Production Planning for Parallel Machine Working System using Makespan Determination

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***Abstract*—This work developed the method used to plan the production for parallel machines, aiming to achieve the maximum working efficiency condition. There are two types of machines, regular and special. The same type of machine might work independently and/or dependently depending on the processes required but for the different type, machine will work dependently. The model based on the determination of machine makespan was applied in order to assign the works providing the maximum daily working time or the minimum idle time of each machine. The constrained conditions were determined, based on the nature of the production of interest. The developed method could establish the efficient plan according to the former study and provide the working efficiency being higher than 90%, comparing to original plan having efficiency only 60-70%.**

***Index Terms*— Production Planning, Makespan, Parallel Machine, Production Efficiency**

I. Introduction

The parallel machine working system is defined as the production using the number of machines working at the same time for completing the same or the different work [1][2]. Those machines may work dependently or independently [3][4]. In order to achieve a good working efficiency for a production mainly operated by the machines, it is very important to have a suitable plan. The determination of machine working under the various conditions has been studied widely using the various methods in order to achieve a good efficiency [3]-[8].

The makespan determination has been one condition used to measure the efficiency or the performance of the machine [9]. It is also used to plan the work for several types of the production such as flow shop, job shop, assembly line, parallel machine, etc,. As the makespan is determined in the meaning of the total completed working time, it has been normally used in the manner of how to minimize the machine makespan so that the system could finish the production as fast as possible [5], [10], [11]. The former study [3] has succeeded to plan the work for non-patterned production system using stock cutting model. The more the assignment of the work to machine, the more the working efficiency will be. For one working day, the more the assigned work, the more the work completion will be. To achieve a good daily working efficiency, the system should be able to complete the work as much as possible. In this case, if the work completion is considered in the term of working time; therefore, the planning via the makespan determination in the manner of maximizing the machine working time is interested.

With the nature of the production system of interest, it is a parallel machine which works independently and dependently [1]-[3], [8], [12]. One worker operates one machine. The same process can be performed using more than one machine [3]. Such manner could imply the conditions that (1) the whole processes are brake to sub-process and (2) the sub-processes are assigned to the machine, according to makespan problem [9], [11]. Finally, the plan providing the maximum efficiency of the machine is determined.

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II. Model Development

A. Model Concept

This research aims to construct the production planning model and used it to arrange the work to a machine. Such work means to the process or the sub-process required for completing the product. The working time for each one is specific as it is the standard time. The idea is that each machine will have the same maximum available working time which will be occupied by the time of assigned sub-processes as shown in Fig. 1. The working processes might be dependent and independent, as shown in Fig. 2, based on the product style. In this case, it is an attempt that the sub-processes need to be arranged or assigned to each machine so that the maximum efficiency or the makespan of the machine would be equal to the machine maximum available time.

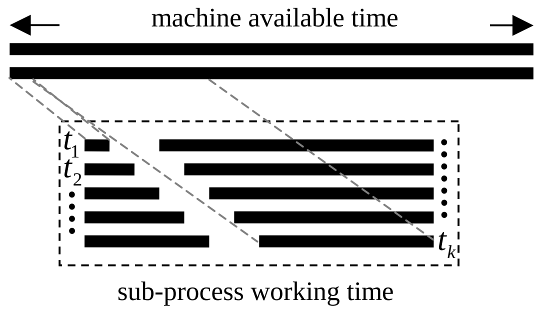

Fig.1 Production Model Diagram

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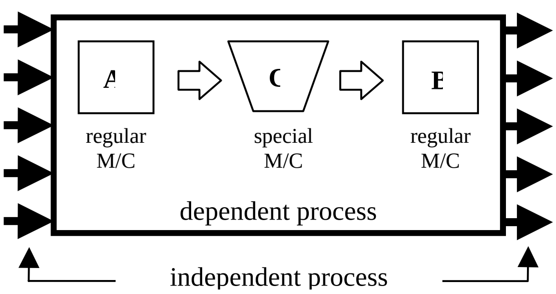

Fig.2 Dependent and Independent Working Diagram

B. Data Preparation

With the nature of production system of interest, there will be a number of processes required to complete one product whereas the method of all processes are the same, sewing method. There are two types of machines including regular and special ones. The processes required might be different or the same when the product style is changed. Applying the comparative working time determination technique, the same the working detail, the same the time consumed will be [14]. In this case, working time data are needed to suitably prepare in the form of processing time data structure (the standard process and its processing time). This research suggests that the whole processes required for all products are clearly declared at first, then, the processing time of each process is collected and prepared in the form of the data file, as shown in Table I. The numbers of "0" and "1" are declared for non-selected and selected processes, respectively. Then, this file will be used as an input data of processes required for the product of interest during production planning calculation.

