A New Hybrid Multitoning Based on the Direct Binary Search

Xia Zhuge, Yuki Hirano, and Koji Nakano *

Abstract— **Halftoning is an important task to convert a gray scale image into a binary image with black and white pixels. The Direct Binary Search (DBS) is one of the halftoning methods that can generate high quality binary images for halftone areas of the original gray scale images. However, binary images generated by the DBS have clippings, that is, have no tone in the highlight and shadow areas. The first contribution of this paper is to clarify the reason why the DBS generates binary images with clippings and to present a hybrid halftoning method based on the DBS that reproduces the tone of the original images. The key idea is to use the void-and-cluster method to the highlight and the shadow areas and then use the DBS to the other areas. The second contribution is to extend our hybrid halftoning method to generate L-level multitone images, in which every pixel has intensity levels $\frac{0}{L-1}$, $\frac{1}{L-1}$, $\ldots$, $\frac{L-1}{L-1}$. The resulting L-level images are so good that they reproduce the tones and the details of the original gray scale images very well.**
Keywords: Image processing, Halftoning, Direct binary search, Void-and-Cluster, Multilevel halftoning

1 Introduction

A gray scale image is a two dimensional matrix of pixels taking a real number in the range $[0, 1]$. Usually a gray scale image has 8-bit depth, that is, each pixel taking one of the real numbers $\frac{0}{255}, \frac{1}{255}, \ldots, \frac{255}{255}$, which correspond to pixel intensities. *A binary image* is also a two dimensional matrix of pixels taking a binary value 0 (black) or 1(white). *Halftoning* is an important task to convert a gray scale image into a binary image [2]. This task is necessary when a monochrome or color image is printed by a printer with limited number of ink colors.

A multitone image is an intermediate of a gray scale image and a binary image. In an L-level multitone image, each pixel takes one of the real numbers $\frac{0}{L-1}$, $\frac{1}{L-1}$, $\ldots$, $\frac{L-1}{L-1}$. Clearly, a 2-level gray scale image is a binary image. Usually, L is small, say, $L = 3$, 4, or 5. The task of *multitoning* is to generate a multitone image for a given gray scale image. A multitone image is used to print inkjet printer with light colors. For example, some inkjet printers support multi-size dots, which can be achieved by adjusting the amount of ink injected from the nozzle. For example, each nozzle of some inkjet printers can inject $1pl$ (pico liter), $2pl$, and $3pl$ ink. To print a gray scale image using this printer, we convert it to a 4-level gray scale image. For pixels with intensity $\frac{1}{3}$, $\frac{2}{3}$, and $\frac{3}{3}$, the nozzle injects $1pl$, $2pl$ and $3pl$ ink, respectively.

Many halftoning techniques including Error Diffusion [4], Dot Diffusion[5], Ordered Dither using the Bayer threshold array[3] and the Void-and-Cluster threshold array [8], Direct Binary Search (DBS) [1, 6], Local Exhaustive Search (LES)[7], have been presented. The most well-known halftoning algorithm is The Error Diffusion [4] method that propagates rounding errors to unprocessed neighboring pixels according to some fixed ratios. Error Diffusion preserves the average intensity level between the original input image and the binary output image. It is also quite fast and often produces good results. However, the Error Diffusion may generate worm artifacts, sequences of pixels like a worm, especially in the areas of uniform intensity. Another drawback of the Error Diffusion is that the pixel values are propagated to neighbors and the resulting images are defocused.

The Ordered Dither [3] and the Void-and-Cluster [8] use threshold arrays to generate a binary image from an original gray scale image. Each pixel of the original gray scale image is compared with an element of the threshold array. From the result of the comparison, the pixel value of the corresponding pixel of the binary image is determined. Binary images generated by the Ordered Dither method using the Bayer threshold array [3] has periodic artifacts arranged in a two dimensional grid. The Ordered Dither method using the Void-and-Cluster threshold array generates better binary images with no artifact, but the resulting images are defocused and lose the details. Figures 1 and 2 show the binary images for "Lena" and ramp images obtained by the Void-and-Cluster.

