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OFDM Transmission Simulation using GNU Octave

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Abstract — **Orthogonal Frequency Division Multiplexing (OFDM) is a multicarrier technology for high data rate communication system. The basic principle of OFDM is to divide the available spectrum into parallel channels in order to transmit data on these channels at a low rate. The OFDM concept is based on the fact that the channels referred to as carriers are orthogonal to each other. Also, the frequency responses of the parallel channels are overlapping. The aim of this paper is to simulate, using GNU Octave, an OFDM transmission under Additive White Gaussian Noise (AWGN) and/or Rayleigh fading and to analyze the effects of these phenomena.**

Keywords – OFDM, AWGN, Rayleigh fading channel, BER.

I. Introduction

Presently, the growing evolution of wireless communication systems demands a technology capable of conveying data at high speed. Wireless broadband applications require a downlink access evolution. Since the communications future is wireless, both research and testing aim at improving the techniques of transmission. The current 3G systems employ a Wideband Code Division Multiple Access (WCDMA), which is already reaching its downlink access limits. The system requirements are growing, e.g. LTE [1], [2], and therefore an enhanced access scheme is necessary. Likewise, technologies such as 802.11a/g/n [3] use OFDM [4], [5] to enhance their data transmission rate. Thus, OFDM is expected to be the transport technology of the next generation wireless communication [6].

The OFDM technique involves assembling the data (input) into parallel channels. These channels perform a modulation, e.g. 64-QAM. Each parallel channel of complex data modulated performs an inverse FFT (IFFT). Before the serial transmission, a guard interval (GI) is added in order to combat the effects of intersymbol interference (ISI). As it is well-known, ISI is an effect of a multipath propagation and of the reflected delayed waves that arrive at receiver. At the receiver an FFT is performed in order to recover data. The OFDM flowchart is depicted in Figure 1 and a summarized multicarrier transmitter is displayed in Figure 2.

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GNU Octave [7] is a high-level language program for numerical computations. Furthermore, it is mostly compatible with MATLAB®. The use of GNU Octave provides open source and powerful tools. In spite of its limitations, it is most useful for the purpose of this paper.

The remainder of this paper is organized as follows. Section II describes in a brief way the OFDM model and an understanding of the mathematical algorithms involved in this communication technique will be developed. Section III presents an OFDM transmission simulation using 64-QAM. Finally, conclusions are drawn in Section IV.

II. OFDM Model

Considering Figure 1 as reference of an OFDM system, we can state that a baseband OFDM symbol with K carriers may be written as:

$$s(t) = \sum_{k=0}^{K-1} x_k \gamma_k(t) \tag{1}$$

, where x_k is a complex modulated symbol associated with a carrier $\gamma_k(t)$ of the following form:

$$\gamma_k(t) = e^{j2\pi k \Delta f\, t} \tag{2}$$

From the above equation, $f_k=k\Delta f$ is the channel frequency and Δf is the carrier spacing.

Let us consider a clean channel, so at the receiver the received signal is multiplied by $e^{-j2\pi r\Delta ft}$. Let us assume that the OFDM symbol has a duration T, and because of OFDM intercarriers orthogonality, the signal may be recovered as follows:

$$\frac{1}{T}\int_0^T e^{j2\pi\Delta f(k-r)t}\,dt = \begin{cases} 1 & k = r \\ 0 & k \neq r \end{cases} \tag{3}$$

Thus, the baseband signal may be rewritten of the following form:

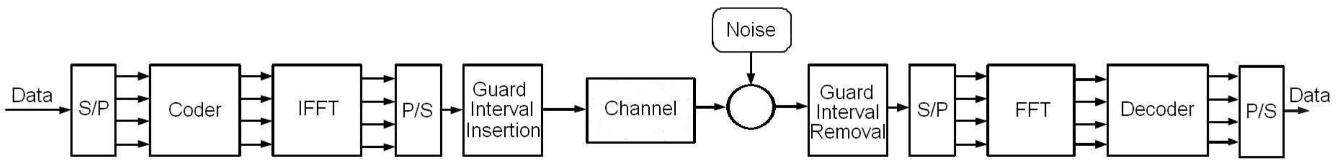

Figure 1. OFDM Flowchart.