Table I Working Time Data Table

Process No.	Standard time (sec.)	Product 1	Product 2	Product 3	Product 4
1	40.88	1	1	0	1
2	31.38	1	0	1	1
3	67.91	0	0	1	0
4	89.29	1	1	1	1
5	102.09	1	1	1	1
6	49.81	1	1	1	1
7	108.13	0	0	1	1
8	40.50	0	1	0	0
9	49.75	1	0	0	1
10	63.75	1	1	1	0
11	79.83	1	0	1	0
12	57.91	1	0	1	0
13	124.75	1	0	0	1
14*	68.34	0	1	1	1
15*	74.16	0	1	0	1

* Process 14th and 15th are used in special machine.

C. Model development

The arrangement of the sub-process to each machine is determined. The number of production arranged to the process is used as the decision variable to maximize the utilization, implying the makespan or the efficiency, of the machine [14], [15].

The machine efficiency depends on the process and the number of production assigned to the machine. To achieve the maximum efficiency, the available time need to be replaced by the assigned process as much as possible. From the definition of working efficiency [14], the production efficiency *(Ef)* can be written as in (1).

$$\%Ef = \left(\left(\sum_{k=1}^{K} t_k \cdot N \right) \Big/ TT \right) \times 100 \qquad (1)$$

where t_k is processing time of process k and N is total number of product or order number. Equation (1), then, will be extensively used to develop the planning model as following procedures;

Step1. Total number of machines determination

In reality, there are two categories of production system. The first one is that the work can be performed without sequential required or the machines can work independently. The second one is the sequential required process or the dependent process. In this case, the system will consist of machines working dependently and/or independently. Therefore, the model will be developed based on the latter condition but it is still effective for the first condition.

Each machine has the same daily available working time. The working time of each process is constant as it is the standard time. Therefore, the number of regular and special machines required can be determined using (2) and (3), respectively, under the condition that the available time *(TT)* will be replaced by the working time of each machine within one working day *(WH)*.

$$I = \left\lceil \left(\sum_{k=1}^{K} t_k \cdot p_k \right) \Big/ WH \right\rceil \qquad (2)$$

$$M = \left\lceil \left(\sum_{o=1}^{O} t_o \cdot p_o \right) \Big/ WH \right\rceil \qquad (3)$$

where p_k is total number of production in process k using regular machine

p_o is total number of production in process o using special machines

I is the number of regular machine

M is the number of special machine

t_k is processing time of process k for regular machines

t_o is processing time of process o for special machines

WH is daily working hours

Step2. Planning model

From (1), in order to get the maximum efficiency of the production, the term of output or total completion time, should be maximized. Such condition will be achieved when the total completion time or the makespan is maximized. From the previous discussion, the maximum work should be achieved for both types of machines so that the efficiency of the system would be maximized. To determine the total completion time, one more decision variable is defined. It is the number of production in process assigned to machine. In this case, x_{ik} is the number of production in process k assigned to regular machine i and y_{mo} is the number of production in process o assigned to special machine m. Therefore, the objective can be written as in (4). Then, production planning model can be formulated and solved using linear programming approach [12].

The constraints for the model are determined from the nature of production of interest. Constraints in (5) and (6) are that each machine can not work longer than daily working hours. Equations (7) and (8) represent the constraint that every process must be finished but not necessary found within one machine. The number of production in processes *k* and *o* must equal to an order number (*N*) as constraints in (9) and (10). The sequential required is determined by (11). All productions are an integer as specified in (12). Additional parameters of production planning model are listed below:

- WH = working hours (8 hrs/day or 28800 sec/day)
- N = total number of product or order number.
- K = total number of processes using regular machine
- O = total number of processes using special machine
- p_k = total number of production of each process for regular machines
- p_o = total number of production of each process for special machines
- N = total number of product or order number.

$$\text{Maximize } \sum_{i=1}^{I}\sum_{k=1}^{K} x_{ik}\cdot t_k + \sum_{m=1}^{M}\sum_{o=1}^{O} y_{mo}\cdot t_o \quad \text{,for all } i \text{ and } m \qquad (4)$$

(maximize makespan of total completion time)