In many cases, the DBS generates a better quality images than the Error Diffusion and the Void-and-Cluster. The key idea of the DBS is to find a binary image whose projected image onto human eyes is very close to the original image. The projected image is computed by applying a Gaussian filter, which approximates the characteristic of

*Department of Information Engineering, Hiroshima University, Kagamiyama, Higashi-Hiroshima 739-8527, Japan. Email: {zhugexia,hiranoyuki,nakano}@cs.hiroshima-u.ac.jp

Figure 1: 256 × 256 Lena using the Void-and-Cluster

Figure 3: 256 × 256 Lena using the DBS

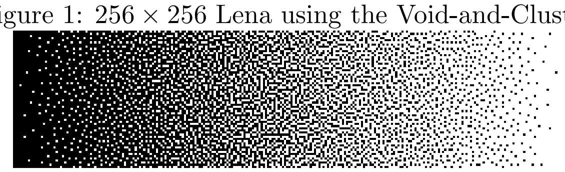

Figure 2: 256×64 ramp image using the Void-and-Cluster

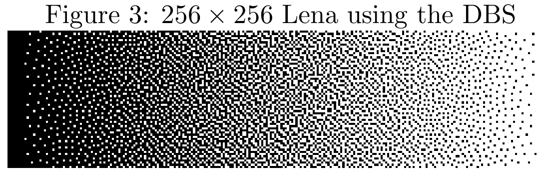

Figure 4: 64 × 256 a ramp image using the DBS

the human visual system. We define the total error of the binary image to be the sum of the differences of the intensity levels over all pixels between the original image and the projected image. In the DBS, a pixel value is flipped if the resulting image has smaller total error. Also, neighboring pixel values are swapped if the total error of the resulting image decreases. The DBS generates a sharp binary image, especially, for halftone areas. However, the generated binary image by the DBS has no tone in the highlight and the shadow areas. Figures 3 and 4 show the binary images generated by the DBS. The resulting image has *clippings*, that is, the highlight and the shadow areas have no dot and lose the tone of the original image. For example, eight columns from the leftmost of Figure 4 have no white dot, although the original image has tone. Also, there is no black dot in the eight columns from the rightmost.

The first contribution of this paper is to clarify the reason why the DBS generates binary image with clippings and present a new halftoning method based on the DBS. The key idea is to use the Void-and-Cluster halftoning method to the highlight and the shadow areas of the original gray scale image. We preserve black pixels in the highlight areas and white pixels in the shadow areas, and apply DBS to the whole image. The resulting binary images have no clipping and reproduces the original tones very well.

Our second contribution is to extend our hybrid halftoning method to generate a L-level multitone image. For this purpose, we first present a DBS-based multitoning method. Let p be the intensity of a pixel of the original gray scale image, and i be an integer such that $\frac{i}{L-1} \leq p \leq \frac{i+1}{L-1}$ holds. The intensity of the corresponding pixel of the binary image is rounded to $\frac{i}{L-1}$ or $\frac{i+1}{L-1}$. We use the DBS to determine if each pixel is rounded to $\frac{i}{L-1}$ or $\frac{i+1}{L-1}$. Figure 9 and 10 show the resulting 3-level multitone images. The readers should have no difficulty to see that the boundary areas, that is, the areas of pixels whose intensity is close to $\frac{1}{2}$ have no tone. In general, for L-level multitioning, the resulting multitone image has no tone in the pixel areas whose intensity is close to $\frac{i}{L-1}$ for integers i. To reproduce the tone correctly, we use the void-and-cluster for such areas, and then apply the DBS. Using this hybrid multitoning method, we can generate a high quality multitone image that reproduces the tones and the details of the original gray scale image.

2 The Ordered Dither and The Direct Binary Search

The main purpose of this section is to review the Ordered Dither [3, 8] and the Direct Binary Search [1], which are key ingredients of our new hybrid halftoning and multitoning methods.

