Proceedings of the International MultiConference of Engineers and Computer Scientists 2010 Vol III,
IMECS 2010, March 17 - 19, 2010, Hong Kong

A Stochastic Approach to Modeling the Managerial Information Processing

Dejun Xie *and Dung Nguyen †

Abstract—**This work concerns the stochastic modeling of business information received by a company from managerial's point of view. A mean reverting stochastic process is proposed to model the percent of information that a company can receive at any time t. Explicit iterative formulas are provided under certain assumptions. Numerical scheme is derived for computing the total effective information received for a given period.**

Keywords: managerial, information processing, stochastic modeling, mean-reverting process, numerical simulation

1 Introduction

A company's managerial function can be described as an information processing system, where the external world is treated as information generator and the management team of the company as information receiver. External information, in forms such as industry regulations, market competition, feedback from client, and credit ratings, creates a business climate in which that the company need to fit. The task of a manager is to process the received information and make sensible decisions in order to accomplish the executive goal. To respond to the external information in a timely manner is crucial for the company to survive the more than ever competitive market in the e-commerce era. Because of the important implications in real economy, there exist considerable studies devoted to such topics. Many of the previous studies are from the point of view of organizational learning, the semantic of which can be found in [7], for instance. Similar approaches are applied in [4, 5, 12], [9]. A more complete literature review relevant to organizational learning can be found in [6]. Here we are more interested in the stochastic modeling of information flow in itself and providing a rigorous mathematical model to calculate how much information a company may lose or receive in a given period of time and given number of qualified managers.

Our study is built on the assumption that the processing ability of any manager is not infinite. First, let us consider a very special case where the external world, as an information generator, generates N bits of information periodically at every T unit time. and the manager has an information processing capability of greater or equal to N/T, which means that the manager is able to process at least N bits of information in the time interval T. It is assumed that the information transmitting time from the external world to the manager is negligible. In this case, no information loss at the manager's office would occur. Hence no deputy managers are needed. Here a deputy manager could mean a consultant, a secretary, or a vice manager.

However, in real situations, the external information arrives at random rates and at random speed. It may arrive at the managerial office any time before or after office hours. Here we are interested in what percent of information a company may lose at any given number of business managers z and time t. We seek a method to compute what quantity of the total information that a company may receive or miss in a infinite period of time, given the function (could be stochastic) governing information growth. Introduce, say, $q(z,t)$ be the percentage of information loss we are interested in. Intuitively q is decreasing in z for fixed t, since more managers will increase the capability of information processing. In the following sections, we first study a discrete case, where z is assumed to be non-negative integers. Under some assumptions, one can see explicit iteration formulas for computing information inefficiency can be obtained. Then we focus on the continuous case, and propose a stochastic differential equation for modeling the percent of information that a company can receive in a specified time duration. The analytical features of the process, including the approximated transitional density functions, are provided. An algorithm is derived for computing the singular integrations appeared in the model. We finished the paper by providing numerical simulations and implications.

2 Discrete Information Processing

Let's descretize the time interval $[0, T_0]$ by a vector of points $(t_1, t_2, t_3, ..., t_k, ...)$. Since the inflow of information is of stochastic nature in both time and quantity,

*Department of Mathematics, University of Delaware, Newark, DE 19716, USA. Email: dxie@math.udel.edu
†Katz Graduate School of Business, University of Pittsburgh, Pittsburgh, PA 15260, USA. Email: nguyen@pitt.edu

so we first introduce a two dimensional random vector $\xi = (\xi_1, \xi_2)$, the sample space of which is $[0,\infty) \times [0,\infty)$, where $\xi_1 = a \in [0,\infty)$ means information inflow occurs at time a, and $\xi_2 = b \in [0,\infty)$ means the quantity of information inflow is b. Define by $\rho(\tau, x) = 1 - \bar{\rho}(\tau, x)$ the joint probability density function of ξ, we seek to find a recursive system determining the probability of manager insufficiency at any given time and given number of managers.

