# Experimental implementation of Pole Placement Techniques for Active Vibration Control of Smart Structures

Rajiv Kumar[1]

***Abstracts--* Fixed controllers can become even unstable, with large changes in system parameters. This problem can be avoided using robust control and adaptive control design techniques. To obtain robust performance, it is desirable that the closed loop poles of the perturbed structure remain at prespecified locations for a range of system parameters. In the present study, the controllers based on adaptive and robust pole placement method are implemented on smart structures. It was observed that, adaptive pole placement controllers are noise tolerant but require high actuator voltages to maintain stability. However, robust pole placement controllers require comparatively small amplitude of control voltage to maintain stability, but are noise sensitive.**

***Index Terms* – Pole placement, active control, adaptive control, robust control**

## I. INTRODUCTION

Un-modeled dynamics, component degradation, changing configuration and changing payloads can destabilize a fixed gain controller based on original system (i.e. nominal) model. This led to adaptive and robust control techniques. An intensive effort is being done to implement the adaptive control techniques to adaptive vibration control of smart structures.

1 Manuscript was received on 23 Dec 2009. The author is with National Institute of Technology Jalandhar, INDIA-144011 (Phone: 919463846067, E-mail: rajivsharma1972@yahoo.com

In this direction, Zeng et al [1] applied output feedback variable structure adaptive control to a flexible spacecraft. By using the neural network based adaptive control strategy; Yaun et al [2] controlled the composite beam vibrations subjected to sudden de-lamination. . Shaw [3] used self tuning regulators combined with Minimum variance controller to control a spring mass system.

Using classical positive position feedback control strategy, Rew et al [4] suppressed multi-modal vibrations of flexible structures.. By using the adaptive predictive control strategy, Bai et al [5] suppressed rotor vibrations. More recently, Lim et al [6] used adaptive bang-bang control for the vibration control of civil structures while seismic vibrations occur. Lee and Eillot [7] controlled a flexible smart beam subjected to step disturbance using adaptive feed forward control. Crassidis et al [8] controlled the vibrations of a beam using H infinity control theory. Other researchers like Liu et al. [9] also applied H infinity robust control theory for control of plate vibration by covering it with a controllable constrained layer damping layer. In 2004, Xie et al. [10] also applied H infinity robust control theory for vibration control of a thin plate covered with a controllable constrained layer damping layer.

Robust performance means that the performance parameters like percentage overshoot and settling times remains almost same, even though the system parameters are perturbed from nominal values. *This can be achieved by fixing the closed loop (CL) poles at certain fixed position.* With this, the settling

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time of the disturbed system remains near a particular value, even if the system parameters are subjected to change. Two inverted L – structures, with different geometries are taken for study. The tip load keeps on varying to change the system parameters. The CL poles are fixed in the complex plane so that desired robust performance is obtained.

## II. MATHEMATICAL MODELING OF SMART STRUCTURES

### *A. FEM Modeling*

The schematic diagram of the proposed structure (i.e. inverted L) is shown in the fig 1. The structure is mounted with two piezoelectric patches bonded on its surface acting as sensors and actuators. One of which are used as actuator and the other one as a sensor. The geometrical and mechanical properties of the structure are listed in table I. The Lagrange's equations of motion for linear systems are given as below

$$\sum_{i=1}^{n} \left[ m_{ji}\,\ddot{\Delta}_i(t) + c_{ji}\,\dot{\Delta}_i + k_{ji}\,\Delta_i(t) \right] = Q_j(t) \qquad i,j=1,2,.....,n \tag{1}$$

where $\Delta(t)$, $\dot{\Delta}(t)$ and $\ddot{\Delta}(t)$ are the physical displacement, velocity and acceleration respectively. Fig.1 shows the geometry and boundary conditions of the structural system. The eigenvalue problem can be solved to give the natural frequencies and mode shapes for various tip loads ranging from 0g – 20g. These modal parameters can be used to construct the system matrices [9, 10].