Suppose that an original gray-scale image $A = (a_{i,j})$ of

ISBN: 978-988-98671-8-8

size $n \times n$ is given[1], where $a_{i,j}$ denotes the intensity level at position (i,j) $(1 \leq i,j \leq n)$ taking a real number in the range $[0,1]$. The goal of halftoning is to find a binary image $B = (b_{i,j})$ of the same size that reproduces the original image A, where each $b_{i,j}$ is either 0(black) or 1(white). The ordered dither uses a threshold array $T = (t_{i,j})$ of size $m \times m$, with each element taking a real number in the range $[0,1)$. More specifically the pixel value of each pixel $b_{i,j}$ is determined by the following formula:

$$b_{i,j} = \begin{cases} 0 & \text{if } a_{i,j} \leq t_{i \bmod m, j \bmod m} \\ 1 & \text{if } a_{i,j} > t_{i \bmod m, j \bmod m} \end{cases}$$

The Bayer halftoning uses the Bayer threshold array [3] and the Void-and-Cluster halftoning uses a threshold array obtained by the Void-and-Cluster [8]. Figures 1 and 2 show the binary images obtained using the Void-and-Cluster.

The idea of the DBS is to measure the goodness of the output binary image B using the Gaussian filter that approximates the characteristic of the human visual system. Let $V = (v_{k,l})$ denote a Gaussian filter, i.e. a 2-dimensional symmetric matrix of size $(2w+1) \times (2w+1)$, where each non-negative real number $v_{k,l}$ $(-w \leq k,l \leq w)$ is determined by a 2-dimensional Gaussian distribution such that their sum is 1. In other words,

$$v_{k,l} = c \cdot e^{-\frac{k^2+l^2}{2\sigma^2}} \tag{1}$$

where σ is a parameter of the Gaussian distribution and c is a fixed real number to satisfy $\sum_{-w \leq k,l \leq w} v_{k,l} = 1$. Let $R = (r_{i,j})$ be the projected gray-scale image of a binary image $B = (b_{i,j})$ obtained by applying the Gaussian filter as follows:

$$r_{i,j} = \sum_{-w \leq k,l \leq w} v_{k,l} b_{i+k,j+l} \quad (1 \leq i,j \leq n) \tag{2}$$

Clearly, from $\sum_{-w \leq k,l \leq w} v_{k,l} = 1$ and $v_{k,l}$ is non-negative, each $r_{i,j}$ takes a real number in the range $[0,1]$ and thus, the projected image R is a gray-scale image. We can say that a binary image B is a good approximation of original image A if the difference between A and R is small enough. Hence, we are going to define the error of B as follows. Error $e_{i,j}$ at each pixel location (i,j) is defined by

$$e_{i,j} = |a_{i,j} - r_{i,j}|, \tag{3}$$

and the total error is defined by

$$Error(A,B) = \sum_{1 \leq i,j \leq n} e_{i,j}. \tag{4}$$

Since the Gaussian filter approximates the characteristics of the human visual system, we can think that image B reproduces original gray-scale image A if $Error(A,B)$ is small enough. The best binary image that reproduces A is a binary image B which is given by the following formula:

$$B^* = \arg\min_B Error(A,B). \tag{5}$$

It is very hard to find the optimal binary image B^* for a given gray-scale image A. The idea of the DBS is to find a near optimal binary image B such that $Error(A,B)$ is sufficiently small. For this purpose, the DBS repeats the iterative improvement of binary image B. The value of a particular pixel $b_{i,j}$ is modified by the following two operations:

Flipping This operation is to flip the value of $b_{i,j}$, that is, $b_{i,j} \leftarrow 1 - b_{i,j}$. The value of $b_{i,j}$ is flipped if $Error(A,B)$ becomes smaller.

Swapping Let $b_{i',j'}$ be a neighbor pixel of $b_{i,j}$, that is, either $|i-i'| \leq 1$ or $|j-j'| \leq 1$. This operation is to exchange the values of $b_{i,j}$ and $b_{i',j'}$, that is $b_{i,j} \leftrightarrow b_{i',j'}$. Swap operation is performed if $Error(A,B)$ takes a smaller value.

