

Measuring Critical thinking in Problem Solving through Online Discussion Forums in First Year University Mathematics

S. M. Jacob and H. K. Sam

Abstract—The first year in university offers an exciting and confusing experience for students as they transit from the secondary school environment. The adaptation is tested through new learning environments, and higher expectations. The inevitable gap created seems pronounced in Mathematics too. The present study focuses on a group of first year university students who take up a Mathematics subject. They are exposed to the rigors of open ended or ill structured problem solving through online discussion forums, as a mechanism to initiate and encourage higher order mathematical thinking and a culture of inquiry. Their thought processes are analyzed through a critical thinking model; their responses to the said approach are sought through a survey. The problem being pursued by the authors here is to measure critical thinking during problem solving in Mathematics among the first year university students. The online discussion forums were used as a medium to expose students to the facets of open ended/ill structured problem solving.

Index Terms—critical thinking, online discussion forums, postings, problem solving.

I. INTRODUCTION

Mathematics is not only a subject but a way of thinking. Mathematics educators have examined the thinking process. Rather than static knowledge detached from other domains and day to day events, mathematics is viewed as problem solving, reasoning and communicating so that students are empowered to confidently explore, conjecture and reason logically about the world around them[1]. It was suggested by the NCTM 2000 that learning goals should incorporate values that reflect mathematics for life, mathematics for the workplace...mathematics for the scientific and technical community. The influence of critical thinking skills or metacognition on mathematical problem solving has attracted research from Ennis [2] and Schoenfeld [3]. In contrast to the traditional text book dominated approach, the mathematics classrooms are encouraged to be a place where discussion and collaboration are valued in building a climate of intellectual challenge. Such reform oriented classrooms are

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described as communities of mathematical inquiry-where students learn to speak and act mathematically by participation in mathematical discussion and solving new or unfamiliar problems [4]. In short, sociocultural perspectives on learning have caused a reform in mathematics education. This is in line with the Vygotskian school of thought which claims that human thinking is inherently social in its origins [5].

The present study was inspired by the authors' efforts to encourage students to apply critical thinking in problem solving sessions in Mathematics classes. The objectives were (1) to suggest a model to measure and report on engagement of critical thinking among the online discussion forum postings; (2) to measure students' engagement in critical thinking using the model; and (3) to evaluate student reactions to the problem solving sessions on the discussion forum using a survey instrument.

II. LITERATURE REVIEW

A. Problem Solving

Gagne said, "The central point of education is to teach people to think, to use their rational powers, to become better problem solvers" [6]. The term "problem solving" in educational settings, though not specified by Gagne, involves the presentation and solution of well-structured textbook problems which are a far cry from the ill-structured problems encountered in everyday and professional contexts. Mathematics classrooms provide classic and consistent usage of well-structured problems. Most problem solving models [7] have the following steps of occurrence: (1) representing problems (2) searching for solutions and (3) implementing them. When a problem is given, the problem solver attempts to construct an appropriate problem representation. The success of the application of the problem to the situation is dependent on the correct recognition of the problem type. If this fails, then the strategy used could be a means-ends analysis as was found from the research on mathematics word problems[8]. The task is complete once the generated solution is satisfied by the context to which it is applied to. If the solution fails, then it calls for a return to the problem space, and redefinition of the problem or another choice of method of solving. The above mentioned process does not apply to ill-structured problems that are encountered in everyday and professional contexts. Ill- structured problems are characterized by the fact that [9] any or all of the three components - an initial state, operators, a goal state is not

well specified. Solving ill-structured problems not only calls on the meta cognitive skills of the problem solvers, but also epistemic knowledge of the validity of alternative solutions. This brings in the need to provide supports to novice problem solvers to help them in the task of solving ill-structured problems [10].

B. Asynchronous Online Learning

Vygotsky [5] emphasized a sociocultural perspective in which students use language and social discourse to make sense of the world. Interaction and discussion of ideas with partners when guidelines are given (e.g., describing observations clearly, reasoning about causes and effects, posing precise questions, formulating hypotheses, critically examining competing explanations, and summarizing results) during science inquiry activities provide a scaffold for the development of reasoning and scientific understanding [11]. Science (and math) investigations should link students to scientists through data sharing, critiquing, and direct communication, and involve the argumentative processes scientists use to achieve common understandings. Computer-supported collaborative learning communities investigating model-eliciting problems help students pose and explore conjectures, understand mathematical concepts, and improve mathematical models [12].