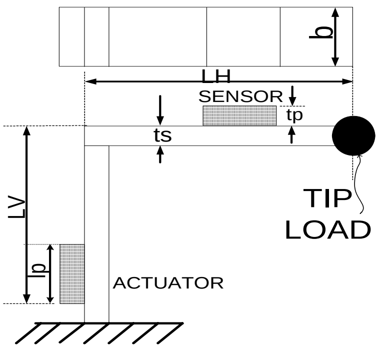


Fig.1: Schematic diagram of the inverted L structure

Table I: Geometrical and mechanical properties of the structure

| Property | Material: Steel | Material: PZT |
|---|---|---|
| Length of Horizontal limb(mm) | $L_H$= 100 | ------- |
| Length of Vertical Limb(mm) | $L_V$=100 | -------- |
| Thickness(mm) | $t_s$=1 | $t_p$=1 |
| Length(mm) | --- | $l_p$=20 |
| Width(mm) | B=10 | b=10 |
| Young's Modulus(Mpa) | $E_s$=210 | $E_p$=64 |
| Density(Kg/m$^3$) | $\rho_s$ =7800 | $\rho_p$=5670 |
| Distance of sensor from Free end i.e. x (mm) | 60 | |
| Distance of actuator from Fixed end i.e. y (mm) | 20 | |
| Distance of primary source of disturbance from jointed point i.e. z (mm) | 20 | |

### *B. Piezoelectric Sensing and Actuation*

When bending moment is given to the structure mounted with PZT patch, certain electrical charge is developed in the patch [11]. Certain voltage is developed by this charge. This developed voltage is a function of the strain developed in the flexible structure on which this PZT patch is attached. On the other hand if a voltage V is applied to a patch attached on a distributed structure, a bending moment is produced [12]. This bending moment is used to reduce the vibrations.

## III. ADAPTIVE POLE PLACEMENT FEEDBACK CONTROLLERS

In transfer function form, the structural system can be represented as a ratio of two polynomials **G**=**B**/**A**. An output feedback is applied to the system which has a transfer function given by **H**=**G**/**F**. The overall transfer function of the system is given by

$$\mathbf{T} = \frac{\mathbf{P}}{\mathbf{Q}} = \frac{\mathbf{G}}{1+\mathbf{GH}} = \frac{\mathbf{BF}}{\mathbf{AF}+\mathbf{BG}} \tag{2}$$

which has CL zeros in **P** and CL poles in **Q**. The co-efficient of the polynomial equation **Q** are called the coefficients of CL characteristic equation. Since a s and b s are not available but their estimate is available i.e. α s and β s. This type of controller in which controller parameters are based on the CL poles is called Adaptive Pole Placement Controller (APPC).

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## IV ROBUST POLE PLACEMENT CONTROL

It is well known that in order to design a robust controller the choice of CL poles depends critically on plant transfer function. An arbitrary choice of stable CL poles can lead to a very poor controller design for certain plants. The present approach seeks to find controllers which minimize (in some sense) the sensitivity of CL poles to perturbations in plant or system parameters. In the present work, only Single-Input, Single-Output case is analyzed and implemented.

Let $\mathbf{q}=[q_0\ q_1\ \ldots\ldots q_{2n-1}]$ be the polynomial co-efficient of $\mathbf{Q}(z^{-1})$. Defining $\mathbf{v}=[a_0\ a_1 \ldots a_n\ b_0\ b_1 \ldots b_n]$ as the system vector, $\mathbf{x}=[h_0\ h_1 \ldots h_{n-1}\ g_0\ g_1 \ldots g_{n-1}]$ as the controller vector and $\boldsymbol{\theta}$ as a 2n x (2n+2) matrix of shifted controller parameters (Sylvester form) i.e.

$$\theta=\begin{bmatrix} h_0 & 0 & & 0 & g_0 & 0 & & 0 \\ h_1 & & & & g_1 & & & \\ \vdots & & & & \vdots & & & \\ h_{n-1} & \cdots & h_0 & 0 & g_{n-1} & \cdots & g_0 & 0 \\ 0 & h_{n-1} & & h_0 & 0 & g_{n-1} & & g_0 \\ \vdots & & & \vdots & \vdots & & & \vdots \\ 0 & \cdots & & h_{n-1} & 0 & \cdots & & g_{n-1} \end{bmatrix} \quad (3)$$

