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Modified Fuzzy Possibilistic C-means

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***Abstract*— Clustering (or cluster analysis) has been used widely in pattern recognition, image processing, and data analysis. It aims to organize a collection of data items into clusters, such that items within a cluster are more similar to each other than they are items in the other clusters. A Modified fuzzy possibilistic clustering algorithm was developed based on the conventional fuzzy possibilistic c-means (FPCM) to obtain better quality clustering results. Numerical simulations show that the clustering algorithm gives more accurate clustering results than the FCM and FPCM methods.**

***Index Terms*—Fuzzy C-Means, Fuzzy Possibilistic C-Means, Modified Fuzzy Possibilistic C-Means, Possibilistic C-Means.**

I. INTRODUCTION

Data analysis is considered as a very important science in the real world. Cluster analysis is a technique for classifying data; it is a method for finding clusters of a data set with most similarity in the same cluster and most dissimilarity between different clusters. The conventional clustering methods put each point of the data set to exactly one cluster. Since 1965, Zadeh proposed fuzzy sets in order to come closer of the physical world [9]–[10]–[17]. Zadeh introduced the idea of partial memberships described by membership functions. A fuzzy version of clustering appeared; it is Fuzzy C-Means with a weighting exponent $m>1$, that uses the probabilistic constraint that the memberships of a data point across classes sum to one [3]–[4]–[6]. The FCM is sensitive to noise. To mitigate such an effect, Krishnapuram and Keller throw away the constraint of memberships in FCM and propose the Possibilistic C-Means (PCM) algorithm [14].Pal deducted that to classify a data point, cluster centroid has to be closest to the data point, and it is the role of membership. Also for estimating the centroids, the typicality is used for alleviating the undesirable effect of outliers. So Pal defines a clustering algorithm called Fuzzy Possibilistic C-Means that combines the characteristics of both fuzzy and possibilistic *c*-means [7]–[12]. The proposed algorithm called Modified Fuzzy Possibilistic C-Means (MFPCM) aims to give good results relating to the previous algorithms by modifying the Objective function used in FPCM.

The remainder of this paper is organized as follows. In section II, preliminary theory algorithms are presented; some drawbacks of them are also mentioned. In section III, the Modified Fuzzy Possibilistic C-Means is proposed. The proposed MFPCM can solve these drawbacks mentioned in section II, and obtain better quality clustering results. In section IV, we present several examples to assess the performance of MFPCM. The comparisons are made between FCM, FPCM and MFPCM. Finally, conclusions are made in Section V.

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II. PRELIMINARY THEORY

The Fuzzy c-means (FCM) can be seen as the fuzzified version of the k-means algorithm. It is a method of clustering which allows one piece of data to belong to two or more clusters. This method (developed by Dunn in 1973 [3] and Modified by Bezdek in 1981 [7]) is frequently used in pattern recognition. The algorithm is an iterative clustering method that produces an optimal c partition by minimizing the weighted within group sum of squared error objective function J_{FCM}:

$$J_{FCM}(V,\mathbf{U},X) = \sum_{i=1}^{c}\sum_{j=1}^{n} u_{ij}^{m} d?\mathbf{x_j},v_i), \quad 1 \prec m \prec +\infty \quad (1)$$

Where X = $\{x_1, x_2,...,x_n\} \subseteq R^p$ is the data set in the p-dimensional vector space, p is the number of data items, c is the number of clusters with $2 \leq c \leq n\text{-}1$. V = $\{v_1, v_2, . . . , v_c\}$ is the c centers or prototypes of the clusters, v_i is the p-dimension center of the cluster i, and $d^2(x_j, v_i)$ is a distance measure between object x_j and cluster centre v_i. U = $\{\mu_{ij}\}$ represents a fuzzy partition matrix with $u_{ij} = u_i(x_j)$ is the degree of membership of x_j in the *ith* cluster; x_j is the *j*th of p-dimensional measured data. The fuzzy partition matrix satisfies:

