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A Fast Image Inpainting Algorithm Based on TV Model

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Abstract—**A fast image inpainting algorithm based on TV model is proposed on the basis of analysis of local characteristics, which shows the more information around damaged pixels appears, the faster the information diffuses. The algorithm first stratifies and filters the pixels around damaged region according to priority, and then iteratively inpaint the damaged pixels from outside to inside on the grounds of priority again. Experiment shows that the inpainting speed of the algorithm is faster and greater impact.**

Index Terms—**Image inpainting, TV model, priority, peak signal to noise ratio(PSNR).**

I. Introduction

Image inpainting is an important part of image restoration, which aims to automatically restore missing information of image according to information around damaged region. It is mainly used for heritage conservation, restoration of old photographs, removal of occlusions and so on .

In general, image inpainting methods can be divided into two categories: structure-based inpainting and texture-based inpainting. Structure-based inpainting refers to the process of image inpainting which employs information around damaged region to estimate isophote from coarse to fine and diffuses information by diffusion mechanism. The method includes the following models: BSCB model[1], TV model[2], CDD model[3] and elastica model[4] and so on. These models have a good inpainting effect on the small-scale and non-texture damaged region such as the scratchs, the creases and the spots and so on, but it is easy to cause the blurriness when they are used to inpaint relatively large damaged region. The above essentially solve partial differential equations (PDE) describing information diffusion, while the numerical solution of PDEs requires a large number of iterations, so that the inpainting speed is very slow. Therefore, how to improve the solution speed of these models has become a very valuable research. At present, the fast structure-based image inpainting algorithm has fast marching method[5] and Oliveira method[6] and so on. Fast marching method uses the weighted-average method to inpaint damaged image so that the edge-preserving is not ideal. Oliveira method is faster, but does not maintain the isophotes' directions in the inpainting process, and therefore it was almost no ability to restore the details of the edges. The methods of texture-based inpainting is to fill missing information by texture synthesis techniques, and are suitable for inpainting relatively larger damaged region. It can also be divided into image inpainting based on decomposition and image inpainting based on texture synthesis.

It is well-known that image inpainting is an ill-posed problem[3], which manifested information around damaged region is insufficient to fully restore missing information. In image processing, the most commonly used method to eliminate morbidness is to join regularization items, whose basic idea is to use prior knowledge of physical problems, adding more constraints to the problem, which makes the solution of the problem continuously depend on the observational data, and additional items must be in line with prior knowledge of physical meaning[7]. Inspired by the above, we make full use of information around damaged region to iteratively inpaint the damaged region quickly in the inpainting template fixed-size case. The basic idea is as follows:

Step 1. Segment the damaged region.

Step 2. Find the edge of damaged region.

Step 3. Calculate the priority of the pixels on the edge and sort in accordance with the priority, if the priority of the pixel is greater than a certain threshold T, then reserve the pixel, else delete it.

Step 4. Store the reserved pixels according to the order of priority as a layer.

Step 5. Update the damaged region.

Step 6. Repeat the Step 2, the Step 3, the Step4, the Step 5 until the area of damaged region is zero.

Step7. Iteratively inpaint according to the size of priority from outside to inside.

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II. Image Inpainting Algorithm Based on TV Model

Image inpainting based on TV model[2] was proposed by T.Chan et al in 2002, described as follows.

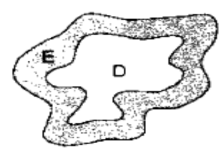

Fig 1:Region to be inpainted

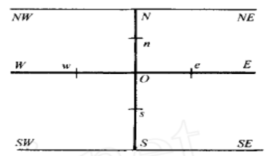

Fig 2:Target pixel and its neighborhood

In Fig 1, D is defined as region to be inpainted, and it is

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generally ring, E is the outer neighborhood of D, u denotes target pixel value, the cost function was defined as:

$$r(u)=\int_{E\cup D}|\nabla u|dxdy \quad (1)$$

and it must meet the following noise constraint conditions:

$$\frac{1}{A(E)}\int_{E}\left|u-u^{0}\right|^{2}dxdy=\sigma^{2} \quad (2)$$

In the formula (2), $A(E)$ is the area of damaged region, pixel value in the E is contaminated by Guassian white noise whose standard deviation is σ. r is a non-negative real function and ∇u denotes the gradient. It is thus clear that the formula (1) is to keep the edge of region to be inpainted smooth and the formula (2) is to make the inpainting process good noise robustness. In this way, the problem turns into solving the extremum of the formula (2) in the formula (1) condition. Using Lagrange multiplier method gets the following function:

$$L(u)=\int|\nabla u|dxdy+\frac{\lambda}{2}\int\left|u-u^{0}\right|^{2}dxdy \quad (3)$$

