Robust Fixed-Structure Cascade Controller for a Quadratic Boost Converter

Pitsanu Srithongchai and Somyot Kaitwanidvilai

***Abstract*— In this paper, a new technique for designing a robust cascade controller for a quadratic boost converter is proposed. A performance index in H infinity loop shaping control, stability margin, is adopted as the objective function in our optimization control problem; GA is used to solve this problem to evaluate the optimal controller. Conditions of cascade control are adopted as constraints in our optimization problem. In addition, pre-compensator weight which is normally difficult to be selected is simultaneously determined with the controller. Comparative study with the conventional H infinity loop shaping is presented. Finally, simulation results verify the effectiveness of the proposed technique.**

***Index Terms*— H-infinity loop shaping control, Genetic algorithm, Quadratic boost converter, Fixed-structure robust control.**

I. Introduction

Many advantages of quadratic DC-DC converter such as reduction of the resonance mode in DC-DC converter, simple structure of circuit, etc. have been presented in previous research works [1-3]. The design of robust controller for this converter is needed to be further developed for enhancing both performance and robustness of the controlled system. Although standard technique such as H infinity optimal control provides a feasible way to design a robust controller; however, the resulting controller in this approach is normally complicated with high order, making it difficult to implement in practice. In addition, weight selection in this technique is normally carried out by trial and error method which is not an easy task. To overcome this problem, we propose an algorithm, a robust cascade controller designed by GA, to design a robust controller for a quadratic DC-DC buck converter. In the proposed technique, inverse of infinity norm from disturbances to states is formulated as the fitness function in GA. The advantages of simple structure, controller structure selectable and robustness are achieved by the proposed technique. In addition, performance weight, which is normally difficult to be obtained, is simultaneously determined by GA. This reduces the difficulty of weight selection in the conventional robust loop shaping design.

The remainder of this paper is shown as follows. Section 2 illustrates the converter model and conventional H infinity loop shaping technique. Section 3 describes the proposed technique. GA is also briefly described in this section. Simulations and results are shown in Section 4. Finally, Section 5 concludes the paper.

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The authors are with the Faculty of Engineering, King Mongkut's Institute of Technology Ladkrabang, Bangkok 10520,Thailand. They are also with the Center of Excellence for Innovative Energy Systems, King Mongkut's Institute of Technology Ladkrabang, Bangkok, 10520. e-mail: kksomyot@kmitl.ac.th, drsomyotk@gmail.com

II. Converter Model and Conventional Loop Shaping Technique

A. Converter Model

Fig. 1 shows the dynamic model of a quadratic converter.

The dynamic model of current and voltage loops in the cascade control scheme can be expressed as [1-2]:

$$G_{IL} = \frac{\tilde{i}_s(s)}{\tilde{d}(s)} = K_1 \frac{a_3 s^3 + a_2 s^2 + a_1 s + a_0}{s^4 + b_3 s^3 + b_2 s^2 + b_1 s + b_0}, \qquad (1)$$

$$G_V \; \frac{\tilde{v}(s)}{\tilde{i}(s)} = \frac{m_3 s^3 + m_2 s^2 + m_1 s + m_0}{a_3 s^3 + a_2 s^2 + a_1 s + a_0} \qquad (2)$$

Where $K_1 = \frac{E}{(1-U)^3 R}$, $a_3 = \frac{R(1-D)^2}{L_1} + \frac{R(1-D)}{L_2}$,

$$a_2 = \frac{(1-D)^2}{L_1 C_2} + \frac{2(1-D)}{L_2 C_2} + \frac{1}{L_1 C_1} - \frac{1}{L_2 C_1 (1-D)}$$

$$a_1 = \frac{2(2-D)(1-D)^2 R}{L_1 C_1 L_2} + \frac{R(1-D)^4}{L_2 C_2 L_1} + \frac{1}{L_1 C_1 C_2 R} - \frac{1}{L_2 C_1 C_2 R(1-D)},$$

$$a_0 = \frac{4(1-D)^2 + 3(1-D)^3}{L_1 L_2 C_1 C_2}, \quad b_3 = \frac{1}{RC_2}$$

$$b_2 = \frac{1}{L_2 C_1} + \frac{(1-D)^2}{L_2 C_2} + \frac{(1-D)^2}{L_1 C_1}, \quad b_1 = \frac{1}{L_2 C_2 C_1 R} + \frac{(1-D)^2}{L_1 C_2 C_1 R},$$

$$b_0 = \frac{(1-D)^4}{L_2 C_2 C_1 L_1}, \quad m_3 = -\frac{E}{RC_2 (1-D)^3}, \quad m_2 = \frac{E}{L_2 C_2 (1-D)},$$

$$m_1 = \frac{-E(2L_1 + L_2(1-D)^2)}{RL_1 C_1 L_2 C_2 (1-D)^3}, \quad m_0 = \frac{2E(1-D)}{L_1 C_1 L_2 C_2}$$

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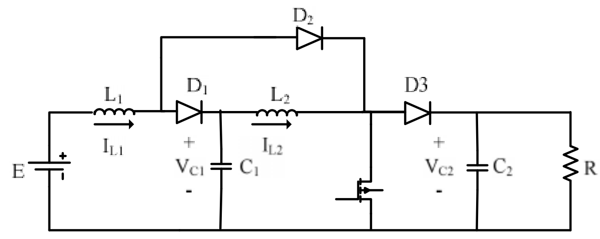

Fig. 1 a quadratic boost converter

D, d are nominal duty cycle and duty cycle, respectively; L1, L2, C1, C2, R are the component values of the converter shown in Fig. 1; E is the nominal input voltage; G_{IL} and G_v are the dynamic models of current and voltage loops; $i_s = i_{L1}+i_{L2}$.