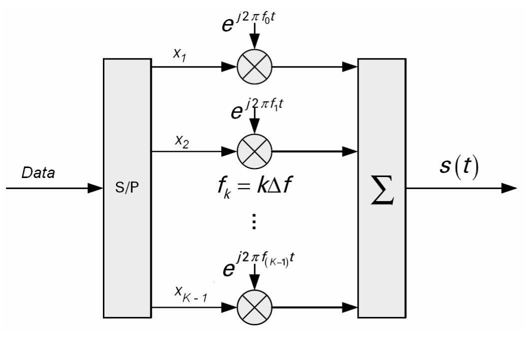

Figure 2. Summarized OFDM transmitter.

$$s(t) = \sum_{k=0}^{K-1} \sum_{n=-\infty}^{\infty} x_k \gamma_k(t) rec(t - nT) \quad (4)$$

, where *rect*(*t*) is the rectangle function of the OFDM symbol filter. However, in a common environment, the signal is contaminated mainly by multipath effects, noise and fading. For the purpose of the simulation, AWGN and frequency-flat Rayleigh fading are considered as damaging effects.

For the 64-QAM modulation scheme, the equations of theoretical Bit Error Ratio (BER) under AWGN and one-path Rayleigh fading respectively are as follows:

$$AWGN_{64QAM} = \frac{7}{4}\operatorname{erfc}\left(\sqrt{\frac{E_b}{7N_0}}\right) - \frac{49}{384}\operatorname{erfc}^2\left(\sqrt{\frac{E_b}{7N_0}}\right) \quad (5)$$

$$RF_{64QAM} = \frac{7}{4}\left(1 - \frac{1}{\sqrt{1 + 7N_0/E_b}}\right) \quad (6)$$

From the above equations, BER depends on the ratio of bit energy to noise power spectral density (E_b/N_0). Furthermore, there is a relationship between the ratio of symbol energy to noise power spectral density E_s/N_0 and E_b/N_0. This relationship depends on the code rate R_C and the size of the modulation alphabet S, e.g. for 64-QAM this is equal to 6. Thus, the value of E_s/N_0 is computed as follows:

$$\frac{E_s}{N_0}[dB] = \frac{E_b}{N_0}[dB] + 10\log(R_C \log_2 S) \quad (7)$$

A. Rayleigh Fading Channel

In order to model a frequency-flat Rayleigh fading channel; Clarke introduced the following formulation [8], [9]:

$$g(t) = g_I(t)\cos 2\pi f_c t - g_Q(t)\operatorname{sen} 2\pi f_c t \quad (8)$$

, where the in-phase and quadrature components are given by:

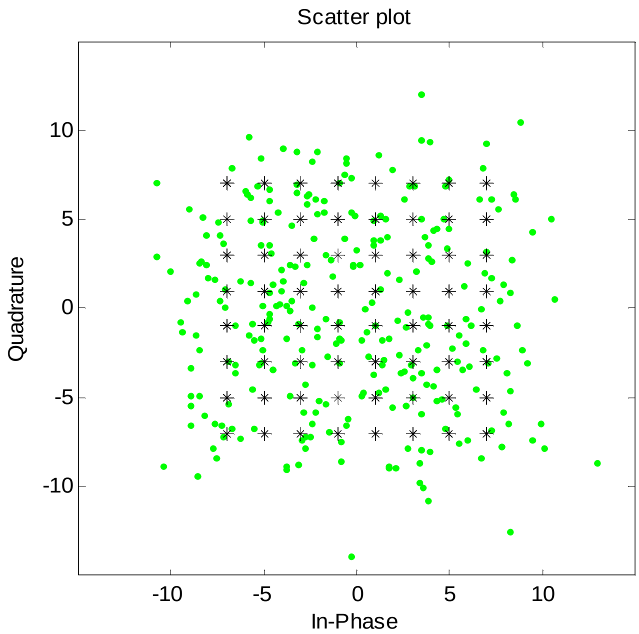

Figure 3. Six OFDM symbols. Green dots: received data (AWGN). Black asterisks: original transmitted data. E_b/N_0 = 0 dB.