For simplicity, we assume that the manager can only start to process the information at discrete times $t = t_1, t_2, t_3$, etc. This assumption is justified for the following reasons. First, managers can not make decisions continuously. Typically, business reports and documentations arrive in the managerial's office at a fixed pace, say, every hour or twice a day. The executives then sign documents, or provide opinions, within fixed but small amount of time. Second, the time needed for decision making itself is negligible compared to the duration between decision making times. In another word, we assume information continuously accumulates while managers' processing is periodic and instant, even though their capacity may not allow them to consume all the information accumulated by those instants. When the number managers needed exceeds the number of managers available, an information loss occurs, thus poses a risk for the company. It is clear that even the information processing speed of the managers is faster than the average speed of information received, information loss still can occur. Additional managers are needed to reduce the information loss.

Suppose there have m managers in the company. Let $\psi(\xi_2(s) \le C(m))$ defines the probability of the first $m, (m \le I)$, managers are needed to accommodate the incoming information at time s. Similarly, $\psi(\xi_2(s) > C(m))$ defines the probability of more than m managers are needed to accommodate the incoming information, hence the probability of information loss if only m managers are hired, at time s. We are interested in recursive formulas for computing $\psi(s, \le m)$ and $\psi(s, > m)$ for $m = 1, 2, ...i, ..., I$. We would like to remark that C could be time dependent, but it suffices to illustrate the main idea using a time homogeneous C. Consider the information loss phenomenon as a stochastic process. Without loss of generality, suppose $s > t \ge 0$. Partition the space $[0,\infty)$ into the union of the the disjoint sets $\Omega_1 = [0, C(1)], \Omega_2 = [C(1), C(2)], ..., \Omega_i = [C(i-1), C(i)], ..., \Omega_{m+1} = [C(m), \infty)$, use the conditional probability density at time t, we have an iterative algorithm for computing the probability of incoming information levels at next time step s:

$$\psi(\xi_2(s) \in \Omega_1) = \sum_{i=1}^{m+1} \psi(\xi_2(t) \in \Omega_i) \int_t^s \int_{\Omega_1} \rho(\tau, x | \xi_2(t) \in \Omega_i) dx d\tau$$

$$\psi(\xi_2(s) \in \Omega_2) = \sum_{i=1}^{m+1} \psi(\xi_2(t) \in \Omega_i) \int_t^s \int_{\Omega_2} \rho(\tau, x | \xi_2(t) \in \Omega_i) dx d\tau$$

$$...$$

$$\psi(\xi_2(s) \in \Omega_j) = \sum_{i=1}^{m+1} \psi(\xi_2(t) \in \Omega_i) \int_t^s \int_{\Omega_j} \rho(\tau, x | \xi_2(t) \in \Omega_i) dx d\tau$$

$$\psi(\xi_2(s) \in \Omega_{m+1}) = \sum_{i=1}^{m+1} \psi(\xi_2(t) \in \Omega_i) \int_t^s \int_{\Omega_{m+1}} \rho(\tau, x | \xi_2(t) \in \Omega_i) dx d\tau$$

When information inflow as a function of x and as a function of t are statistically independent, $\rho(x, t)$ may be expressed as a product of two independent distributions [3]. In this scenario, one can approximate, by standard central limit theorem (see, for instance, [3], [1]), the inflow of information measured in the number of mangers needed for processing such information, by standard normal distribution. In the same spirit, one can approximate the marginal density in t using, say, a Winer process [17]. Higher computational efficiency is expected with these reasonable approximations.

3 Stochastic Modeling Information Process

To have a model more conforming to the real business world, one needs to compute the total percent of available information that a company can actually receive. Only after such information is obtained shall the probability of information loss by managers be meaningful. Let p_t be the percent of all external information received by the company at time $t > 0$, we propose that p_t be governed by the stochastic process

$$dp_t = k(\mu - p_t)dt + \sigma\sqrt{(1-p_t)p_t}dW_t, \qquad (1)$$

where $k, \mu, \sigma > 0$ are constants, and W_t is standard Brownian motion. To compute the total amount percent of information received by the company, one need to evaluate the following integrals:

$$T := E[\int_0^t Q(\tau) p_\tau d\tau], \qquad (2)$$

where $Q(t)$ defines the total information governing the business at time t. Without loss of generality, we assume $Q(\tau)$ follows the stochastic differential equation

$$\frac{dQ(t)}{Q(t)} = rdt + \omega dWt, \qquad (3)$$

where the drift parameter $r > 0$ generates the long term growth and ω measures the volatility of the growth. Known as geometric Brownian motion, the model is widely used in the field of financial option pricing, as

ISBN: 978-988-18210-5-8
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

used for modeling the movement of stock prices [11].