$$0 \prec \sum_{j=1}^{n} \mu_{ij} \prec n, \forall i \in \{1,..., c\} \quad (2)$$

$$\sum_{i=1}^{c} \mu_{ij} = 1, \forall j \in \{1,..., n\} \quad (3)$$

The parameter m is a weighting exponent on each fuzzy membership and determines the amount of fuzziness of the resulting classification; it is a fixed number greater than one. The objective function J_{FCM} can be minimized under the constraint of U. specifically, taking of J_{FCM} with respect to u_{ij} and v_i and zeroing then respectively, tow necessary but not sufficient conditions for J_{FCM} to be at its local extrema will be as the following:

$$u_{ij} = \left[\sum_{k=1}^{c}\left(\frac{d?\mathbf{x_j},v_i)}{d?\mathbf{x_j},v_k)}\right)^{2/(m-1)}\right]^{-1}, 1 \leq i \leq c, \quad 1 \leq j \leq n. \quad (4)$$

$$v_i = \frac{\sum_{k=1}^{n} \mu_{ik}^{m} x_k}{\sum_{k=1}^{n} \mu_{ik}^{m}}, 1 \le i \le c. \quad (5)$$

Although FCM is a very useful clustering method, its memberships do not always correspond well to the degree of belonging of the data, and may be inaccurate in a noisy environment, because the real data unavoidably involves some noises. To improve this weakness of FCM, and to produce memberships that have a good explanation for the degree of belonging for the data, Krishnapuram and Keller [13] relaxed the constrained condition (3) of the fuzzy *c*-partition to obtain a possibilistic type of membership function and propose PCM for unsupervised clustering. The component generated by the PCM corresponds to a dense region in the data set; each cluster is independent of the other clusters in the PCM strategy. The objective function of the PCM can be formulated as follows:

$$J_{PCM}(V,\mathbf{U},X) = \sum_{i=1}^{c}\sum_{j=1}^{n} u_{ij}^{m} d?\mathbf{x_j}, v_\mathbf{i}) + \sum_{i=1}^{c} \eta_i \sum_{j=1}^{n} (1-u_{ij})^m \quad (6)$$

where

$$\eta_i = \frac{\sum_{j=1}^{n} \mu_{ij}^{m} \left\| x_j - v_i \right\|^2}{\sum_{j=1}^{n} \mu_{ij}^{m}} \quad (7)$$

is the scale parameter at the ith cluster,

$$u_{ij} = \frac{1}{1 + \left(\frac{d^2(\mathbf{x}_j, v_i)}{\eta_i} \right)^{\frac{1}{m-1}}} \quad (8)$$

is the possibilistic typicality value of training sample x_j belonging to the cluster i. m $\in$ $[1,\infty)$ is a weighting factor called the possibilistic parameter. Typical of other cluster approaches, the PCM also depends on initialization. In PCM techniques, the clusters do not have a lot of mobility, since each data point is classified as only one cluster at a time rather than all the clusters simultaneously. Therefore, a suitable initialization is required for the algorithms to converge to nearly global minimum.

Pal defines a clustering algorithm that combines the characteristics of both fuzzy and possibilistic *c*-means [7]: Memberships and typicalities are important for the correct feature of data substructure in clustering problem. Thus, an objective function in the FPCM depending on both memberships and typicalities can be shown as:

$$J_{FPCM}(U,T,V) = \sum_{i=1}^{c}\sum_{j=1}^{n} (u_{ij}^{m} + t^{\eta}) d?\mathbf{x_j}, v_\mathbf{i}) \quad (9)$$

with the following constraints :

$$\sum_{i=1}^{c} \mu_{ij} = 1, \forall j \in \{1,...,n\} \quad (3)$$

$$\sum_{j=1}^{n} t_{ij} = 1, \forall i \in \{1,...,c\} \quad (10)$$

A solution of the objective function can be obtained via an iterative process where the degrees of membership, typicality and the cluster centers are update via:

