakawa et al. [14]. We set generation gap $G = 0.9$, population size $N_{GA} = 150$, and terminal generation $T_{GA} = 2000$. Probabilities of crossover, mutation, and inversion are $p_C = 0.9$, $p_M = 0.01$, and $p_I = 0.03$, respectively. Probability of mutation proposed in Section 5.4 is $p_L = 0.03$.

In order to compare the computational results of our solving method with other computational results, we use the following two solving method to solve the examples of the CFLPs. One is to solve (10) by using a branch and bound method. Because the enumeration method needs much computational time for finding a strict optimal solution of (10) if m and n are large, we set 5000 seconds to the upper bound of its computational time for each example. The other is to solve (7) directly by using the genetic algorithm for numerical optimization for constrained problem (GENOCOP), proposed by Koziel and Michalewicz [8]. We set that its population size is 100 and its terminal condition is satisfied if its computational time is more than our solving method.

Now we apply the three solving methods to several examples of the CFLPs, where each of these algorithms is implemented 20 times for each example by using DELL Optiplex GX620 (CPU: Pentium(R) 4 2.33GHz, RAM: 512MB).

First, we apply the three solving methods for the CFLPs that it is fixed that $m = 1$. The computational results of our solving method, branch and bound method, and GENOCOP are shown in Tables 1-3.

From Tables 1-3, our solving method can find the same good solutions as the solutions of branch and bound method at the CFLPs with $n = 20, 40, 60, 80$ and better solutions than branch and bound method at the CFLPs with $n = 100$ with a shorter computational time, and ours solving method can find better solutions than GENOCOP at the all CFLPs with a shorter computational time. Since the computational time of our solving method is not suddenly increased for the increase of n, it is shown that our solving method is also efficient for large scale CFLPs.

Next, we apply the three solving methods for the CFLPs that it is fixed that $n = 50$. The computational results of our solving method, branch and bound method, and GENOCOP are shown in Tables 4-6.

Table 1: Computational results by genetic algorithm ($m = 1$)

n	20	40	60	80	100
Best	50	91	138	175	214
Mean	50.0	91.0	137.1	175.0	214.0
Worst	50	91	136	175	214
CPU times	7.131	14.27	19.57	28.35	35.26

Table 2: Computational results by branch and bound method ($m = 1$)

n	20	40	60	80	100
Optimal	50	91	138	175	202
CPU times	0.121	1.839	123.5	1529	5000

Table 3: Computational results by GENOCOP ($m = 1$)

n	20	40	60	80	100
Best	50	91	138	175	214
Mean	50.0	90.9	136.4	173.2	210.0
Worst	50	88	136	171	207
CPU times	10.0	20.0	30.0	40.0	50.0

Table 4: Computational results by genetic algorithm ($n = 50$)

m	1	2	3	4	5
Best	124	172	218	229	243
Mean	124.0	171.5	214.1	225.0	241.4
Worst	124	164	205	213	232
CPU times	21.13	39.16	66.12	103.3	155.8

Table 5: Computational results by branch and bound method ($n = 50$)

m	1	2	3	4	5
Optimal	124	172	218	221	228
CPU times	15.07	229.2	3467	5000	5000

Table 6: Computational results by GENOCOP ($n = 50$)

m	1	2	3	4	5
Best	124	164	214	214	228
Mean	123.6	164.0	201.2	207.5	218.9
Worst	120	164	196	196	214
CPU times	30.0	50.0	70.0	110.0	160.0

From the discussion of computational time of (10) in Section 4, the computational time for finding a strict optimal solution of (10) is exponentially increased for m. From Table 5, the branch and bound method cannot find strict optimal solutions of the CFLPs with $m = 4, 5$ within 5000 seconds. On the other hand, our solving method can find better solutions of these CFLPs than the branch and bound method and GENOCOP with a shorter computational time.