Clearly, flipping and swapping operations improves the binary image B. In the DBS, these operations are executed in the raster scan order. Further, this raster scan order improvement is repeated until no more improvement by flipping or swapping operations for a pixel is possible. Figures 3 and 4 show the resulting binary images using the DBS. Although the DBS generates high-quality binary images. it does not work very well in the highlight and the shadow areas. It has *clippings*, that is, the highlight and the shadow areas have no dot and lose the tone of the original image.

Let us consider the reason why the shadow area has no white pixel. Let A be a constant tone gray scale image of size $(2w+1) \times (2w+1)$ such that $a_{i,j} = d$ $(0 \leq i,j \leq 2w+1)$ for some real number d. Also, let B be a initial binary image with each pixel taking value 0. Suppose that the center pixel $b_{w,w}$ of B is flipped, and let B' be the resulting binary image. Let $e(d)$ be the improvement of B' over B in terms of the error for A, that is,

$$\begin{aligned} e(d) &= Error(A,B') - Error(A,B) \\ &= \sum_{-w \leq k,l \leq w} |d - v_{k,l}| - \sum_{-w \leq k,l \leq w} d \\ &= \sum_{-w \leq k,l \leq w} |d - v_{k,l}| - d(2w+1)^2 \end{aligned}$$

If d is much smaller than the minimum of $v_{k,l}$, then

$$\begin{aligned} e(d) &= \sum_{-w \leq k,l \leq w} (v_{k,l} - d) - d(2w+1)^2 \\ &= 1 - 2d(2w+1)^2 > 0. \end{aligned}$$

[1]For simplicity, we assume that images are square.

On the other hand, if d is larger than the maximum of $v_{k,l}$, then

$$\begin{aligned} e(d) &= \sum_{-w\le k,l\le w} (d - v_{k,l}) - d(2w+1)^2 \\ &= -\sum_{-w\le k,l\le w} v_{k,l} = -1. \end{aligned}$$

It should have no difficulty to confirm that function $e(d)$ is monotonically decreasing. Thus, there exists a real number D such that $e(D) = 0$. Also, clearly, $e(d) > 0$ for all $d < D$. Hence, the flipping operation increases the error and does not improves the binary image. Therefore, B is the best binary image with the minimum error, although it has no white pixel. It follows that the shadow area consisting of pixels with intensity smaller than D has no white pixel in the corresponding areas of the binary image if we use the DBS. By the same reason, the highlight area with intensity larger than $1 - D$ has no black pixel.

3 Our hybrid halftoning using the DBS and the Void-and-Cluster

This section is devoted to show our new hybrid halftoning method. The key idea of our hybrid halftoning method is to use the Void-and-Cluster method for the pixels with intensity smaller than D or larger than $1 - D$ and then use the DBS.

Suppose that a gray scale image $A = (a_{i,j})$ to be halftoned is given. For a given parameter σ of a Gaussian filter, we first compute the threshold value D satisfying $e(D) = 0$. Since the function e is monotonically decreasing, it is easy to find such value D by an obvious binary search over the argument of e. We first use the Void-and-Cluster to A and obtain a binary image $B = (b_{i,j})$. Next, we assign label *determined/undetermined* to every pixel as follows:

> $b_{i,j}$ is determined, if ($a_{i,j} < D$ and $b_{i,j} = 1$) or ($a_{i,j} > 1 - D$ and $b_{i,j} = 0$), and
> $b_{i,j}$ is undetermined, otherwise.

In other words, if $b_{i,j}$ is a white pixel in the shadow area or a black pixel in the highlight area, then it is a determined pixel. Next, the DBS is executed for all undetermined pixel, that is, flipping and swapping operations repeated in the raster scan order until no more improvement of the error is possible.