Asynchronous online course delivery has seen the most significant growth in the last decade [13]. Their marketing logo is “learn anytime anywhere”. Online learning is grounded in a collaborative constructivist view of teaching and learning. Constructivist approaches to learning strive to create environments where learners actively participate in the environment in ways that are intended to help them construct their own knowledge, rather than having the teacher interpret the world and ensure that students understand the world as they have been told. In constructivist environments, learners are active in the sense that they must participate and interact with the surrounding environment, resulting in their own view of the subject. Collaboration is the hallmark of constructivism [14]. Asynchronous learning environments allow learners to interact anytime, from any place. The Community of Inquiry Model [15] talks of the cognitive presence, teaching presence and social presence in a community of inquiry. And cognitive presence is vitally important for the development of critical thinking skills particularly in asynchronous text based communication. According to Garrison and Anderson [15], the educational transaction experience has two purposes: to personally construct meaning, and “to refine and confirm this understanding collaboratively within a community of learners”. Socially constructing meaning within a learning community is the heart of collaborative, constructivist learning. Tertiary educational courses make heavy use of the companies designing online Learning Management Systems (LMS) such as Blackboard (<http://www.blackboard.com>), WebCT (<http://www.webct.com>). These systems provide avenues for online asynchronous discussions also known as discussion forums. Research has shown the potential of computer conferencing for creating an educational community of inquiry and mediating critical reflection and discourse. Discussion forums have the unique capacity to

support higher-order constructivist learning and the development of a learning community [16]. The term ‘discussion forum’ would be used to mean ‘asynchronous online learning forum’ in this paper.

C. Critical Thinking and the Existing Models

The rational activities of critical thinking are often associated with problem solving. Critical thinking is a form of problem solving, but a major difference between the two is that critical thinking involves reasoning about open ended or “ill structured” problems, while problem solving is usually considered narrow in scope [17]. Kurfis [17] continues to suggest that the overlap between them is substantial enough to justify close examination of problem solving and related processes for insight regarding critical thinking. Much more than analyzing arguments, critical thinking is a larger process which includes not only discovery (the intuitive and creative processes), but also justification (the evaluative and logical-reasoning processes). Critical or higher order thinking has consistently been seen as a necessary condition for education. Paul and Elder [18] believe “Critical thinking is a process by which the thinker improves the quality of his or her thinking by skillfully taking charge of the structures inherent in thinking and imposing intellectual standards upon them.” According to Paul and Elder, a well cultivated critical thinker “Raises vital questions and problems; Formulates them clearly and precisely; Gathers and assesses relevant information; Arrives at well-reasoned conclusions and solutions, testing them against relevant criteria and standards; Thinks open-mindedly within alternative systems of thought, recognizing and assessing, as need be, their assumptions, implications, and practical consequences; Communicates effectively with others.”

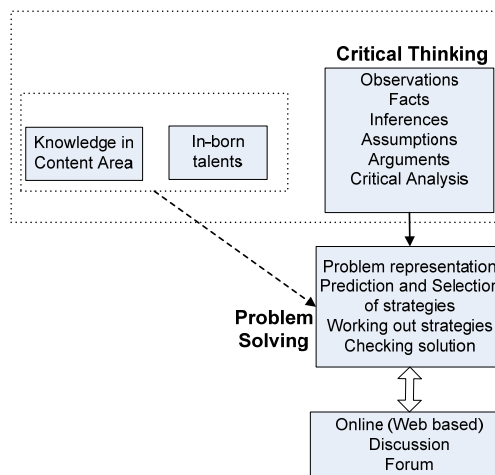


Figure 1. The overview of domains of Critical Thinking and Problem Solving done through Online discussion forums

The figure 1 shown by the authors depicts that problem solving ability is controlled by various factors like previous knowledge of the topic, inborn talents and critical thinking skills. The authors are concerned about only how critical thinking skills affect the problem solving abilities. The diagram shows that the various facets of critical thinking contribute to the different stages of problem solving.