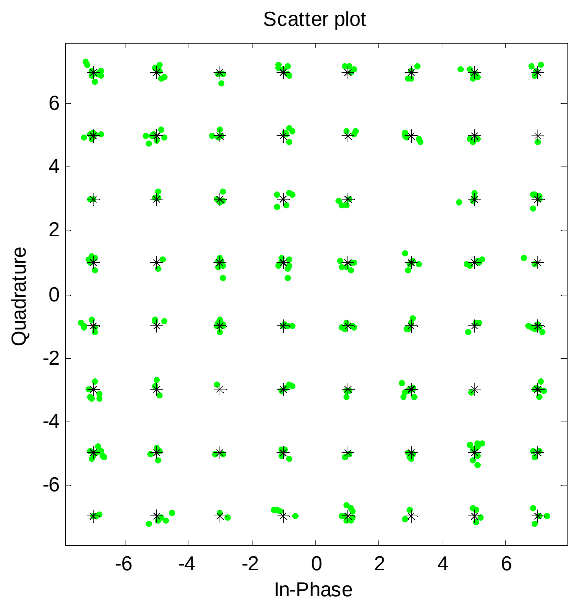

Figure 4. Six OFDM symbols. Green dots: received data (AWGN). Black asterisks: original transmitted data. E_b/N_0 = 23 dB.

$$g_I(t) = \sum_{m=1}^{M} A_m(t)\cos\left[2\pi f_c \tau_m(t)(1 - \theta_{d,m}(t))\right] \quad (9)$$

$$g_Q(t) = \sum_{m=1}^{M} A_m(t)\operatorname{sen}\left[2\pi f_c \tau_m(t)(1 - \theta_{d,m}(t))\right] \quad (10)$$

From the above equations, M is the number of multipaths, $m \in M$, f_c is the carrier frequency, τ_m is the delay of the m^{th} path, $\theta_{d,m}$ is the Doppler phase shift of the m^{th} path. Furthermore, the Doppler phase shift is given as follows:

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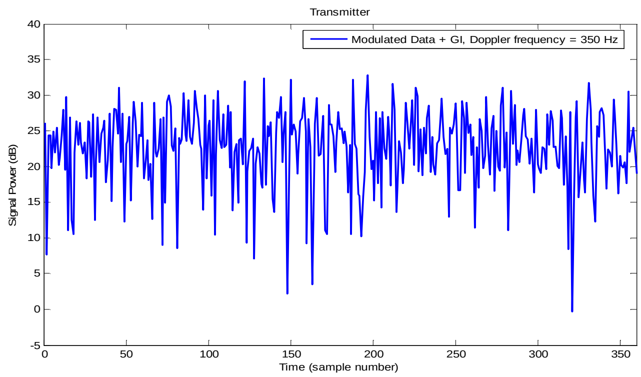

Figure 5. Signal power of six OFDM symbols transmitted.

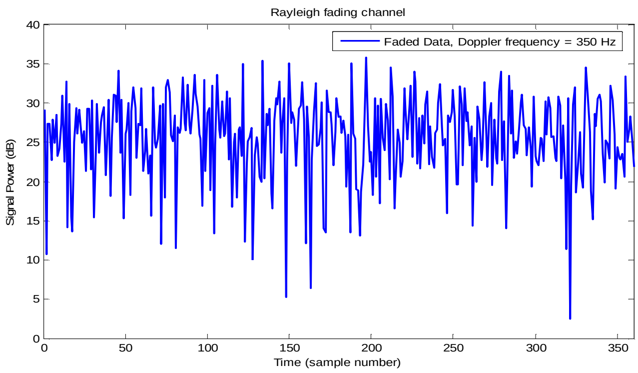

Figure 6. Signal power of six OFDM symbols under Rayleigh channel.