Figure 5 and Figure 6 show the resulting images obtained using our hybrid halftoning method. To obtain these images, we use the Gaussian filter with parameter $\sigma = 1.2$ and the threshold value $D = \frac{7}{255}$, because $e(d) > 0$ for $d \le \frac{7}{255}$, and $e(d) < 0$ for $d \ge \frac{8}{255}$. We can see clearly the resulting binary image has dots in the highlight and shadow areas.

Figure 5: 256×256 Lena using our hybrid halftoning

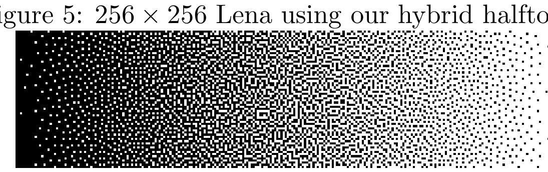

Figure 6: 256×64 a ramp image our hybrid halftoning

4 Multitoning using the Void-and-Cluster and the DBS

The main purpose of this section is to show how we apply the Void-and-Cluster and the DBS for multitoning.

Recall that, the task of multitoning is to generate, for a given gray scale image $A = (a_{i,j})$, an L-level multitone image $M = (m_{i,j})$ with each pixel taking a value in $\{\frac{0}{L-1}, \frac{1}{L-1}, \ldots, \frac{L-1}{L-1}\}$. In our multitoning methods, each pixel of A is rounded to obtain M. That is, $m_{i,j}$ takes a value either $\frac{\lfloor a_{i,j}(L-1)\rfloor}{L-1}$ (round down) or $\frac{\lceil a_{i,j}(L-1)\rceil}{L-1}$ (round up). For the purpose of determining if "round down" or "round up", we use binary image $B = (b_{i,j})$ obtained by halftoning, such that "round down" if $b_{i,j} = 0$ and "round up" if $b_{i,j} = 1$.

Using the idea above, it is not difficult to see how a multitone image obtained by the Void-and-Cluster as follows. First, for a given gray scale image A, we compute a binary image B using the following formula:

$$b_{i,j} = \begin{cases} 0 & \text{if } \mathrm{frac}(a_{i,j} \cdot (L-1)) \le t_{i \bmod m, j \bmod m} \\ 1 & \text{if } \mathrm{frac}(a_{i,j} \cdot (L-1)) > t_{i \bmod m, j \bmod m} \end{cases}$$

In the formula, "frac" is a function removing the integer part of the argument. After that, every pixel $m_{i,j}$ of an L-level multitone image M is determined as follows:

$$m_{i,j} = \begin{cases} \frac{\lfloor a_{i,j}(L-1)\rfloor}{L-1} & \text{if } b_{i,j} = 0 \\ \frac{\lceil a_{i,j}(L-1)\rceil}{L-1} & \text{if } b_{i,j} = 1 \end{cases}$$