To our best knowledge there is no closed form analytical expression for the transitional probability density function determined by (1). But one can at least use Monte-Carlo simulation to graph the density for given p_0 and t. In Figure 2, the plot on the left is one simulated sample path for the mean-reverting process, and the plot on the right is the interpolated density of function $p(y)$ for $t = 0.1$, starting from $p_0 = 0.5$, using 10,000 simulations. For both plots in Figure 2, the parameter values used for simulations are $k = 1, \mu = 0.5, \sigma = 0.2$, and $dt = 0.01$.

In a complete competitive market (see, [13], for instance) where the companies are so many that each company's market share is very low, it is reasonable to believe that each company's information received is only a very small fraction of the whole set of information. In this case, one can let $p_t \to 0$, and approximate the above process (1) by the famous CIR model [14], i.e.,

$$dp_t = k(\mu - p_t)dt + \sigma\sqrt{p_t}dW_t. \tag{4}$$

The benefit of such an approximation, from mathematical analysis point of view, is that the transitional density for CIR model is explicitly known, i.e.

$$p_t(x) = \gamma\chi_{\lambda,\eta}(\gamma x),$$

where

$$\begin{aligned}
\gamma &= \frac{4k}{\sigma^2(1-e^{-4kt})}, \\
\lambda &= \frac{4k\theta}{\sigma^2}, \\
\eta &= \gamma p_0 e^{-kt},
\end{aligned}$$

and $\chi(a, b)$ denotes the chi-square distribution with parameter a measuring the degrees of freedom and b measuring the concentricity. To illustrate the transitional density, we simulate two numerical plots in Figure 3, one for small $t = 0.1$ and the other for large $t = 10$, using the same parameter values $k = 1, \mu = 0.5, \sigma = 0.2$, and $dt = 0.01$.

4 Numerical Implementation

To numerically compute the total amount of information received by the company defined by the equation (2), an efficient integration quadrature is necessary since the integrand is usually highly singular. Examples of such integrals and relevant quadratures for their evaluations can be found in [11], for instance. When p_t follows CIR model, the right hand side of (2) can be reduced to the integration defined by

$$X(k, l, p, x) := \int_x^\infty y^k e^{-ly} I_p(\sqrt{y})dy, \tag{5}$$

where $l, p \in R$ and $k > 0$ are given constants, and I_p's are usually special mathematical functions determined by underlying processes. We remark this type of integral arises naturally in many physical problems including, for instance, the valuation of fixed rate mortgages where the underlying market interest follows a given stochastic model model [8].

We shall evaluate $X(x)$ in terms of incomplete gamma functions and modified Bessel functions of the first kind. To this end we first introduce some basic facts about incomplete gamma functions and modified Bessel functions. The incomplete gamma function of order α (see [15] and [16], for instance) is defined by

$$\gamma(\alpha, x) = \int_0^x t^{\alpha-1}e^{-t}dt. \tag{6}$$

and the differentiation of the incomplete gamma function gives

$$\frac{d}{dx}\gamma(\alpha, x) = x^{\alpha-1}e^{-x}. \tag{7}$$

The modified Bessel function [15] of the first kind of order ν is defined as

$$I_\nu(x) = \frac{(\frac{x}{2})^\nu}{\sqrt{\pi}\Gamma(\nu+\frac{1}{2})}\int_{-1}^1 (1-t^2)^{\nu-\frac{1}{2}}e^{\pm xt}dt \qquad \nu > -\frac{1}{2}. \tag{8}$$

and it has integral relation of

$$\int_0^x t^{-(\nu-1)}I_\nu(t)dt = x^{-(\nu-1)}I_{\nu-1}(x) - \frac{2^{-(\nu-1)}}{\Gamma(\nu)} \tag{9}$$

and differentiation relation of

$$x^{\frac{\nu-1}{2}}I_{\nu-1}(2\sqrt{x}) = \frac{d}{dx}[x^{\frac{\nu}{2}}I_\nu(2\sqrt{x})] \tag{10}$$