Then it is very straight forward to write the CL pole placement equation as $\boldsymbol{\theta}\ \mathbf{v}=\mathbf{q}$. The pole placement problem becomes that of determining the controller vector $\mathbf{x}$ (whose components are in $\boldsymbol{\theta}$) such that $\boldsymbol{\theta}\ \mathbf{v}=\mathbf{q}$. When the system vector $\mathbf{v}$ is uncertain (i.e. assumed to lie in a range), the controller $\mathbf{x}$ obtained from the above equation may not be able to stabilize the CL system for perturbations of $\mathbf{v}$ from its nominal value [17].

In order to develop a robust solution, it will initially assumed that a s and b s have independent interval coefficients; hence the system vector $\mathbf{v}$ becomes an interval vector $[\mathbf{v}^-, \mathbf{v}^+]$. If we assume that there is certain flexibility in desired pole locations so that $\mathbf{q}$ becomes an interval vector $[\mathbf{q}^-, \mathbf{q}^+]$. The controller can be designed by choosing the nominal system vector $\mathbf{v}^0$, a nominal desired pole vector $\mathbf{q}^0$, system error vector $\boldsymbol{\mu}$ and a pole assignment flexibility error vector $\boldsymbol{\varepsilon}$ as follows [17]

$$\begin{aligned} &v^-=v^0-\mu \quad \text{and } v^+=v^0+\mu \\ &q^-=q^0-\varepsilon \quad \text{and } q^+=q^0+\varepsilon \end{aligned} \quad (4)$$

Then the set of all robust pole placement controllers is given by

$$S \triangleq \left\{x:\left|\theta(x)\,v\ -\ q^0\right|\ \le \varepsilon\ \ \forall v:\left|v-v^0\right| \le \mu\right\} \quad (5)$$

where the mod operation is taken to be component wise. To find a time invariant robust controller in S that will take every system vector $\mathbf{v} \in [\mathbf{v}^-, \mathbf{v}^+]$ and map into any CL system vector $\mathbf{q} \in [\mathbf{q}^-, \mathbf{q}^+]$ can be posed as robust pole placement problem as

$$\begin{aligned} &\text{minimize } f(x)=\left\|\mathbf{x}\ -\mathbf{x}^0\right\|\ ;\ x \in \mathbb{R}^{2n} \\ &\text{subjected to } \left|\theta(x)\,v\ -\ q^0\right|\ \le \varepsilon\ \ \forall v:\left|v-v^0\right| \le \mu \end{aligned} \quad (6)$$

where $\mathbf{x}^0$ can be any desired controller. $\mathbf{x}^0$ can be found by solving the pole-placement problem for the nominal system and nominal desired pole positions. i.e. $\boldsymbol{\theta}^0\ \mathbf{v}^0=\mathbf{q}^0$. The robust pole-placement seeks to find controller vector $\mathbf{x}^*$ which guarantees that the CL polynomial coefficients $\mathbf{q}$ remain within the prescribed regions for all prescribed uncertainties in the system vector $\mathbf{v}$. In order to present the above optimization problem into more tractable mathematical optimization form, the following results are needed.

Fig 2 shows the flow Chart for implementing the robust pole placement controller. The nominal system is chosen corresponding to structure-I and zero gram tip load. Then a certain initial region for pole perturbation i.e $\boldsymbol{\varepsilon}$ is specified. After that system parameter error interval vector $\boldsymbol{\mu}$ is specified corresponding to 5g tip load. Now the robust controller is calculated. If no feasible controller is obtained, the pole perturbation vector $\boldsymbol{\varepsilon}$ is enlarged till feasible controller is obtained. Then the desired simulations are carried out to check the performance parameters of the system. If the desired performance is obtained, in the next iteration, system parameter error interval vector $\boldsymbol{\mu}$ is enlarged until all the systems corresponding to various tip loads come in the domain of the robust controller. Finally the calculated continuous time controller is transformed into digital controller corresponding to a desirable sampling rate [19].