$$\theta_{d,m}(t) = 2\pi f_d t \tag{11}$$

, where the Doppler shift has the following form: $f_d = (v/\lambda)\cos\alpha_m$, with λ the carrier wavelength, α_m is the angle between the line-of-sight path and the direction of motion, and v is the mobile speed. Besides, one can approximate $g_I(t)$ and $g_Q(t)$ as jointly Gaussian process by the central limit theorem when M is large. Moreover, it is considered as a zero-mean Gaussian process since $E[g_I(t)]=E[g_Q(t)]=0$. Considering that g_I and g_Q are samples, with zero mean and variance σ^2, of $g_I(t)$ and $g_Q(t)$ respectively, R is a Rayleigh random variable if: $R = \sqrt{g_I^2 + g_Q^2}$. The Rayleigh distribution has the following probability density function:

$$p_R(r) = r/\sigma^2 \, e^{-r^2/2\sigma^2} \quad , r \geq 0 \tag{12}$$

III. OFDM Transmission Simulation

In this section, an OFDM transmission simulation is performed using GNU Octave. The code employed throughout this paper was programmed by the author.

In order to perform the simulation; we followed the flowchart in Figure 1. Two scenarios are considered, AWGN channel and Rayleigh fading channel. The levels of AWGN and Rayleigh fading are given by E_b/N_0. Likewise, as it is well known, the multipath environment is considered as an intrinsic property of the propagation.

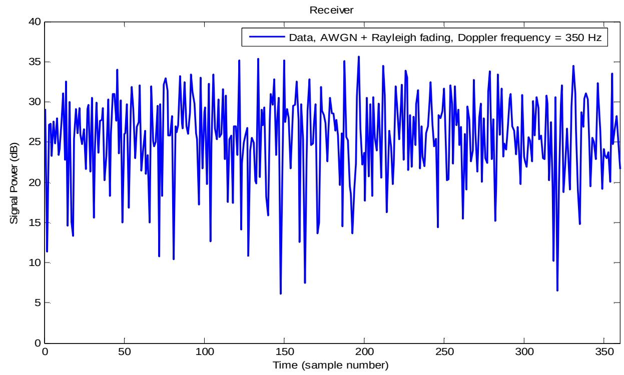

Figure 7. Signal power of six OFDM symbols received.

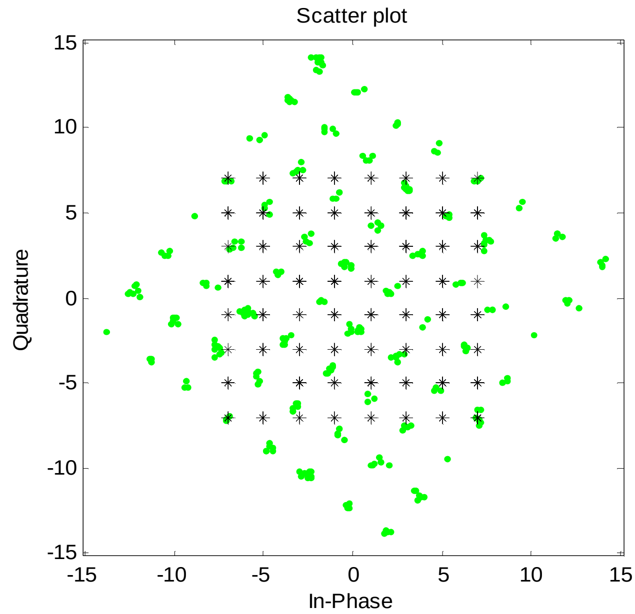

Figure 8. OFDM symbols. Green dots: received data (Rayleigh fading and AWGN). Black asterisks: transmitted data. E_b/N_0 = 23 dB.