B. Conventional H∞ Loop Shaping Control

H infinity loop shaping control was first introduced by McFarlane [4]. In this design, desired open loop shape in frequency domain is specified by shaping the open loop of the system, G, with the weighting functions, pre-compensator (W_1) and post-compensator (W_2). The shaped plant can be written as:

$$G_s = W_1 G W_2 \tag{3}$$

$$G_\Delta = (N_s + \Delta_{Ns})(M_s + \Delta_{Ms})^{-1} \tag{4}$$

where Δ_{Ns} and Δ_{Ms} are the uncertainty transfer functions in the nominator and denominator factors, respectively. G_Δ is the shaped plant with uncertainty. $\|\Delta_{Ns}, \Delta_{Ms}\|_\infty \le \varepsilon$, where ε is the stability margin. The design steps of H infinity loop shaping can be briefly described as follows:

Step 1 Specify the pre- and post-compensator weights for achieving the desired open loop shape.
Step 2 Find the optimal stability margin (ε_{opt}) by solving the following equation.

$$\gamma_{opt} = \varepsilon_{opt}^{-1} = \inf_{stab\,K} \left\| \begin{bmatrix} I \\ K \end{bmatrix} (I - G_s K)^{-1} M_s^{-1} \right\|_\infty \tag{5}$$

If (ε_{opt}) is too low, then go to Step 1 to select the new weights.
Step 3 Select the stability margin ($\varepsilon<\varepsilon_{opt}$) and then synthesize the controller, K_∞, by solving the following inequality.

$$\begin{aligned} \|T_{zw}\|_\infty &= \left\| \begin{bmatrix} I \\ K_\infty \end{bmatrix} (I - G_s K_\infty)^{-1} M_s^{-1} \right\|_\infty \\ &= \left\| \begin{bmatrix} I \\ K_\infty \end{bmatrix} (I - G_s K_\infty)^{-1} [I \quad G_s] \right\|_\infty \le \varepsilon^{-1} \end{aligned} \tag{6}$$

Step 4 Final controller (K) is

$$K = W_1 K_\infty W_2 \tag{7}$$

C. Fixed-Structure Robust Loop Shaping Control

Although robust loop shaping technique is an efficient technique to design a robust controller; however, the final controller designed by this approach is usually high order and complicated. To overcome this problem, we propose a GA based fixed-structure robust loop shaping control to design a fixed-structure robust controller. The proposed technique can be described as follows:

The structure of the proposed controller is shown in Fig. 2.

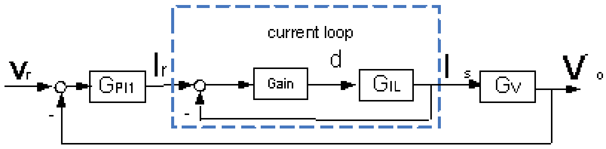

Fig. 2 cascade control scheme for a quadratic boost converter.

In this paper, we selected PI controller as the outer loop controller and P controller as the inner loop controller. Thus, if $G_{innerloop}$ is the plant of close loop system of the current loop, thus $G_s = W_1 G_{inner\,loop} G_v$ and the stability margin in (6) can be written as:

$$\left\| \begin{bmatrix} I \\ W_1^{-1} G_{PI1} \end{bmatrix} (I - G_s W_1^{-1} G_{PI1})^{-1} [I \quad G_s] \right\|_\infty^{-1} \tag{8}$$

Consequently, our design objective is to find the optimal gains in the current loop, the PI controller (G_{PI1}) and the weight W_1 such that the stability margin in (8) is maximized. To achieve the cascade controller, one constraint which is "the bandwidth of inner loop must be much higher than that of the outer loop" is added to the optimization. The following steps are the proposed design.
Step 1 Specify the structures of weight and controller. Select the post-compensator weight as I.
Step 2 The structure of the voltage loop controller is $G_{PI1}(p)$, thus, based on (7),

$$K_\infty = W(x)^{-1} G_{PI1}(p) \tag{9}$$

By substituting (9) into (6), the infinity norm of transfer function from disturbances to states, subjected to be minimized, can be written as:

$$J_{cost} = \frac{1}{\varepsilon} = \gamma = \|T_{zw}\|_\infty = \left\| \begin{bmatrix} I \\ W(x)^{-1} G_{PI1} \end{bmatrix} (I - G_s W(x)^{-1} G_{PI1}(p))^{-1} [I \quad G_s] \right\|_\infty \tag{10}$$

Where $G_s = W_1 G_{inner\,loop} G_v$.