According to Euler-Lagrange equation we can see that u minimizing $L(u)$ should meet the following condition:

$$-div\left(\frac{\nabla u}{|\nabla u|}\right)+\lambda_{e}(u-u^{0})=0,\lambda_{e}=\begin{cases}\lambda,(x,y)\in E\\ 0,(x,y)\in D\end{cases} \quad (4)$$

So this is total variation model of image inpainting, and the difference method is adopted for its numerical solution. In Fig 2, o is target pixel, and $\Lambda=\{E,N,W,S\}$ denotes its neighborhood, and $\{e,n,w,s\}$ denotes its half-pixel neighborhood. If v is denoted as: $v=(v^{1},v^{2})=\frac{\nabla u}{|\nabla u|}$, then its divergence can be approximately denoted as:

$$\nabla\cdot v=\frac{\partial v^{1}}{\partial x}+\frac{\partial v^{2}}{\partial y}\approx\frac{v_{e}^{1}-v_{w}^{1}}{h}+\frac{v_{n}^{2}-v_{s}^{2}}{h} \quad (5)$$

Where h is step, and it is generally 1. The half-pixel gradient can be computed to take point e as the example.

$$v_{e}^{1}=\frac{1}{|\nabla u_{e}|}\left[\frac{\partial u}{\partial x}\right]_{e}\approx\frac{1}{|\nabla u_{e}|}\frac{u_{E}-u_{o}}{h} \quad (6)$$

$$|\nabla u_{e}|\approx\frac{1}{h}\sqrt{(u_{E}-u_{o})^{2}+(u_{NE}-u_{SE})^{2}} \quad (7)$$

The solution method of other half-pixel gradient is the same as the half-pixel e.Finally, $v_{e}^{1},v_{w}^{1},v_{n}^{2},v_{s}^{2}$ and the formula(5)substituted in the formula (4) can derived:

$$\sum_{p\in\Lambda}\frac{1}{|\nabla u_{p}|}(u_{o}-u_{p})+\lambda_{e}(u_{o}-u_{o}^{0})=0 \quad (8)$$

In this paper, we define $w_{p}=\frac{1}{\sqrt{|\nabla u_{p}|^{2}+\alpha^{2}}},p\in A$ in which α is a diminutiveness to curb the weight and can avoid the disturbance of $w_{p},h_{op}=\frac{w_{p}}{\sum_{p\in\Lambda}w_{p}+\lambda_{e}(o)}$ and $h_{op}=\frac{w_{p}}{\sum_{p\in\Lambda}w_{p}+\lambda_{e}(o)}$.Accor -ding to the above, the formula(8)can be denoted as $u_{o}=\sum_{p\in\Lambda}h_{op}u_{p}+h_{oo}u_{o}^{0}$. By Guass-Jacobi iteration algorithm, damaged pixel value can be denoted as $u_{o}^{(n)}=\sum_{p\in\Lambda}h_{op}^{(n-1)}u_{P}^{(n-1)}+h_{oo}^{(n-1)}u_{o}^{(n-1)}$.The pixels in the damaged region can be derived from the formula by iterative solution. When iterations reach stability, image inpainting is over.

III. FAST IMAGE INPAINTING ALGORITHM BASED ON TV MODEL

Image inpainting algorithm based on TV model is essentially a weighted-average algorithm. The smaller the difference between target-pixel and neighborhood-pixels is, the greater weight is, on the contrast, the greater the difference is, the smaller the weight is. Iteration is in fact a process of anisotropic information diffusion. Experiments show that there are more non-damaged pixels around damaged pixel ,then the speed of the diffusion is faster, which means the converged speed is faster and inpainting is also faster. Thus, this paper try to divide the damaged region of image into a number of layers, and then, doing the inpainting layer by layer. However, there is still a problem that there are some pixels whose non-damaged neighborhood-pixels is less in each layer. Therefore, it will slow down the diffusion speed of information and reduce the reliability. In order to solve this problem, a threshold T was set , whose purpose is to filter these pixels that are placed in the next layer of the current layer. At that moment, there will be more available information around the pixel, so information diffusion will be faster and stronger. However, the traditional image inpainting method based on TV model iteratively inpaint damaged pixels line by line, and it did not treat damaged region as a whole, which will inevitably result in the available information around damaged region has not been maximized. In particular, when damaged region is relatively narrow and horizontal, the problem that information diffusion is slow is more prominent. Therefore, image inpainting requires more iterations.