Let us consider an OFDM transmitter which meets the requirements according to Figure 1. GI is inserted to avoid ISI. The chosen modulation is 64-QAM and the number of orthogonal carriers is K=48.

As a first example, let us consider that six OFDM symbols are transmitted to the air with E_b/N_0 equal to 0 dB. Through the path, the modulated data are contaminated by AWGN. So, Figure 3 depicts the quadrature and in-phase of both original transmitted data (black asterisks), and received data (green dots). As we can see, the data were altered in amplitude, but not in phase. The multipath effects are reduced because of GI. Also, each channel of an OFDM symbol suffers different amplitude fluctuations in an AWGN path.

However, the AWGN effects are reduced in an OFDM symbol according to ratio E_b/N_0. Now, the ratio E_b/N_0 is 23 dB. Figure 4 displays the quadrature and in-phase of both the original transmitted data (black asterisks), and received data (green dots). Then, the AWGN effects and BER are in direct relation. The dashed blue line in Figure 11 is the relationship between E_b/N_0 and BER under AWGN using 64-QAM.

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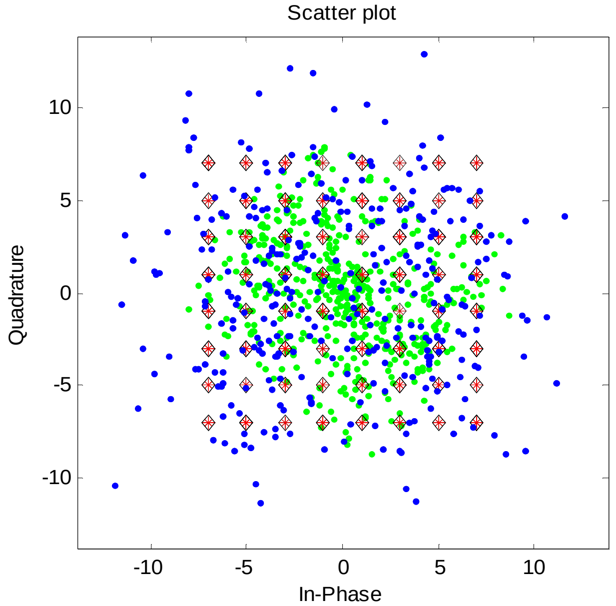

Figure 9. OFDM symbols. Green dots: received data (Rayleigh fading and AWGN). Blue dots: recovered data (PACE). Red diamonds: transmitted data. E_b/N_0 = 0 dB.

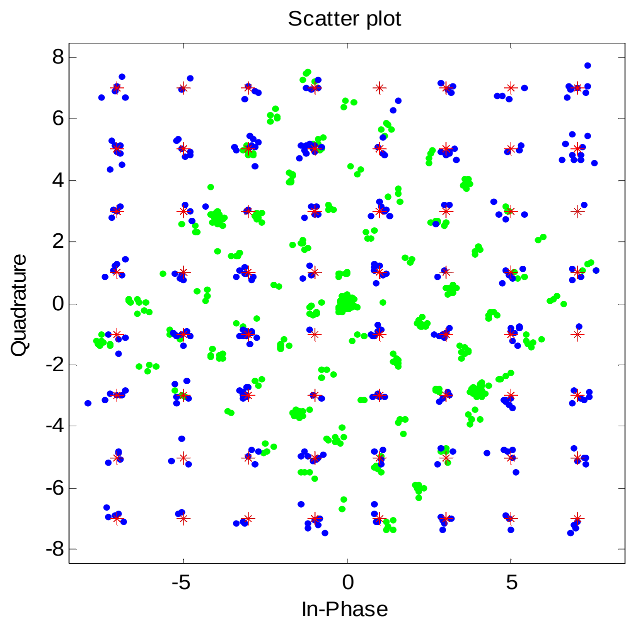

Figure 10. OFDM symbols. Green dots: received data (Rayleigh fading and AWGN). Blue dots: recovered data (PACE). Red asterisks: transmitted data. E_b/N_0 = 23 dB

