Instability of a Class of Nonlinear Discrete Dynamical Systems

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***Abstract*—We give an instability result of the nonlinear discrete dynamical system**

$$X(n+1)=(AE+B)X(n)+F(X(n))X(n),$$
$$(n,X)\in N\times R^m$$

by means of the analysis and computing method.

***Index Terms*—discrete system, initial value, integer, instability.**

I. INTRODUCTION

We do research on the nonlinear discrete dynamical system

$$X(n+1)=(AE+B)X(n)+F(X(n))X(n) \qquad (1)$$

where A is a constant, E is an $n\times n$ identity matrix, B is an $n\times n$ anti-symmetric matrix, $B\neq 0$, $X\in R^m$, $R=(-\infty,+\infty)$, R^m is an *m*-dimensional linear vector space over the reals with norm $\|\cdot\|$, $n\in N$, N is a set of nonnegative integer, $F: R^m\to R$ is a given continuous scalar function. There is a unique solution of the system (1) through initial value (n_0, X_0).

The instability problem of discrete dynamical system (1) is very important [1—4]. In this paper, an instability result of the system (1) is given.

II. ANALYSIS

Let

$$V(X)=(x_1,x_2,\cdots,x_m)\begin{pmatrix}x_1\\x_2\\\vdots\\x_m\end{pmatrix}=x_1^2+x_2^2+\cdots+x_m^2=X^TX.$$

Thus, if

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$$X(n)=X(n,n_0,X_0)$$

is a solution of dynamical system (1) through (n_0, X_0), then the difference $\Delta V_{(1)}$ of V along the solution $X(n)$ satisfies

$$\begin{aligned}\Delta V_{(1)}&=V(X(n+1))-V(X(n))\\&=X^T(n+1)X(n+1)-X^T(n)X(n).\end{aligned}$$

Using (1), we obtain

$$\begin{aligned}\Delta V_{(1)}&=X^T(n+1)X(n+1)-X^T(n)X(n)\\&=[(AE+B)X(n)+F(X(n))X(n)]^T\times\\&\quad[(AE+B)X(n)+F(X(n))X(n)]-X^T(n)X(n).\end{aligned}$$

Using the equality

$$(C+D)^T=C^T+D^T \text{ for the matrix } C \text{ and } D,$$

we have

$$\begin{aligned}\Delta V_{(1)}&=[(AE+B)X(n)+F(X(n))X(n)]^T\times\\&\quad[(AE+B)X(n)+F(X(n))X(n)]-\\&\quad X^T(n)X(n)\\&=\{[(AE+B)X(n)]^T+[F(X(n))X(n)]^T\}\times\\&\quad[(AE+B)X(n)+F(X(n))X(n)]-\\&\quad X^T(n)X(n).\end{aligned}$$

Using the equality $(CD)^T=D^TC^T$, we have

$$\begin{aligned}\Delta V_{(1)}&=\{[(AE+B)X(n)]^T+[F(X(n))X(n)]^T\}\times\\&\quad[(AE+B)X(n)+F(X(n))X(n)]-\\&\quad X^T(n)X(n)\\&=\{[X^T(n)(AE+B)^T]+[F(X(n))X(n)]^T\}\times\\&\quad[(AE+B)X(n)+F(X(n))X(n)]-\\&\quad X^T(n)X(n)\\&=\{[X^T(n)(AE+B^T)]+[F(X(n))X^T(n)]\}\times\\&\quad[(AE+B)X(n)+F(X(n))X(n)]-\\&\quad X^T(n)X(n).\end{aligned}$$

Using the equality

$(C_1+D_1)D = C_1 D + D_1 D$ for the matrix C_1, D_1 and D,

we have

$$\begin{aligned}\Delta V_{(1)} &= \{[X^T(n)(AE+B^T)] + [F(X(n))X^T(n)]\}\times \\ &\quad [(AE+B)X(n) + F(X(n))X(n)] - \\ &\quad X^T(n)X(n) \\ &= X^T(n)(AE+B^T)(AE+B)X(n) + \\ &\quad X^T(n)(AE+B^T)F(X(n))X(n) + \\ &\quad F(X(n))X^T(n)(AE+B)X(n) + \\ &\quad F(X(n))X^T(n)F(X(n))X(n) - \\ &\quad X^T(n)X(n).\end{aligned}$$

Using the equality

$(k_1 C k_2)D = (k_1 k_2)CD$ for the constant k_1 and k_2,

we have

$$\begin{aligned}\Delta V_{(1)} &= X^T(n)(AE+B^T)(AE+B)X(n) + \\ &\quad X^T(n)(AE+B^T)F(X(n))X(n) + \\ &\quad F(X(n))X^T(n)(AE+B)X(n) + \\ &\quad F(X(n))X^T(n)F(X(n))X(n) - \\ &\quad X^T(n)X(n). \\ &= A^2X^T(n)X(n) + AX^T(n)B^TX(n) + \\ &\quad AX^T(n)BX(n) + X^T(n)B^TBX(n) + \\ &\quad AF(X(n))X^T(n)X(n) + \\ &\quad F(X(n))X^T(n)B^TX(n) + \\ &\quad AF(X(n))X^T(n)X(n) + \\ &\quad F(X(n))X^T(n)BX(n) + \\ &\quad F^2(X(n))X^T(n)X(n) - X^T(n)X(n).\end{aligned}$$

B is an $n\times n$ anti-symmetric matrix. Therefore,

$$X^T(n)BX(n) = 0,\ X^T(n)B^TX(n) = 0,$$

we obtain

$$\begin{aligned}\Delta V_{(1)} &= A^2X^T(n)X(n) + AX^T(n)B^TX(n) + \\ &\quad AX^T(n)BX(n) + X^T(n)B^TBX(n) + \\ &\quad AF(X(n))X^T(n)X(n) + \\ &\quad F(X(n))X^T(n)B^TX(n) + \\ &\quad AF(X(n))X^T(n)X(n) + \\ &\quad F(X(n))X^T(n)BX(n) + \\ &\quad F^2(X(n))X^T(n)X(n) - X^T(n)X(n) \\ &= A^2X^T(n)X(n) + X^T(n)B^TBX(n) + \\ &\quad 2AF(X(n))X^T(n)X(n) + \\ &\quad F^2(X(n))X^T(n)X(n) - X^T(n)X(n) \\ &= [A^2 + 2AF(X(n)) + F^2(X(n)) - 1]X^T(n)X(n) \\ &\quad + [BX(n)]^T BX(n) \\ &= \{[A+F(X(n))]^2 - 1]\}X^T(n)X(n) \\ &\quad + [BX(n)]^T BX(n).\end{aligned}$$

If $[A+F(X)]^2 \geq 1$ for $X \in R^m$, then the difference $\Delta V_{(1)}$ of V along the solution $X(n)$ satisfies

$$\Delta V_{(1)} > 0 \quad \text{for } BX \neq 0.$$

Thus, the nonlinear discrete dynamical system (1) is unstable.

III. Main Result

From analysis above, we have result as follow:

If $(A+F(X))^2 \geq 1$ for $X \in R^m$, then the system (1) is unstable.

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