The more available information around damaged pixels, the faster information spreading, the convergence speed is also faster. The following experiments will prove this idea. Fig 3 is an experimental image and local pixel value of R component of the image, and the blank in Fig 3 denotes damaged pixels. Fig 4 indicate the degree of iterative times and information diffusion of (c),(d),(e),(f),(g) and (h) in Fig 3.

(a)

123	137	133
119	128	109
46	47	46

(b)

123	137	
119		

(c)

123	137	133
119		

(d)

123	137	133
119		109

(e)

123	137	133
119		109
46		

(f)

123	137	133
119		109
46	47	

(g)

123	137	133
119		109
46	47	46

(h)

Fig 3. Expermental image and local pixel value of R component of the image

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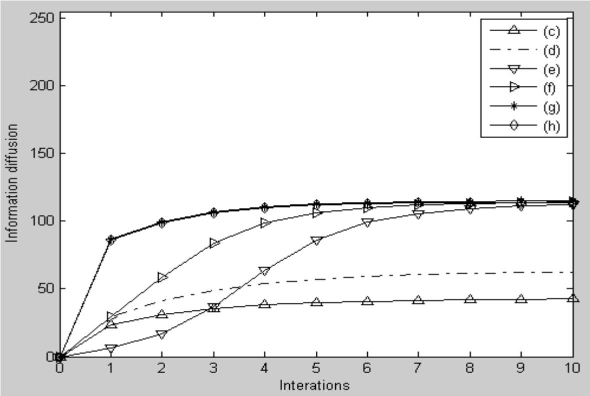

Fig 4. The relation bewteen iterations and information diffusion

As can be seen from Fig 4, the more available information around damaged pixels is, the faster information spreading is. This is the reason why computing priority in the paper, to ensure that iterative damaged pixel have more available information, making the diffusing information on each iteration as much as possible. Finally, the convergence is faster.

The inpainting template M of 3×3 is selected in experiments. Priority p is calculated as follows: $p=\sum_{s\in M} A$, A and s denote weight and the pixels in the template, respectively. If s is a damaged pixel, then $A=0$; else if s is directly adjacent to the core, $A=2$, and indirectly adjacent to the core, $A=1$. Fig 5 show the flowchart of the algorithm.

IV. EXPERIMENT RESULT AND ANALYSIS

Thangka image is used as experimental subjects in this paper. Through the inpainting of damaged Thangka images verifies the effectiveness of the proposed method. The experimental results shown in Fig6 and Fig7, where T=7, α =0.01.