From these differential and integral relations, we have, by integral by parts, that

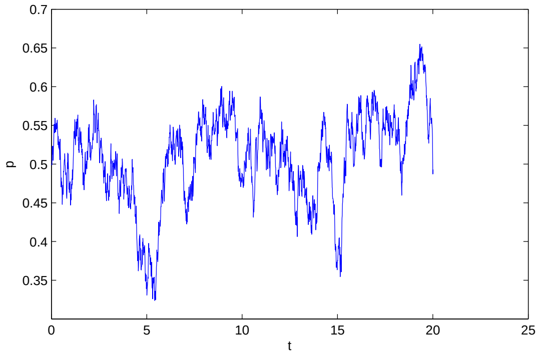
$$\begin{aligned}
X(x) &= \int_0^x y^k e^{-ly}I_p(\sqrt{y})dy \\
&= 4\int_0^{x/4}(4z)^k e^{-4lz}I_p(2\sqrt{z})dz \\
&= 4^{k+1}\int_0^{x/4} z^{k-\frac{p}{2}}e^{-4lz}z^{\frac{p}{2}}I_p(2\sqrt{z})dz \\
&= 4^{k+1}\int_0^{x/4} z^{k-\frac{p}{2}}e^{-4lz}d[z^{\frac{p+1}{2}}I_{p+1}(2\sqrt{z})] \\
&= 4^{k+1}[z^{k-\frac{p}{2}}e^{-4lz}z^{\frac{p+1}{2}}I_{p+1}(2\sqrt{z})]|_0^{x/4} - P
\end{aligned}$$

where

$$P = 4^{k+1}\int_0^{x/4} z^{\frac{p+1}{2}}I_{p+1}(2\sqrt{z})d[z^{k-\frac{p}{2}}e^{-4lz}]$$

$$= 4^{k+1}(k-\frac{p}{2})\int_0^{x/4} z^{\frac{p+1}{2}}I_{p+1}(2\sqrt{z})z^{k-\frac{p}{2}-1}dz$$

ISBN: 978-988-18210-5-8
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)



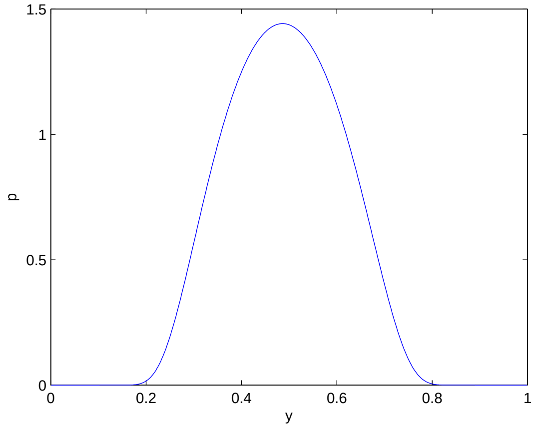


Figure 1: Simulated sample path of p_t and the probability density function at fixed t.

$$-4l4^{k+1}\int_0^{x/4} z^{\frac{p+1}{2}} I_{p+1}(2\sqrt{z}) z^{k-\frac{p}{2}} e^{-4lz} dz$$
$$= 4^{k+1}(k-\frac{p}{2})\int_0^{x/4} z^{k-\frac{1}{2}} I_{p+1}(2\sqrt{z}) dz$$
$$-4^{k+2}l\int_0^{x/4} z^{k+\frac{1}{2}} I_{p+1}(2\sqrt{z}) e^{-4lz} dz,$$

thus we arrive at an recursive relation of X function, on basis of which the integral of arbitrary order of Bessel and expenentials functions can be evaluated.