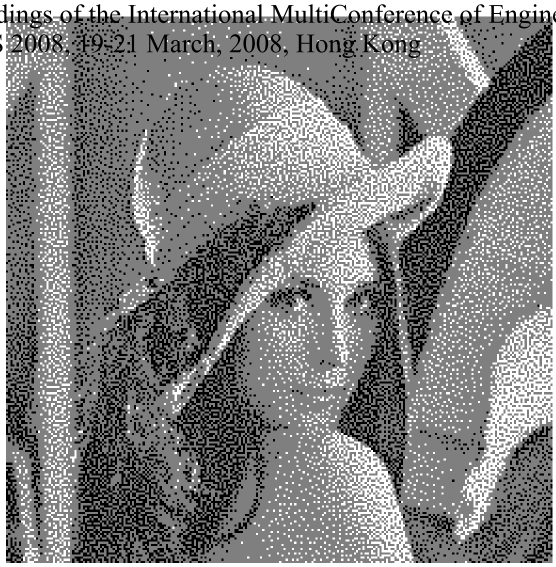

Figure 7: 256 × 256 Lena using the multitone Void-and-Cluster

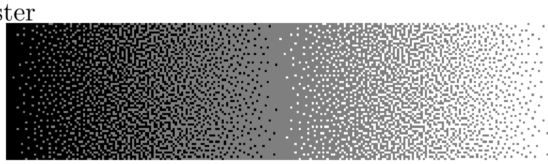

Figure 8: 256 × 64 a ramp image using the multitone Void-and-Cluster

Figure 9: 256 × 256 Lena using multitone DBS

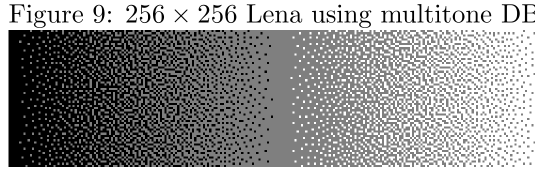

Figure 10: 256 × 64 a ramp image using multitone DBS

Figure 7 and 8 show binary images obtained by the multitone version of the Void-and-Cluster.

An L-level multitone image can be obtained using the DBS in a similar way. In other words, we compute a binary image that determines rounding up or rounding down. Let B be the current binary image thus obtained by the DBS. The corresponding L-level multitone image M can be computed in the same way as the Void-and-Cluster. We then compute the projected image R of the L-level multitone image M and the total error of M with respect to the original gray scale image A using the Eq. (3) and (4). Similarly to the DBS for halftoning, we repeatedly execute flipping and swapping operations for B to find a near optimal multitone image M with small total error. These operations are repeated until no more improvement possible.

Two 3-level multitoning examples with DBS, Lena and the ramp gray image, are shown in Figure 9 and Figure 10 respectively. The resulting images have no tone in the highlight and the shadow areas as well as the halftone areas with intensity levels close to $\frac{1}{2}$. If we use this DBS-based multitoning method to obtain an L-level multitone image, the resulting image has no tone with intensity levels close to $\frac{0}{L-1}, \frac{1}{L-1}, \ldots, \frac{L-1}{L-1}$.

Let us compute the threshold value D' that determine the areas which has no dot using the DBS-based multitoning for obtaining an L-level multitone image. Again, let $V = v_{k,l}$ denote a Gaussian filter of size $(2w+1) \times (2w+1)$. For a fixed real number $d \geq 0$, let $v'_{k,l} = |d - \frac{v_{k,l}}{L-1}|$. Let $e'(d)$ be the function such that

$$\begin{aligned} e'(d) &= \sum_{-w \leq k,l \leq w} v'_{k,l} - \sum_{-w \leq k,l \leq w} d \\ &= \sum_{-w \leq k,l \leq w} |d - \frac{v_{k,l}}{L-1}| - d(2w+1)^2. \end{aligned}$$

By the same argument of the DBS for halftoning, we can find a threshold value D' such that $e(D') = 0$ and $e'(d) > 0$ for all $d < D'$. For such D', the resulting multitone image has no dots for the areas with intensity levels below D' and above $1 - D'$. Further, the resulting image has no tone for the areas with intensity levels in the ranges $[\frac{1}{L-1} - D', \frac{1}{L-1} + D']$, $[\frac{2}{L-1} - D', \frac{2}{L-1} + D']$, …, $[\frac{L-1}{L-1} - D', \frac{L-1}{L-1} + D']$. For example, in Figures 9 and 10, 3-level multitone images have no tone for the areas with intensity $[0, D']$, $[0.5 - D', 0.5 + D']$ and $[1 - D', 1]$.

5 Our hybrid multitoning using DBS and the Void-and-Cluster

This section shows our hybrid multitoning method using the DBS and the Void-and-Cluster.

Similarly to our hybrid halftoning method, we first determine the binary image using the multitoning version of the Void-and-Cluster and then assign determined/undetermined labels to every pixel. For every undetermined pixels we use the multitoning version of the DBS. The details are spelled out as follows.