Garrison et al [15] have initiated the concept of a “community of inquiry” and have worked to show the

importance of asynchronous online communications as a means to create and sustain cognitive presence and as a vehicle for engaging in critical thinking. Garrison's Practical Inquiry model of critical thinking follows the four phases of triggers, exploration, integration and resolution. Fahy [19] has compared his Transcript Analysis Tool against Garrison's model and confirmed that both models reveal different aspects of online interaction.

III. RESEARCH METHODOLOGY

A. The Proposed Model

The model for the current study was adapted from the model proposed and tested by Perkins and Murphy [20]. Perkins and Murphy had developed and used a model to measure and report individual engagement in critical thinking in a course in Education. There are few studies which focus on reporting individual engagement in critical thinking and the authors of this study were looking for the same. Paul and Elder [18] have defined precisely the qualities of a well cultivated thinker in their book. The authors have used the category names from the model of Perkins and Murphy and made use of the concepts of Paul and Elder to describe the categories to come up with the model which identifies critical thinking in problem solving as shown in Table 1. The two models used are among the latest models tested so far, hence their relevance to the study. The indicators have been modified, in comparison to the model by Perkins and Murphy to suit the identification and measurement of the critical thinking during the stages of problem solving in Mathematics. The model includes the indicators and description to each category.

TABLE I. MODEL FOR IDENTIFYING ENGAGEMENT IN CRITICAL THINKING DURING PROBLEM SOLVING

Clarification			
Formulates the problem precisely and clearly.			
Analyses, negotiates or discusses the scope of the problem	Identifies one or more underlying assumptions in the parts of the problem	Identifies relationships among the different parts of the problem	Defines or criticizes the definition of relevant terms
Assessment			
Raises vital questions and problems within the problem.			
Gathers and assesses relevant information.	Provides or asks for reasons that proffered evidence is valid or relevant.	Make value judgment on the assessment criteria or argument or situation.	
Inference			
Reasons out based on relevant criteria and standards			
Makes appropriate deductions from discussed results.	Arrives at well thought out conclusions	Makes generalizations from relevant results.	Frames relationships among the different parts of the problem.
Strategies			
Thinks and suggests open mindedly within alternative systems of thought.			
Propose specific steps to lead to the solution.	Discuss possible steps.	Evaluate possible steps.	Predicts outcomes of proposed steps.

B. The Research Study

This study is the first stage of a research project set out to investigate the effectiveness of using Discussion Forums as a

medium to promote the critical thinking skills in the problem solving sessions of Mathematics classes in a first year university setup. The first year university students go through a transition stage from school into university. Hence they need a lot of support in terms of coping up with new learning environments and taking ownership of their learning. The particular university has a full fledged LMS namely the Blackboard Learning System (BBL) in place. The Discussion Forum is one of the widely used features on the BBL. The course chosen for the research project was a compulsory Mathematics course in the Bachelor of Engineering program offered by the university. A percentage of their final course grade was assigned to the students for participation in the Discussion Forum. The sessions for this study were run around the middle of the semester in 2007. The students were being exposed to such problem solving sessions on the BBL for the first time in their university life. An open ended problem was posted on the Discussion Forum. The problem was not fully ill structured but could be classified as open ended because the goals were defined and there were given some constraints. But it did possess multiple paths and solutions. A deadline of few weeks was given to the students to discuss and solve the problem over the forum. Late postings were ignored.



Figure 2. The sample online (web-based) discussion forum screen shot

Around 48 students took part in the problem solving sessions put up on the Black Board Learning System and there were 119 postings generated as in fig. 2. The discussion was moderated by the course lecturer (the first author) very minimally. Instead the lecturer asked a few students in the same course (whose mathematical abilities and initiatives were known to be fairly good) to direct and moderate the discussion. The lecturer stepped in to encourage students to present new ideas and questions. Though this did encourage the interactive dialogues, the resultant postings did not promote deepening the dialogues. The postings of only 8 students were studied and the others kept aside for reasons like not enough number of postings, or extremely brief or atypical postings. The insufficient and non standard responses were expected since the students are novices to this culture of discussion forums in problem solving. Also the lack of scaffolding by the lecturer could have been the reason of extremely brief postings.

The Discussion Forum postings were downloaded. Each posting was considered a unit and was coded into the four categories, using the indicators in the model as guides. In

cases where more than one critical thinking process appeared within a posting, only one code was associated, which seemed to be the most important in the context. But some postings did not receive a code since they were personal or social in nature, and not part of the discussion and analysis of the problem.