$$u_{ij} = \left[\sum_{k=1}^{c} \left(\frac{d?\mathbf{x_j}, v_\mathbf{i})}{d?\mathbf{x_j}, v_\mathbf{k})} \right)^{2/(m-1)} \right]^{-1}, 1 \le i \le c, \quad 1 \le j \le n \quad (4)$$

$$t_{ij} = \left[\sum_{k=1}^{n} \left(\frac{d?\mathbf{x_j}, v_\mathbf{i})}{d?\mathbf{x_j}, v_\mathbf{k})} \right)^{2/(\eta-1)} \right]^{-1}, 1 \le i \le c, \quad 1 \le j \le n \quad (11)$$

$$v_\mathbf{i} = \frac{\sum_{k=1}^{n} (u_{ik}^{m} + t_{ik}^{\eta}) \mathbf{x_k}}{\sum_{k=1}^{n} (u_{ik}^{m} + t_{ik}^{\eta})}, 1 \le i \le c \quad (12)$$

PFCM produces memberships and possibilities simultaneously, along with the usual point prototypes or cluster centers for each cluster. PFCM is a hybridization of possibilistic c-means (PCM) and fuzzy c-means (FCM) that often avoids various problems of PCM, FCM and FPCM. PFCM solves the noise sensitivity defect of FCM, overcomes the coincident clusters problem of PCM. But the noise data have an influence on the estimation of centroids.

III. PROPOSED MODIFIED FUZZY POSSIBILISTIC CLUSTERING ALGORITHM

The choice of an appropriate objective function is the key to the success of the cluster analysis and to obtain better quality clustering results; so the clustering optimization is based on objective function [16]. To meet a suitable objective function, we started from the following set of requirements: The distance between clusters and the data points assigned to them should be minimized and the distance between clusters should to be maximized [5]. The attraction between data and clusters is modeled by term (9); it is the formula of the objective function. Also Wen-Liang Hung proposed a new algorithm called Modified Suppressed Fuzzy c-means (MS-FCM), which significantly ameliorates the performance of FCM due to a prototype-driven learning of parameter α [15]. The learning process of α is based on an exponential separation strength between clusters and is updated at each iteration. The formula of this parameter is:

$$\alpha = \exp\left(-\min_{i \ne k} \frac{\left\| v_i - v_k \right\|^2}{\beta} \right) \quad (13)$$

where β is a normalized term so that we choose β as a sample variance. That is, we define β:

$$\beta = \frac{\sum_{j=1}^{n} \left\| x_j - \overline{x} \right\|^2}{n} \quad \text{where } \overline{x} = \frac{\sum_{j=n}^{n} x_j}{n}.$$

But the remark which must be mentioned here is the common value used for this parameter by all the data at each iteration, which may induce in error. We propose a new parameter which suppresses this common value of α and replaces it by a new parameter like a weight to each vector. Or every point of the data set has a weight in relation to every cluster. Therefore this weight permits to have a better classification especially in the case of noise data. So the weight is calculated as follows:

$$w_{ji} = \exp\left(- \frac{\left\| x_j - v_i \right\|^2}{\left(\sum_{j=1}^{n} \left\| x_j - \bar{v} \right\|^2 \right) * c / n} \right) \quad (14)$$

where w_{ji} is weight of the point j in relation to the class i. this weight is used to modify the fuzzy and typical partition.

All update methods that were discussed in section II are iterative in nature, because it is not possible to optimize any of the objective functions reviewed directly. Or to classify a data point, cluster centroid has to be closest to the data point, it is membership; and for estimating the centroids, the typicality is used for alleviating the undesirable effect of outliers. The objective function is composed of two expressions: the first is the fuzzy function and uses a fuzziness weighting exponent, the second is possibililstic function and uses a typical weighting exponent; but the two coefficients in the objective function are only used as exhibitor of membership and typicality.