(a)Damaged image (b)Inpainting result of TV algorithm

(c) Inpainting result of the proposed algorithm (d)Original image

Fig 6.The experimental result 1

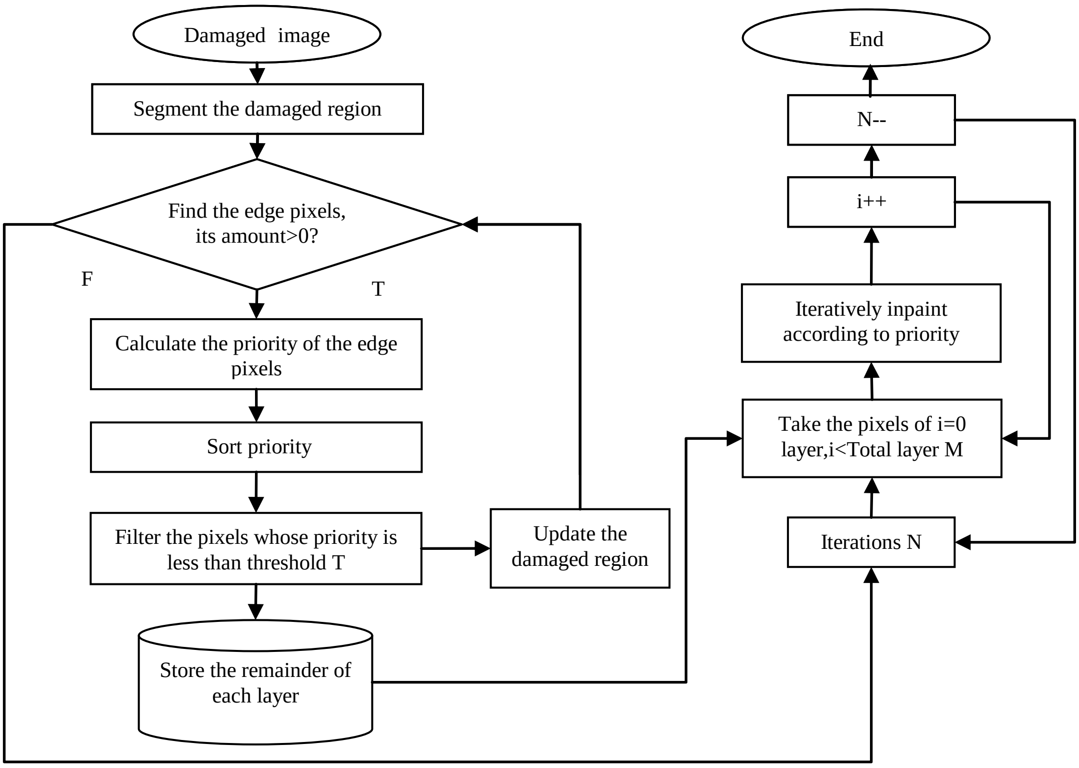

Fig 5. The flowchart of the proposed algorithm

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(a)Damaged image

(b)Inpainting result of TV algorithm

(c) Inpainting result of the proposed algorithm

(d)Original image

Fig 6.The experimental result 2

Generally speaking, the evaluations of inpainting effectiveness have two ways: objective evaluation and subjective evaluation.The objective evaluation method was used for the evaluation of inpainting effectiveness, which is mainly from the fidelity of image restoration to be measured. At present, The widely used objective evaluation methods are: mean square error(MSE)measurement, signal to peak signal to noise ratio (PSNR) measurement and improved signal to noise ratio (ISNR) measurement and so on. In this paper, peak signal to noise ratio (PSNR) was used to evaluate the inpainting results.

As can be seen from Table 1, Table 2 and Table 3, the inpainting speed and inpainting accuracy are improved by the proposed algorithm compared with the traditional algorithm based on TV model. The inpainting effectiveness of the algorithm is superior to the traditional algorithm. This also validates the correctness and rationality of our standpoint.

V. Conclusion

The traditional model of image inpainting based on TV is essentially solving partial differential equations(PDE) describing the diffusion of information, while its numerical solution requires a large number of iterations, so that the inpainting speed is very slow. We make full use of the information around damaged region in the inpainting template fixed-size in the paper, which can make information diffusion stronger and the inpainting speed faster. In fact, the idea we put forward has certain guidance functions to other model based on partial differential equations.

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Table 1. Objective evaluation of experimental result 1

	Damaged area	Iterations	Cost time	R(PSNR)	G(PSNR)	B(PSNR)
b	5619	2000	91s	45.701877	46.172111	45.649765
c	5619	2000	70s	45.738811	46.373402	45.879455

Table 2. Objective evaluation of experimental result 2

	Damaged area	Iterations	Cost time	R(PSNR)	G(PSNR)	B(PSNR)
b	15806	2000	260s	45.466330	44.549387	45.188315
c	15806	2000	196s	45.245876	44.722801	45.202082

Table 3 reflects PSNR changes with increasing iterations when inpainting damaged image in the experiment result 2.

[7]Wu bin,Wu Yadong and Zhang Hongying.Image restruction techniques based on Variational partial differential equations(First Edition)[M],Beijing,Peking University Press, 2008-1,pp.7-8.

Table 3. The relation between iterations and PSNR

	Inpainting algorithm based on TV model			Inpainting algorithm in the paper		
Iterations	R	G	B	R	G	B
10	18.748620	20.306043	23.872961	18.761382	20.322904	23.897729
50	20.686794	22.305174	26.148538	20.710204	22.343455	26.221788
100	22.970758	24.563541	28.569680	23.002391	24.637958	28.683413
300	34.465803	35.622947	39.180603	34.923339	35.979008	39.557426
500	39.085976	39.440427	42.657337	39.091503	39.430762	42.646423
1000	39.292272	39.643974	42.980719	39.291578	39.644114	42.978344

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