IV. RESEARCH FINDINGS

The transcripts or postings were coded using the model defined in Table 1. Table 2 reports the critical thinking engagement of individual participants.

Among the postings generated by the students in the previously defined minimally scaffolded setup, it is evident by looking at the group as a whole that the students tended to engage more in *Clarification* (mean of 38% and S.D. of 21%) closely followed by *Assessment* (mean of 35% and S.D. of 15%). The postings related to *Strategies* only consist of a mean of 7% and a S.D. (standard deviation) of 8%, which shows the clear inconsistency of postings in this category. Inference related postings occupy a mean of 20% and a S.D. of 16%. The variations among the postings seem to be high as shown by the S.D., but could be attributed to the small number of participants.

TABLE II. SUMMARY OF STUDENTS' ENGAGEMENT IN CRITICAL THINKING

	Student Participants								Mean	S.D
	S1	S2	S3	S4	S5	S6	S7	S8		
Total No. of postings	9	14	13	17	8	8	13	7	11	3
Total No. of coded postings	7	13	10	13	7	7	13	7	10	3
% of coded 'Clarification' postings	29	31	30	46	14	71	15	71	38	21
% of coded 'Assessment' postings	43	38	60	38	29	14	46	14	35	15
% of coded 'Inference' postings	14	8	10	8	57	14	31	14	20	16
% of coded 'Strategies' postings	14	23	0	8	0	0	8	0	7	8

There were two hypotheses the authors were interested to confirm in line with the second objective of the study.

Hypothesis 1 – The majority of online discussion forum postings would be in the lower levels of critical thinking, in a general crowd of non trained(not exposed to sessions like this before) students.

This was noted from the findings of Perkins and Murphy [20]. Garrison, Fahy [19] has observed a majority of postings in the *exploration* phase, which is the second category among his four categories to measure engagement in critical thinking. In the present case, the mean number of postings (%) in Table 2 shows a majority in the first and second categories. Thus the hypothesis is true on a group level. On an individual level, it could be noted that student S5 stands out unique with a 57% of postings in the *Inference* category. Also student S2 has the highest number of 23% postings in

the *Strategies* category, as compared to the rest who have very few. Student S1 is another exception who has a fairly distributed % of postings in all the categories.

Hypothesis 2 – Discussion forums with little scaffolding does not generate much higher levels of arguments in problem solving. It was inferred by [10] that course tutors should intervene if students don't make much progress by themselves. This hypothesis was also confirmed from the few number of total coded postings (only 77) and also very few number of postings in the *Inference* and *Strategies* categories as seen in table 2. The authors strongly feel that more inquiry oriented and strategy related postings could have been generated with the frequent intervention of the lecturer. The analysis of the postings based on the codes does provide information to the instructors on which categories of critical thinking needs to be encouraged through instruction and giving feedbacks. Though it is good to notice that a high proportion of students are engaging in clarification and assessment, it is reasonable to expect a higher proportion of postings in inference and strategies because these are skills expected from university level students. A useful step here could be to let the students see the postings collected by the lecturer, and be asked to judge the postings based on the codes suggested by the model and provide the instructor with examples of each. This could reveal the importance of higher order thinking skills to the students.

In short, the model which included four critical thinking processes, proved effective for the identification and measurement of individuals' critical thinking in online Discussion Forums. The inter rater reliability factor was not relevant here, since only the course lecturer was involved in the coding of the postings.

Some examples of the postings in the four categories are shown below. The question was "Two towns are to get their water supply from a river. Both towns are on the same side of the river at distance of 6 km and 18 km respectively from the river bank. If the distance between the points on the river bank nearest to the towns respectively be 10 km, find where may a single pumping station be located to require the least amount of pipe and how much pipe is needed for the same."

Clarification – "First, of course we'll have to draw out the sketch of the towns' position and the river and the pumping station should be at the intersection of the hypotenuses of the right angle triangles. The triangles have their right angles between the side parallel to the river bank and the perpendicular distance of the towns to the river."