As a second instance, we consider an OFDM transmission with E_b/N_0 equal to 23 dB under Rayleigh fading and AWGN. Figure 5 depicts the signal power of both I-channel and Q-channel of six OFDM symbols transmitted. As we can see, the Doppler frequency used was 350 Hz. The guard interval, GI, was inserted to avoid ISI. However, during the travel, the transmitted data are contaminated by Rayleigh fading. The Rayleigh fading simulated makes phase and amplitude of modulated data fluctuate. The signal power of Rayleigh faded data is displayed in Figure 6. Also, when the data arrived at the receiver, the resultant data received are contaminated by Rayleigh fading and AWGN. Figure 7 displays the signal power of received data. Likewise, Figure

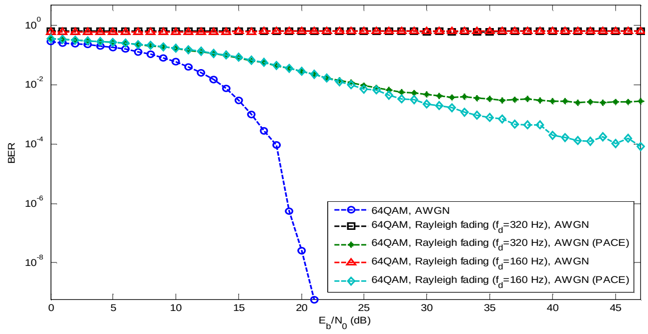

Figure 11. BER performance.

8 shows both the contaminated data under Rayleigh fading and AWGN (green dots), and the original transmitted data (black asterisks).

Nevertheless, in order to thwart the phase fluctuations under Rayleigh fading, pilot-aided channel estimation (PACE) [10] is employed. As a third example, let us consider 52 carriers, 48 of them for data and the rest for pilot-aided symbols in order to use PACE. The multipath conditions are kept; AWGN and Rayleigh fading. Thus, Figure 9 displays an OFDM transmission with E_b/N_0 equal to 5 dB under Rayleigh fading and AWGN. The use of PACE helps the receiver to estimate the changes produced by the communication channel. As it is shown in Figure 9, the green dots correspond to the received data, which has a phase shift. The blue dots in Figure 9 are the recovered data, but they are still noisy. However, the data phase was recovered. From the same figure, we can see the original transmitted data, the red diamonds.

Now, the same environment is kept but with E_b/N_0 equal to 23 dB. Figure 10 shows that the signal recovered is less noisy than the recovered signal shown in Figure 9.

Figure 11 shows four cases wherein it is possible to see the relationship between E_b/N_0 and BER under different multipath effects. In this way, from Figure 11, if PACE is not used, the BER would be very high and thus the quality of transmission would be very poor.

IV. Conclusion

In this paper, an OFDM transmission under multipath effects was analyzed by means of simulations.

In spite of GNU Octave limitations, it is an open source option as a numerical computation tool. To be able to use GNU Octave, some bugs in built-in functions were fixed in order to perform scripts and plots.

The simulation plots depict how modulated data are altered (in phase and amplitude) through a communication channel. Rayleigh fading and AWGN damage the quality of the transmission and this can be seen from the plots performed throughout this paper. The BER performance plot shows the relationship between E_b/N_0 and BER for 64-QAM. The chosen modulation, 64-QAM, is mostly employed in cellular communication systems and IEEE-802.11 systems.

PACE was used as the means of channel estimation in order to recover data with phase fluctuations, and in this way

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to assure transmission QoS. This task becomes easier because of the OFDM scheme and its orthogonal channels.

Also noteworthy, as a proposal for future works to extend this paper is to add turbo coding to the OFDM scheme, to perform simulations, and in addition to simulate an MIMO OFDM transmission. The software employed could be either MATLAB® or GNU Octave.

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