Subject to BW(inner loop) > 5 BW(outer loop). BW is denoted as bandwidth.
Step 3 Use GA to find the optimal parameter, p^*, *Gain* and x^*. The followings briefly describe the GA.

In the proposed technique, GA is adopted in control synthesis. This algorithm applies the concept of chromosomes and the genetic operations of crossover, mutation and reproduction. At each step, called

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generation, fitness value of each chromosome in population is evaluated by using fitness function. Chromosome, which has the maximum fitness value, is kept as a solution in the current generation. The new population of the next generation is obtained by performing the genetic operators such as crossover, mutation, and reproduction. In this paper, a roulette wheel method is used for chromosome selection. In this method, chromosome with high fitness value has high chance to be selected. Operation type selection, mutation, reproduction, or crossover depends on the pre-specified operation's probability. Normally, chromosome in genetic population is coded as binary number. However, for the real number problem, decoding binary number to floating number is applied.

IV. SIMULATION RESULTS

In our study, converter parameters are given as follows: C_1=22 μF, C_2 = 100 μF, L_1 = 90 μH, L_2 = 382 μH, load R = 100 ohms. GA is adopted to find the solution of above optimization problem. Weight parameters, gain in PI controller and gain in P controller, are set as the chromosome in GA. Constraints of time domain specifications, i.e. settling time < 0.05 sec., overshoot < 0.5% are adopted in this optimization problem. When running GA for 49 generations, an optimal solution is obtained.

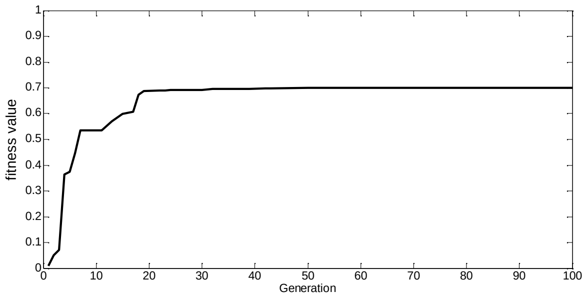

Fig. 3 Fitness versus generations in GA optimization.

Resulting weight and controller are shown in Table 1. Conventional H infinity control with the same weight and inner loop current controller gain is adopted to design the controller for comparison purpose. The full order H infinity controller is designed as (11).

$$Hinf = \frac{(0.5665\, s + 18.05)}{s}\,\frac{(6037 s^7 + 6.499*10^{9} s^6 + 1.223*10^{13} s^5 + 3.214*10^{18} s^4 + 3.013*10^{21} s^3 + 3.968*10^{26} s^2 + 8.896*10^{28} s + 3.166*10^{30})}{(s^8 + 1.084*1^{06} s^7 + 1.068*10^{10} s^6 + 5.491*10^{14} s^5 + 4.799*10^{18} s^4 + 6.984*10^{22} s^3 + 5.482*10^{26} s^2 + 1.138*10^{29} s + 3.113*10^{30})} \quad (11)$$

Table 1 Resulting controllers and their stability margins.

	Proposed controller	Conventional H_∞ loop shaping
Weight	$\frac{(0.5665\, s + 18.05)}{s}$	$\frac{(0.5665\, s + 18.05)}{s}$
Controller (outer loop)	$\frac{0.4418\, s + 53.97}{s}$	Hinf in (11) (8th order controller)
Controller (inner loop)	2.74	2.74
Stability margin	0.70	0.745

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As seen in this table, the stability margin of the proposed controller is almost the same as the conventional full order H infinity controller; however, the order of our controller is much lower than that of the conventional controller. This makes it easy to be implemented in practice. Since analog controller is normally used to design the converter controller, our technique is more feasible than the conventional robust control.

Time domain responses of both controllers are shown in Fig. 3. As seen in this figure, our proposed technique gains better response in terms of fast rise time and fast settling time.

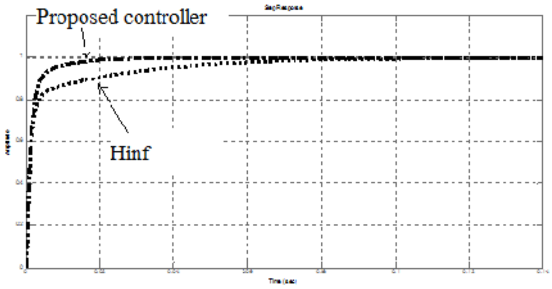

Fig.4 Step response of the proposed controller and Hinf controller.

V. Conclusions

In this paper, the design of high-performance and robust controller for a quadratic DC-DC converter using Genetic Algorithm has been proposed. Results show that the order of the proposed controller is much lower than that of the conventional robust loop shaping controller. In addition, performance weight which is not easy to be specified, can be simultaneously evaluated with the controller by the proposed technique. The tracking performance specifications can be achieved by the proposed controller.

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