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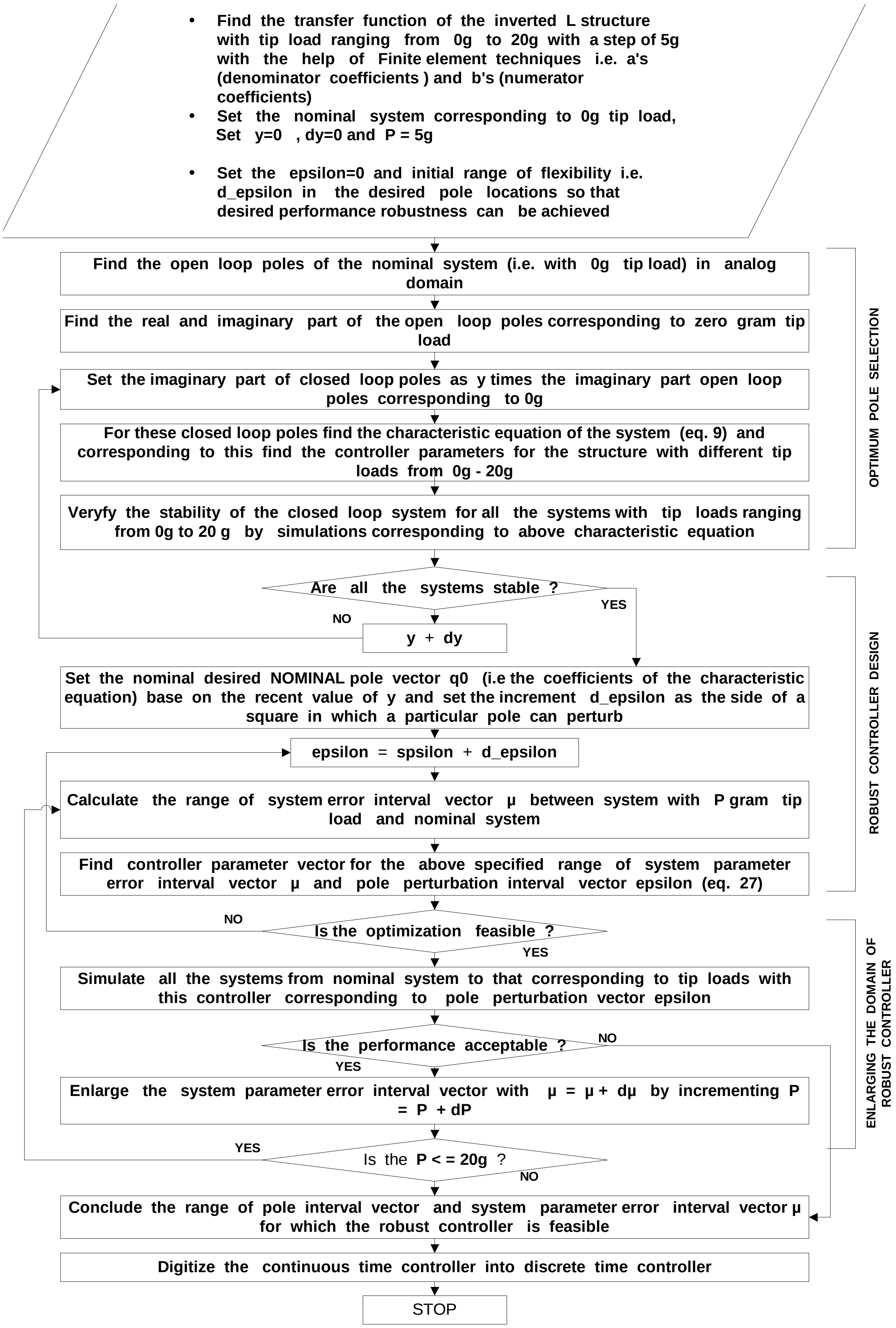


Fig 2 Flow Chart for implementing the Robust pole Placement Control

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## V. IMPLEMENTATION AND VERIFICATION OF CONTROL SYSTEMS

### *A. Experimental Setup and Procedure*

The schematic view of the inverted L structure along with the hardware is shown in the fig 3. The inverted L structure is equipped with 2 PZT patches. To bear the computational burden LABVIEW based real time engine 8187 RT is used.