A new relation, lightly different, enabling a more rapid decrease in the function and increase in the membership and the typicality when they tend toward 1 and decrease this degree when they tend toward 0. This relation is to add Weighting exponent as exhibitor of distance in the two under objective functions. The objective function of the MFPCM can be formulated as follows:

$$J_{MFPCM} = \sum_{i=1}^{c} \sum_{j=1}^{n} (\mu_{ij}^{m} w_{ji}{}^{m} d^{2m}(x_j, v) + t_{ij}^{\eta} w_{ji}{}^{\eta} d^{2\eta}(x_j, v_i)) \quad (15)$$

U = {μ_{ij}} represents a fuzzy partition matrix, is defined as:

$$u_{ij} = \left[\sum_{k=1}^{c} \left(\frac{d?\mathbf{x_j}, v_\mathbf{i})}{d?\mathbf{x_j}, v_\mathbf{k})} \right)^{2m/(m-1)} \right]^{-1} \quad (16)$$

T = {t_{ij}} represents a typical partition matrix, is defined as:

$$t_{ij} = \left[\sum_{k=1}^{n} \left(\frac{d?\mathbf{x_j}, v_\mathbf{i})}{d?\mathbf{x_j}, v_\mathbf{k})} \right)^{2\eta/(\eta-1)} \right]^{-1} \quad (17)$$

V = {v_i} represents c centers of the clusters, is defined as:

$$v_i = \frac{\sum_{j=1}^{n} \left(\mu_{ij}^{m} w_{ji}^{m} + t_{ij}^{\eta} w_{ji}^{\eta} \right) * x_j}{\sum_{j=1}^{n} \left(\mu_{ij}^{m} w_{ji}^{m} + t_{ij}^{\eta} w_{ji}^{\eta} \right)} \quad (18)$$

IV.

VI. Experimental results

In this section, we perform some experiments to compare the performances of these algorithms with some numerical datasets. All algorithms are implemented under the same initial values and stopping conditions. The experiments are all performed on a GENX computer with 2.6 GHz Core (TM) 2 Duo processors using MATLAB (Mathworks, Inc., Natick, MA) version 7.0.4.

A. Example 1 (Data sets in [6])

In the first experiment, we use a two-cluster data set as presented in [6] shown in Fig. 1. To demonstrate the quality of classification of our approach in relation to the other algorithms (FCM, FPCM) in a case data set without outlier. The clustering results of these algorithms are shown in Fig. 1(a)~1(c) respectively, where two clusters from the clustering algorithms are with symbols "+" and "o" ; also the figure shows that our approach is better than others.

Table I. shows that the degrees of membership and typicality are better in our approach. The degrees tend toward 1 when the point is near the center class.

In the second experiment, we use a two-cluster data set with outlier as presented in [6] shown in Fig. 2. The clustering results of these algorithms are shown in Fig. 2(a)~2(c) respectively, shows that our approach is better than others. The last point (0, 10) is an outlier but it doesn't have an influence on centers although it has the same membership degrees. Table II. shows that the degrees of membership and typicality are better in our approach. The point (0, 10) has a degree of typicality nearly equal to 0.

The FCM, FPCM and MFPCM are compared in the two previous experiences, using the following criteria for the cluster centers locations: the mean square error (MSE) of the centers ($MSE = \sqrt{\left\| v_c - v_t \right\|^2}$, where v_c is the computed center and v_t is the true center) [16] and the number of iterations (NI). The cluster centers found by MFPCM are closer the true centers, than the centers found by FCM and FPCM. The number of iterations tends toward the same value.

In the third experiment, we use a three-cluster data set as presented in [6] shown in Fig. 3. The clustering results of these algorithms are shown in Fig. 3(a)~3(c) respectively, shows that our approach is better than others.

After a classifier or a cluster model has been constructed, one would like to know how "good" it is. Quality criteria are fairly easy to find for classifiers, Or according to Borgelt [2], the quality of a clustering result is calculated while using index of performances or validity index that are used to determine the number of classes. So we can say that MFPCM is better than FCM and FPCM while using criteria index of performances.