Assessment – "That's nice but please check the part I highlighted on the attachment...then everything will be fine. Check the expansion below:

$$x^4 - 20x^3 + 136x^2 = 324x^2 - 6480x + 34200 + x^4 - 20x^3 + 136x^2$$

$$324x^2 - 6480x + 34200 = 0$$

Inference – "The calculation would be simpler if we dropped a perpendicular line from town B to the distance between town A and the river (refer fig.3).The pump should be located at the mid section of this perpendicular line. The perpendicular line could also be parallel to the distance between the two points closest to the river. If we use this perspective we are to 24 km as being the least amount of piping you can use."

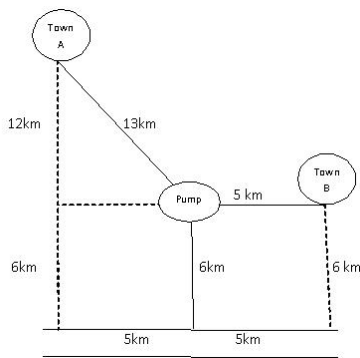


Figure 3. The student diagram from the forum posting

Strategies – “Here is my latest working, please give some comments on it. It is found that the value of h_1+h_2 is decreasing when x is increasing. Let the length of pipe, $L=h_1 + h_2$ (refer fig. 4).

$$L = \sqrt{18^2 + x^2} + \sqrt{6^2 + (10-x)^2}$$

$$\frac{dL}{dx} = \left(\frac{1}{2} (18^2 + x^2)^{-\frac{1}{2}} (2x) \right) + \left(\frac{1}{2} (x^2 - 20x + 136)^{-\frac{1}{2}} (2x - 20) \right)$$

$$\frac{dL}{dx} = \frac{x}{\sqrt{(18^2 + x^2)}} + \frac{x - 10}{\sqrt{(x^2 - 20x + 136)}}$$

Find the critical point, $x=P$, where $\frac{dL}{dx} = 0$.

$$\frac{x}{\sqrt{18^2 + x^2}} = \frac{10 - x}{\sqrt{x^2 - 20x + 136}}$$

$$\frac{x}{10 - x} = \frac{\sqrt{18^2 + x^2}}{\sqrt{x^2 - 20x + 136}}$$

$$\frac{x^2}{x^2 - 20x + 100} = \frac{324 + x^2}{x^2 - 20x + 136}$$

$$x^4 - 20x^3 + 136x^2 = 324x^2 - 6480x + 34200 + x^4 - 20x^3 + 136x^2$$

$$324x^2 - 6480x + 34200 = 0$$

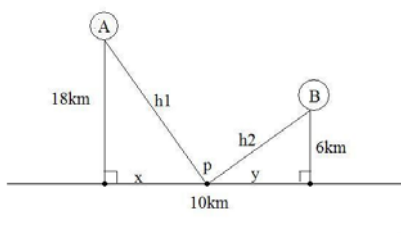


Figure 4. The student diagram from forum posting

A. The Post Discussion Forum Survey

In accordance with the third objective of the study as mentioned in the Introduction of the paper, a survey was done among the students who had participated in the problem solving sessions of the Online Discussion Forum, after the sessions had ended. The specific objectives of the survey were to (1) identify the responses of the students towards the problem posted and (2) the reasons for participating in the sessions and the level of activeness in the sessions.

The first objective was tested through 14 statements given, against which the students were asked to associate their level of agreement. The 14 statements as seen in figure 5 are the following – The problem is concise and precise, the difficulty of the problem is medium, the problem is relevant to students' daily life, The context of the problem is interesting, the problem is inquiry oriented, the problem is creative, the

problem focuses on the procedure rather than the result, the problem is for facilitating students' problem solving, the problem contributes to mathematics understanding, the problem contributes to mathematical communication, the problem contributes to critical thinking, the problem is ill-structured, the problem is reasonable, I look forward to the next problem solving session on Discussion Forum. The possible responses were given using the five point Likert scale starting from Strongly Disagree (=1), Disagree (=2), Neither agree nor disagree (=3), Agree (=4), Strongly Agree (=5). All the statements have a mean score clearly above 3. The highest mean was noticed for two statements namely, the problem contributed to mathematics understanding and critical thinking (mean of 4.1). The eagerness of the students for the next session was also evident from the figure 5. Regarding the second objective of the reasons for participation, the responses were measured using four closed ended statements inviting “Yes” or “No”. The reasons to be chosen from were: (1) I enjoy discussing and/or debating with my class friends. (2) I like working online when compared to paper work/submission. (3) It gives me marks for participation in the forum. (4) It makes me think or explore mathematically and enjoy mathematics.