7 Conclusions and future researches

In this paper, we have proposed a new CFLP on the plane by introducing the A-distance. Because the formulated CFLP is difficult to find a strict optimal solution of the problem directly, we have shown that the CFLP can be reformulated as a combinatorial optimization problem. Since the combinatorial optimization problem is NP-hard, we have proposed an efficient solving method based upon genetic algorithms by utilizing characteristics of the CFLPs. We have applied the solving method to several examples of the CFLPs in order to show its efficiency. Although it is known that the CFLPs with the Euclid distance needs much computational time to find their optimal solutions, the CFLPs with the A-distance can be found their optimal solutions with shorter computational time. This means that CFLPs can be solved efficiently by giving proper directions customers can move.

References

[1] Bruno, G., Improta, G., "Using gravity models for the evaluation of new university site locations: A case study," *Computers & Operations Research*, V35, N2, pp. 436-444, 2/08.

[2] Chvátal, V., *Linear programming*, W.H. Freeman and Company, 1983.

[3] Drezner, Z., "Competitive location strategies for two facilities," *Regional Science and Urban Economics*, V12, pp. 485-493, 11/82.

[4] Fernández, J., Pelegrín, B., Plastria, F., Tóth, B., "Solving a Huff-like competitive location and design model for profit maximization in the plane," *European Journal of Operational Research*, V179, N3, pp. 1274-1287, 6/07.

[5] Hakimi, S.L., "On locating new facilities in a competitive environment," *European Journal of Operational Research*, V12, pp. 29-35, 1/83.

[6] Hotelling, H., "Stability in competition," *The Economic Journal*, V30, pp. 41-57, 3/29.

[7] Jansen, B., de Jong, J.J., Roos, C., Terlaky, T., "Sensitivity analysis in linear programming: just be careful!" *European Journal of Operational Research*, V101, N1, pp. 15-28, 8/97.

[8] Koziel, S., Michalewicz, Z., "Evolutionary algorithms, homomorphous mappings, and constrained Parameter Optimization," *Evolutionary Computation*, V7, N1, pp. 19-44, 04/99.

[9] Martini, H., Schŏbel, A., "Median hyperplanes in normed spaces - a survey," *Discrete Applied Mathematics*, V89, pp. 181-195, 12/98.

[10] Matsutomi, T., Ishii, H., "Minimax location problem with A-distance," *Journal of the Operations Research Society of Japan*, V41, pp. 181-195, 6/98.

[11] Moreno Pérez, J.A., Marcos Moreno Vega, L., Verdegay, J.L., "Fuzzy location problems on networks," *Fuzzy Sets and Systems*, V142, N3, pp. 393-405, 3/04.

[12] Okabe, A., Suzuki, A., "Stability of competition for a large number of firms on a bounded two-dimensional space," *Environment and Planning A*, V19, N8, pp. 1067-1082, 1/87.

[13] Plastria, F., Vanhaverbeke, L., "Discrete models for competitive location with foresight," *Computers & Operations Research*, V35, N3, pp. 683-700, 2/08.

[14] Sakawa, M., Kato, K., Ushiro, S., "An interactive fuzzy satisficing method for multiobjective 0-1 programming problems involving positive and negative coefficients through genetic algorithms with double strings," *Proceedings of the 8th International Fuzzy Systems Association World Congress*, V1, pp. 430-434, 8/99.

[15] Uno, T., Ishii, H., Saito, S., Osumi, S., "Competitive facility location problem. An algorithm for the problem concerning the existence of multi-type customers," *Central European Journal of Operations Research*, V12, N1, pp. 79-85, 12/04.

[16] Uno, T., Katagiri, H., Kato, K., A Fuzzy Model for the Multiobjective Emergency Facility Location Problem with A-Distance *The Open Cybernetics & Systemics Journal*, V1, pp. 21-27, 11/07.

[17] Wendell, R.E., McKelvey, R.D. "New perspective in competitive location theory," *European Journal of Operational Research*, V6, pp. 174-182, 02/81.

[18] Wesolowsky, G.O., "Rectangular distance location under the minimax optimality criterion," *TRANS.SCI.*, V6, pp. 103-113, 2/72.

[19] Widmayer, P., Wu, Y.F., Wong, C.K., "On some distance problems in fixed orientations," *SIAM J.COMPUT.*, V16, pp. 728-746, 8/87.

[20] Zhang, L., Rushton, G., "Optimizing the size and locations of facilities in competitive multi-site service systems," *Computers & Operations Research*, V35, N2, pp. 327-338, 2/08.