B. Example 2 (Data sets in [1])

In the experiment, we tested these methods on well-known data sets from the UCI machine learning repository [1] shown in Table III. The clustering results of these algorithms show that our approach is better than others by using the

Performance Index named Fukuyama-Sugeno index, who supposes that the algorithm which has the minimal value of index is the best in relation to others [12].

V. CONCLUSIONS

In this paper we have presented a Modified fuzzy possibilistic clustering algorithm, which is developed to obtain better quality of clustering results. The objective function is based by adding new weight of data points in relation to every cluster and modifying the exponent of the distance between a point and a class. A comparison of the clustering algorithm and the FCM, FPCM algorithms shows that clustering algorithms will increase the cluster compactness and the separation between clusters. Finally, a numerical example shows that the clustering algorithm gives more accurate clustering results than the FCM and FPCM algorithms for typical problem.

REFERENCES

[1] C. L Blake and C .J Merz, UCI Repository of Machine Learning Databases. University of California, Irvine, CA, USA 1998
[2] C. Borgelt, “Prototype-based classification and clustering”, habilitation June 2005.
[3] J. C. Dunn, “A Fuzzy Relative of the ISODATA Process and its Use in Detecting Compact Well-Separated Clusters”, *J. Cybernetics*, vol. 3, No. 3, pp. 32-57, 1973..
[4] E. R. Ruspini, “A New Approach to Clustering”, *Inform. Control*, vol. 15, no. 1, pp. 22-32, July 1969.
[5] H Timm, C Borgelt, C. Doring, R. Kruse, “An extension to possibilistic fuzzy cluster analysis”, *Fuzzy Sets and Systems* 147 (2004) 3–16.
[6] J. C. Bezdek, J. Keller, R. Krishnapuram and N. R. Pal, “Fuzzy Models and Algorithms for Pattern Recognition and Image Processing”, *Kluwer Academic Publishers*, TA 1650.F89, 1999.
[7] J. C. Bezdek, “Pattern Recognition with Fuzzy Objective Function Algorithms”, New York, Plenum, 1981.
[8] M. Barni, V. Cappellini, A. Mecocci, “Comments on A possibilistic approach to clustering", *IEEE Trans on Fuzzy Systems*, vol. 4: 393-396, 1996.
[9] M R. Berthold and D. J. Hand, “Intelligent Data Analysis”. *Springer-Verlag*, Berlin, Germany 1999.
[10] M. W. Berry, “Survey of Text Mining”, *Springer-Verlag*, New York, NY, USA 2003.
[11] N. R. Pal, K. Pal and J. C. Bezdek, “A mixed c-means clustering model”, *Proceedings of the Sixth IEEE International Conference on Fuzzy Systems*, Vol. 1, pp. 11-21, Jul. 1997.
[12] K. Lung, “A cluster validity index for fuzzy clustering”, *Pattern Recognition Letters* 25(2005) 1275-1291.
[13] R. Krishnapuram and J. Keller, “A possibilistic Approch to Clustering”, *IEEE Trans. on Fuzzy Systems*,vol. 1, n. 2, May 1993
[14] U.M. Fayyad, G. Piatetsky-Shapiro, P. Smyth, and R. Uthurusamy, “Advances in Knowledge Discovery and Data Mining”, *AAAI Press and MIT Press, Menlo Park and Cambridge,* MA, USA 1996.
[15] W.L. Hung, M Yang, D. Chen,” Parameter selection for suppressed fuzzy c-means with an application to MRI segmentation”, *Pattern Recognition Letters* 2005.
[16] Y. Zhonghang, T. Yangang, S. Funchun,S. Zengqi , “Fuzzy Clustering with Novel Serable Criterion”, *Tsinghua Science and Technology* ISSN 1007-0214 09/21 pp 50-53 Volume 11, Number 1, February 2006.
[17] L. Zadeh, “Fuzzy Sets”, *Inform. Control*, 8, pp. 338-353, 1965.