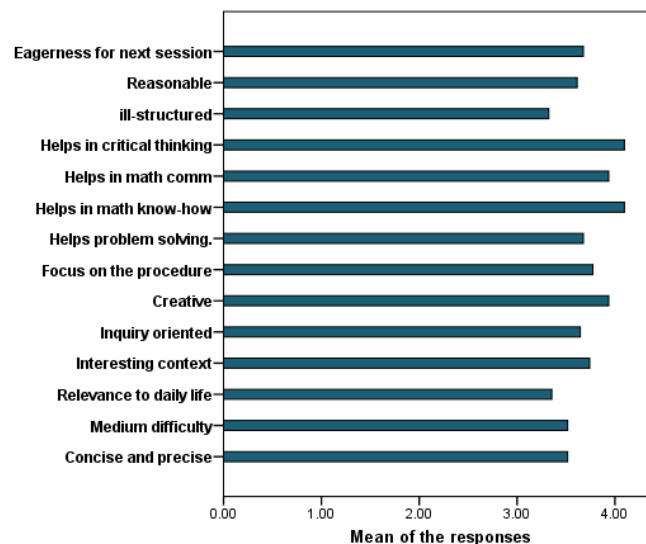


Figure 5. Graph showing the responses to Likert Scale Questions

TABLE III. IT GIVES ME MARKS FOR PARTICIPATION IN FORUM

	Frequency	Percentage
Yes	20	60.6
No	13	39.4
Total	33	100

The most common reasons given as in table 3 and table 4.were “It gives me marks for participation in the forum” voiced by 61% of the students, and “It makes me think or explore mathematically and enjoy mathematics” chosen by 64%.

TABLE IV. IT MAKES ME THINK OR EXPLORE MAHEMATICALLY

	Frequency	Percentage
Yes	21	63.6
No	12	36.4
Total	33	100

Nearly 45% of the students have categorized themselves as “slightly active” in response to the question of the level of activeness on a scale of three-Inactive, Slightly Active, Active in the participation of the Forum sessions as in figure 6. The finding is in line with the few number of quality postings observed in the sessions.

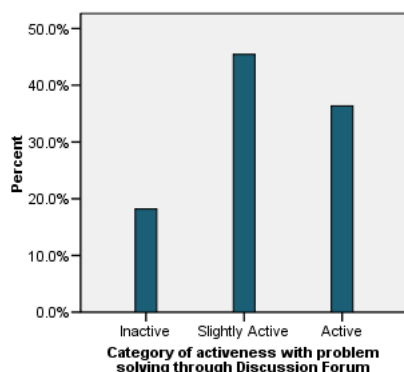


Figure 6. Graph showing the level of activeness in participation

V. CONCLUSIONS AND IMPLICATIONS

The purpose of the study was to test a model of critical thinking that could be used effectively to identify individual profiles of engagement in critical thinking during problem solving sessions over the online discussion forums. The post conference survey revealed students were satisfied with the different dimensions of the problem solving session and were looking forward to the next session. It is a fact that there is limited information revealed in terms of the individual profiles. Future research could be extended to a larger group to generate clear patterns of behavior. The authors believe that with process level scaffolding, which includes human coaching, situation specific guidance and expert participation, the same group of students could be proven to engage in critical thinking in a convincing manner. Clearly it would be complex and time consuming to use the categories of the critical thinking model, especially when large amounts of data need to be analyzed. But the usage of the adapted model here (or any other model) is flexible in that specific categories could be isolated and selected to be used for instruction and evaluation purposes.

There are useful insights available to the instructor from the coding of categories using the model. The differences in the engagement of critical thinking processes observed could be reflective of the processes that the student is comfortable with, or is intellectually ready to use. Such knowledge could be used by the instructor to revise the management of the course content or course delivery in order to encourage a broader range of processes. The knowledge about the differences may be used to provide feedback to the students who seem to be incapable of or uncomfortable with engaging in a particular thinking process. The need for insight and further research into critical thinking processes during problem solving was identified by Perkins and Murphy [20], hence the study is of relevance to researchers and academics in the field. The authors are hopeful of bringing in strategies that emphasize development of life long learning and higher order thinking skills in an effort to acquire and process information within an ever expanding field of knowledge.

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