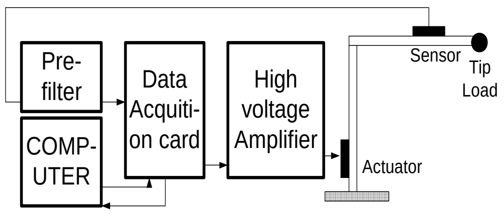


Fig 3 Schematic diagram of the experimental setup

### *B. Simulations and Experimental Results*

First the nominal system is taken pertaining to 0g tip load. CL system poles are fixed taking the system parameters into account. The controller is then designed by solving the Diophantine equation. The tip load is changed from 0g to 15g for the first structure and 0g to 60g for second structure. Maximum available actuator voltage is taken as the 220 volt.

### C. *Performance of Adaptive Pole Placement Controller*

First of all numerical simulations are carried out to understand the dynamics of the CL system with adaptive pole placement controller (APPC) and robust pole placement controller (RPPC) afterwards experimental implementation was done. Part (a) of fig. (4) Shows the response of the adaptive control system for structure-I with zero gram tip load. The OL and CL response is almost the same i.e. no control effectiveness. This is in contrast to the response of the same structural system if non-adaptive controller is applied. This means that any arbitrary pole locations of the CL poles, system gives severely deteriorated performance of adaptive control system for the nominal system (i.e. at 0g tip load). But, if the imaginary part of the CL pole locations is made smaller, performance improves. The optimal performance is obtained when the CL pole is 0.85 times the imaginary part of the OL pole locations (i.e. with a large movement of the pole position towards origin on vertical axis); this defect was clearly eliminated (fig 4b). Similar deterioration in performance was observed if the tip load was changed from 0g to 15g (fig 5a). *The transition response is not so good***.** The amplitude of the CL system increases as compared to OL system during initial time steps. Also the CL settling time is large (i.e. 2.5 second ) as compared with the second case ( with CL settling time of 1.3 second) where the imaginary part of the CL pole locations are made 0.85 times the imaginary part of the OL pole locations (fig 5b). Obviously, higher control voltages will be needed in the later case. By constraining the control voltage to a certain magnitude which is available practically, this problem can be solved. By observing the response in frequency domain (fig 5c), it is observed that, although the first mode amplitude is reduced, the second mode gets excited. However, by using optimal location of CL poles, better performance gets resulted (fig 5d).

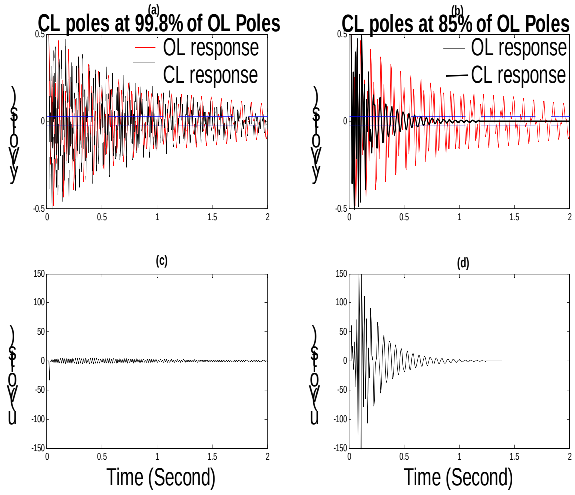


Fig 4 Effect of different positions of closed loop poles on the performance of adaptive controller for nominal system (0g tip load)