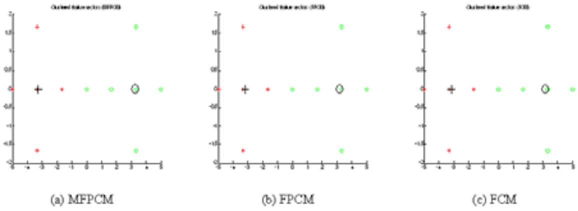

Fig. 1. MFPCM, FPCM, FCM clustering results for the two-cluster data set without an outlier.

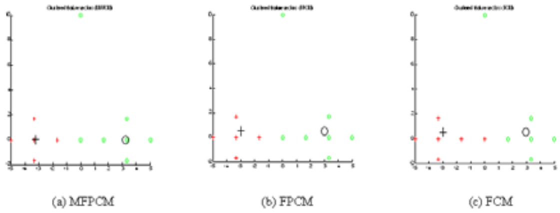

Fig. 2. MFPCM, FPCM, FCM clustering results for the two-cluster data set with an outlier.

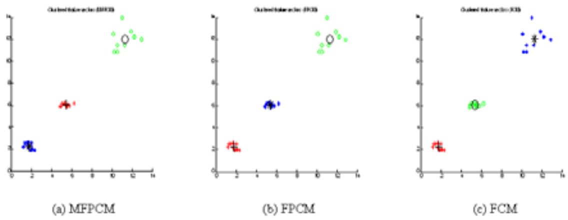

Fig. 3. MFPCM, FPCM, FCM clustering results for the three-cluster data set.

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TABLE I
CENTERS, PERFORMANCE INDEX, MSE, NI, MEMBERSHIPS AND TYPICALITY GENERATED BY FCM, FPCM, MFPCM FOR THE EXPERIMENTS IN FIG. 1.

Vector		FCM		FPCM				MFPCM			
		Centers									
		-3,1674 0,0000		-3,1980 0,0000				-3,2678 0,0000			
		3,1673 0,0000		3,1980 0,0000				3,2678 0,0000			
		Performance Index									
		-69,66		-130,93				-147,88			
		MSE									
		0,24418		0,20083				0,122			
		NI									
		11		14				14			
		Memberships and typicality									
Feature1 X	Feature2 Y	μ_{1j}	μ_{2j}	μ_{1j}	μ_{2j}	t1j	t2j	μ_{1j}	μ_{2j}	t1j	t2j
-5,00	0,00	0,952060	0,047935	0,953910	0,046089	0,006007300	0,000290310	0,998080	0,001923	0,000003017	0,000000006
-3,34	1,67	0,941220	0,058781	0,941890	0,058107	0,006944300	0,000428480	0,996390	0,003605	0,000003479	0,000000013
-3,34	0,00	0,999300	0,000703	0,999530	0,000471	0,967420000	0,000456440	1,000000	0,000000	0,999990000	0,000000014
-3,34	-1,67	0,941220	0,058781	0,941890	0,058107	0,006944300	0,000428480	0,996390	0,003605	0,000003479	0,000000013
-1,67	0,00	0,912560	0,087441	0,910310	0,089691	0,008354900	0,000823330	0,989150	0,010846	0,000004167	0,000000046
0,00	0,00	0,500000	0,500000	0,500000	0,500000	0,001907400	0,001907700	0,500000	0,500000	0,000000238	0,000000240
1,67	0,00	0,087431	0,912570	0,089685	0,910320	0,000823170	0,008356700	0,010842	0,989160	0,000000046	0,000004195
3,34	1,67	0,058780	0,941220	0,058106	0,941890	0,000428400	0,006945600	0,003605	0,996390	0,000000013	0,000003501
3,34	0,00	0,000704	0,999300	0,000472	0,999530	0,000456350	0,967410000	0,000000	1,000000	0,000000014	0,999990000
3,34	-1,67	0,058780	0,941220	0,058106	0,941890	0,000428400	0,006945600	0,003605	0,996390	0,000000013	0,000003501
5,00	0,00	0,047938	0,952060	0,046091	0,953910	0,000290250	0,006008400	0,001924	0,998080	0,000000006	0,000003035

TABLE II
CENTERS, PERFORMANCE INDEX, MSE, NI, MEMBERSHIPS AND TYPICALITY GENERATED BY FCM, FPCM, MFPCM FOR THE EXPERIMENTS IN FIG. 2.