Suppose that a gray scale image $A = (a_{i,j})$ to be multitoned is given. As before, the threshold value D' such that $e(D') = 0$ can be computed by an obvious binary search over the argument of e. We first use the multitoning version of Void-and-Cluster to A and obtain the binary image $B = b_{i,j}$. Recall that the multitone image can be obtained using the binary image B. Next, we assign label *determined/undetermined* to every pixel as follows:

> $b_{i,j}$ is determined, if $(frac(a_{i,j} \cdot (L-1)) < D' \cdot (L-1)$ and $b_{i,j} = 1)$ or $(frac(a_{i,j} \cdot (L-1)) > (1-D') \cdot (L-1)$ and $b_{i,j} = 0)$, and
> $b_{i,j}$ is undetermined, otherwise.

In other words, $b_{i,j}$ is undetermined if it is white (i.e rounding up) and $a_{i,j}$ is in $[\frac{0}{L-1}, \frac{0}{L-1} + D']$, $[\frac{1}{L-1}, \frac{1}{L-1} + D']$, $\ldots$, $[\frac{L-1}{L-1}, \frac{L-1}{L-1} + D']$. It also undetermined if it is black (i.e rounding down) and $a_{i,j}$ is in $[\frac{1}{L-1} - D', \frac{1}{L-1}]$, $[\frac{2}{L-1} - D', \frac{2}{L-1}]$, $\ldots$, $[\frac{L-1}{L-1} - D', \frac{L-1}{L-1}]$. Next, the DBS is executed for all undetermined pixels, that is, flipping and swapping operations repeated in the raster scan order until no more improvement of the error is possible.

The 3-level multitoning results of Lena image and the ramp image by this hybrid way are shown in Figure 11 and Figure 12. We use the Gaussian filter with parameter $\sigma = 1.2$ and the threshold value $D' = \frac{4}{255}$. Comparing them with the images in Figure 9 and Figure 10 shown in Section 4, we can see they reproduces the tones and details of the original images are much better.

Figure 11: 256×256 Lena using our hybrid multitoning method

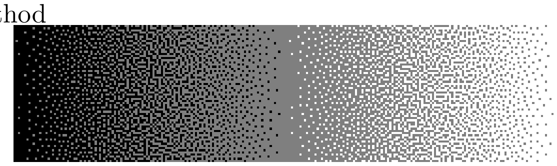

Figure 12: 256×64 ramp image with hybrid multitoning method

6 Conclusions

In this paper, we have presented a hybrid halftoning method of the Direct Binary Search and the Void-and-Cluster. The resulting halftone images have no clipping and reproduce the tone of original gray scale images. We have also shown how to extend the Void-and-Cluster and the Direct Binary Search to generate L-level multitone images. Finally, we have presented a hybrid multitoning method by combining the multitoning versions of the Void-and-Cluster and the Direct Binary Search.

References

[1] M. Analoui and J.P. Allebach. “Model-based halftoning by direct binary search”, In *Proc. SPIE/IS&T Symposium on Electronic Imaging Science and Technology*, volume 1666, pages 96–108, 1992.

[2] D. L. Lau and G. R. Arce. *Modern Digital Halftoning*. Marcel Dekker, 2001.

[3] B.E. Bayer, ”An optimum method for two-level rendition of continuous-tone pictures”, Conference Record, IEEE International Conference on Communications, vol.26, pp.11-15,1973.

[4] R.W. Floyd and L. Steinberg, ”An adaptive algorithm for spatial gray scale”, SID 75 Digest, Society for Information Display, pp.36-37, 1975.

[5] D.E. Knuth, ”Digital halftones by dot diffusion”, ACM Trans. Graphics, vol.6, no.4, pp.245-273, 1987.

[6] S. Bhatt, J. Harlim, J. Lepak, R. Ronkese, J. Sabino and Chai Wah Wu, ”Direct Binary Search with Adapative Search and Swap”, pp.1-9, 2005.

[7] Y. Ito, and K. Nakano, “FM Screening by the Local Exhaustive Search with Hardware Acceleration,” International Journal on Foundations of Computer Science, Vo.16, No.1, pp. 89-104, Feb. 2005

[8] R. Uichney. ”The void-and-cluster method for dither array generation”, Proceedings SPIE, Human Vision, Visual Processing, Digital Displays IV, vol.1913,pp.332-343,1993.