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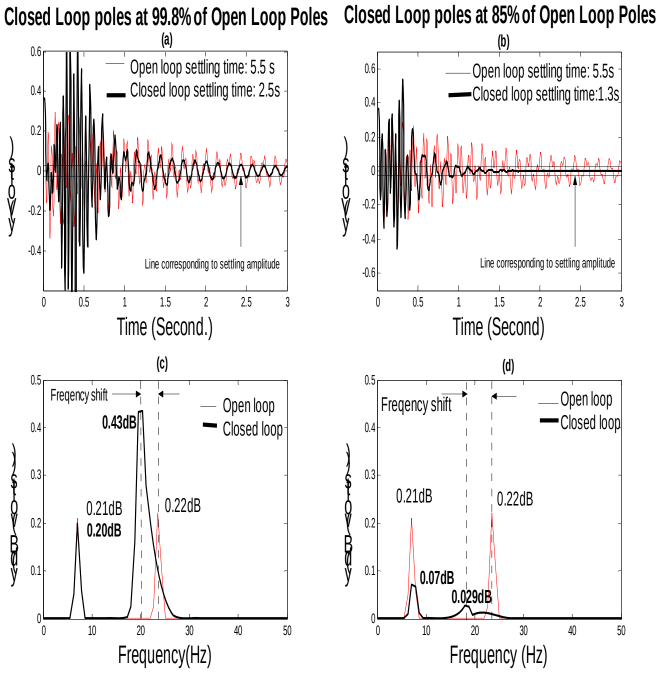


Fig 5 Effect of different positions of closed loop poles on the performance of adaptive controller at 15g tip load

## D. Performance of Robust Pole Placement Controller

Table II shows the limiting amplitude of the actuator voltages for both the structures at different tip loads for adaptive pole placement controller (APPC) and robust pole placement controller (RPPC). For both the structures APPC requires high voltage for stable operations.

TABLE II Performance comparison of adaptive and robust pole placement control

| No. of the structure | Original Weight (OW) | Tip Load | OW/BM | Maximum Actuator Voltage (volts) required | |
|---|---|---|---|---|---|
| | | | | APPC | RPPC |
| Structure I | 10.5 g | 0g | 578 | --- | --- |
| | | 5g | 867 | 150 | 30 |
| | | 10g | 1156 | 180 | 60 |
| | | 15g | 1441 | 220 | 100 |
| | | 20g | --- | UNSTABLE | UNSTABLE |
| Structure II | 57.3 g | 0g | 1127 | -- | -- |
| | | 14g | 1433 | 190 | 30 |
| | | 30g | 1741 | 640 | 25 |
| | | 60g | 2356 | 130 | 30 |

## E. Performance Comparison of Adaptive and Robust Pole Placement Controller

The structure was excited by a constant velocity excitation. A steel ball was thrown from a certain height near the tip or at the mid of the horizontal limb. A certain auto-regressive model of order 12 was used to model the measurement noise. Excellent CL results were obtained (fig 6). So, if the noise can be modeled properly, RPPC is the best choice for both light weight as well as heavy structures.

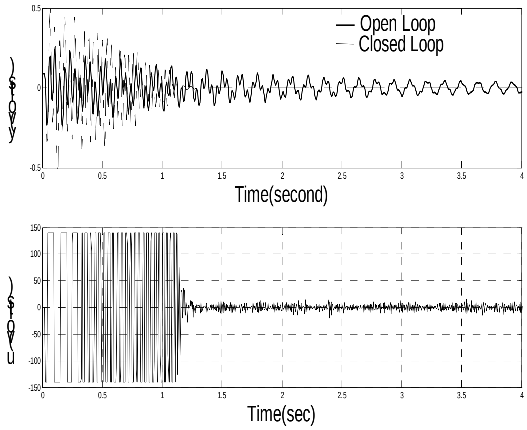


Fig 6 Performance of robust controller at 15g tip load WITH modeling the measurement noise (Experimental)

## VI CONCLUSION

Robust and adaptive control design techniques give controllers which are effective and stable for certain range of system parameters. Adaptive pole placement control is a suitable alternative for vibration control of light weight flexible structures **only**, since it requires very large amplitude of control voltages for maintaining stability, if applied to heavy structures. It works better, even if, certain amount of un-modeled noise is present in the system. However, if the exact noise model is available by using high quality hardware, robust pole placement control is the best alternative. It requires comparatively very less control voltage amplitudes and works for light as well as heavy structures. The position of closed loop poles for designing the pole placement based controllers is an important issue. Study can easily be extended to MIMO cases easily.

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