5 Conclusion and Discussion

A stochastic model of information transfer has been provided in the context of business management. Iterative formulas for computing the probability of information inefficiency have been derived under certain assumptions. A mean reverting stochastic process is proposed to model the percent of information that a company can receive at time t. Both analytical and numerical features of this process are provided. However, the cost pertaining to acquiring the information is not considered, nor is the salary paid to managers. A direction for future research would be to address the limitation by including financial cost in the model.

References

[1] J. M. Keynes, A Treatise On Probability Rough Draft Printing, 2008.

[2] A. Stuart & K. Ord, Advanced Theory of Statistics Wiley, 2009.

[3] C. W. Garidiner, Handbook of Stochastic Methods: For Physics, Chemistry and the Natural Sciences Springer, 1996.

[4] C. Barbara & R. H. McNurlin Information Systems Management in Practice, Prentice Hall PTR, 2001.

[5] G. James, Exploration and Exploitation in Organizational Learning, Organization Science, **2**(1991), 71–87.

[6] M. Dodgson, Organizational Learning: A Review of Some Literatures, Organization Studies, **14**(1993), 375–394.

[7] C. Argyris, Organizational learning and management information systems, ACM SIGMIS Database, **13**(1982), 3–11.

[8] D. Xie, X. Chen & J. Chadam, *Optimal Termination of Mortgages*, European Journal of Applied Mathematics, 2007.

[9] P. G. Huber, Organizational Learning: The Contributing Processes and the Literatures, Organization Science, **2**(1991), 88–115.

[10] P. G. Huber, Information theory and an extension of the maximum likelihood principle, 2nd International Symposium of Information Theory, (1973), 267–281.

[11] X. Chen & J. Chadam, Mathematical analysis of an American put option, SIAM J. Math. Anal., **38** (2007), 1613–1641.

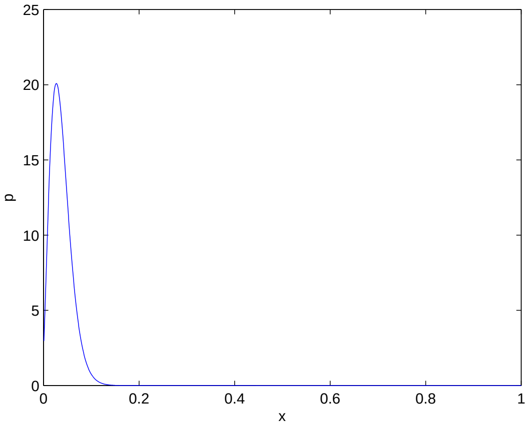
[12] D. T. Sterling, Humanizing Computerized Information Systems, Science, **12**(1975), 1168–1172.

[13] N. G. Mankiw, Macroeconomics, Worth Publishers, 2006.

[14] Cox, John, Ingersoll, Jonathan, & Ross, Stephen, *A theory of the term structure of interest rates*, Econometrica, **53**(1985), 385-407.

[15] A. Jeffrey & D. Zwillinger, *Table of Integrrals, Series, and Products* , 6th Edition.

ISBN: 978-988-18210-5-8
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)



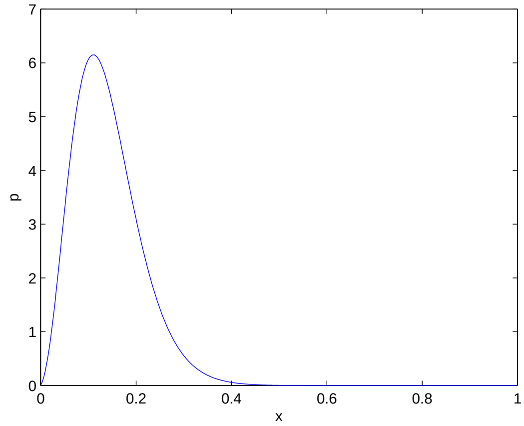


Figure 2: The approximated transitional density functions for small time (left) and large time (right).

[16] J. Spanier & K. B. Oldham, *An Atlas of Functions*
.

[17] B. Oksendal, *Stochastic Differential Equations*, Springer, 5th edition.

ISBN: 978-988-18210-5-8
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)