Vector		FCM		FPCM				MFPCM			
		Centers									
		-2,98540 0,54351		-3,01160 0,50643				-3,2972 0,0017			
		2,98540 0,54354		3,01160 0,50646				3,2972 0,0017			
		Performance Index									
		-11,064		-48,866				-107,45			
		MSE									
		0,76866		0,71622				0.1308			
		NI									
		11		13				13			
		Memberships and typicality									
Feature1 X	Feature2 Y	μ_{1j}	μ_{2j}	μ_{1j}	μ_{2j}	t1j	t2j	μ_{1j}	μ_{2j}	t1j	t2j
-5,00	0,00	0,93636	0,06364	0,93868	0,06132	0,0515460	0,0033676	0,99797	0,00203	0,000007739	0,000000016
-3,34	1,67	0,96731	0,03269	0,96613	0,03387	0,1484700	0,0052047	0,99636	0,00364	0,000009364	0,000000034
-3,34	0,00	0,98966	0,01034	0,99111	0,00889	0,5956900	0,0053453	1,00000	0,00000	0,999960000	0,000000039
-3,34	-1,67	0,89937	0,10063	0,90296	0,09704	0,0447940	0,0048141	0,99633	0,00367	0,000009288	0,000000034
-1,67	0,00	0,91558	0,08442	0,91513	0,08487	0,1055300	0,0097870	0,98952	0,01048	0,000011782	0,000000126
0,00	0,00	0,50000	0,50000	0,50000	0,50000	0,0232690	0,0232690	0,49996	0,50004	0,000000656	0,000000660
1,67	0,00	0,08443	0,91557	0,08487	0,91513	0,0097867	0,1055300	0,01048	0,98952	0,000000125	0,000011857
3,34	1,67	0,03268	0,96732	0,03387	0,96613	0,0052045	0,1484700	0,00364	0,99636	0,000000034	0,000009420
3,34	0,00	0,01034	0,98966	0,00890	0,99111	0,0053452	0,5956800	0,00000	1,00000	0,000000039	0,999960000
3,34	-1,67	0,10064	0,89936	0,09704	0,90296	0,0048139	0,0447940	0,00367	0,99633	0,000000034	0,000009344
5,00	0,00	0,06364	0,93636	0,06133	0,93867	0,0033675	0,0515470	0,00204	0,99797	0,000000016	0,000007783
0,00	10,00	0,50000	0,50000	0,50000	0,50000	0,0021877	0,0021877	0,50000	0,50000	0,000000006	0,000000006

TABLE III PERFORMANCE INDEX GENERATED BY FCM, FPCM, MFPCM FOR DIFFERENT DATASETS.

Data set	number of data	number of clusters	number of data items	Performance Index FCM	Performance Index FPCM	Performance Index MFPCM
Iris	150	3	4	-44527	-46847	-54036
breast-cancer-wisconsin -cont	683	4	9	-6299	-6402	-16623
Wine	178	3	13	-10751000	-11334000	-21260000
Yeast	528	11	10	117,62	119,28	-1071,6
Auto MPG	398	8	3	-197210000	-202020000	-224870000
Balance Scale	625	4	3	1698,20	1711,00	941,51
Buta	345	7	2	81790	77319	20370
glass	214	9	6	-610970	-668590	-789220
hayes	132	5	3	-132980	-141360	-159520
Monk’s Problem	432	7	2	1006,60	1013,00	821,87
Lettre Image	16000	16	26	57556	57457	35644