Subject to

$$\sum_{k=1}^{K} x_{ik}\cdot t_k \le WH \quad \text{, for all } i \qquad (5)$$

(regular machines can not work longer than working hours)

$$\sum_{o=1}^{O} y_{mo}\cdot t_o \le WH \quad \text{, for all } m \qquad (6)$$

(special machines can not work longer than working hours)

$$\sum_{i}^{I} x_{ik} = p_k \quad \text{, for all } k \qquad (7)$$

(every processes must be finished)

$$\sum_{m}^{M} y_{mo} = p_o \quad \text{, for all } o \qquad (8)$$

(every processes must be finished)

$$p_k = N \quad \text{, for all } k \qquad (9)$$

(number of production must equal an order number for regular machine)

$$p_o = N \quad \text{, for all } o \qquad (10)$$

(number of production must equal an order number for special machine)

$$\sum_{k=1}^{K} x_{ik}^{A} \ge \sum_{k=1}^{K} x_{ik}^{B} \quad \text{, for all } i \qquad (11)$$

(sequential process, process A before process B)

$$x_{ik}, y_{mo} \ge 0 \quad \text{, integer} \qquad (12)$$

III. Computation Results

The results are shown based on the type of working system.

A. Independent work

From Table I, product no.1 requires 10 working processes. For the production order of 734 pieces and from (2), the number of machines required is eighteen. This product requires only regular machines then term of y_{mo} in (4) is zero. The constraints based on (5), (7), (9), and (12) are effective. Then, the model is formulated and provides the plan as shown in Table II. The daily working time and efficiencies of each machine are shown in Fig.2 and Fig.3, respectively.

From Fig.2, the maximum makespan (machine working time) is 28800 seconds or 8 hours. The average makespan of the system is 27708.4 seconds providing the working system having the machine efficiency above 95% averaged which is much better than the production following the general plan (60-70%).

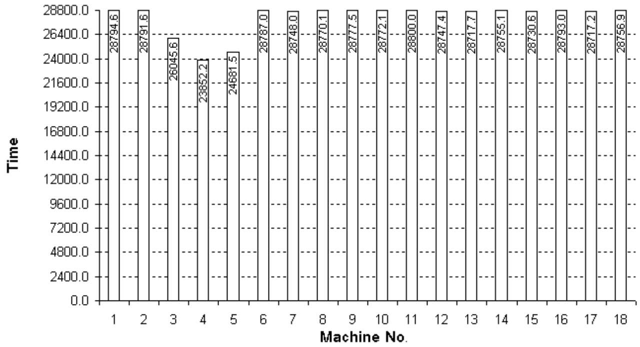

Fig.2 Production time of each machine

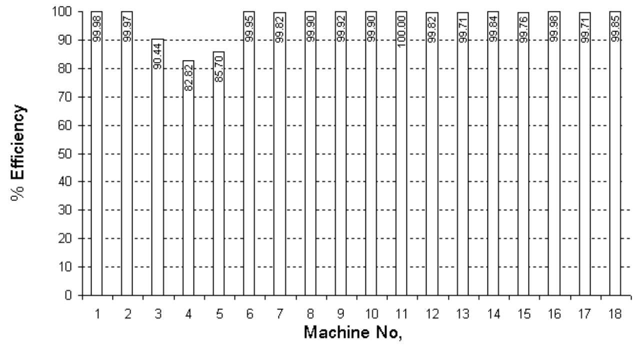

Fig.3 Production efficiency

B. Dependent work

The product no.2 from Table II requires 8 working processes. There are 6 processes using regular machines and 2 processes using special machines. For the production order of 500 pieces and from (2) and (3), the numbers of regular and special machines required are 7 and 3, respectively. From (4) to (12), the model is formulated and provides the plan as shown in Table III. There are sequential requirements that processes no.14 has to be after process no. 1 but before process no.4 and processes no.15 has to be after process no.6 but before process no.8. These imply the dependent processes. The sequences of process are specified using (11). Then, the efficiencies and daily working time of each machine are shown in Fig.4 and Fig.5, respectively.

IV. Discussion

It is found that the plan formulated by the developed model exhibits the possibility that the production system could perform with the efficiency being almost 100% as high as the former work [3] as shown in Fig.6. Plans for the other production conditions, different an order numbers and the number of processes required, are also formulated and reported [16].

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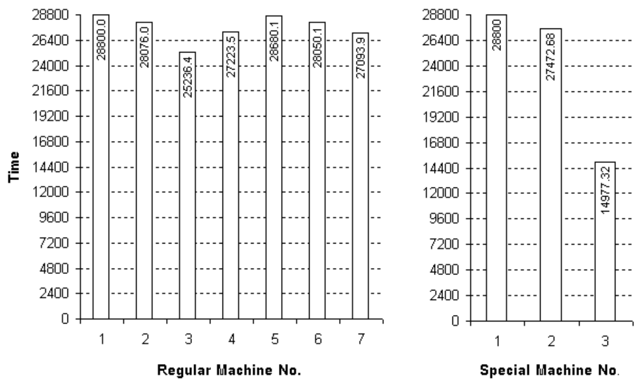
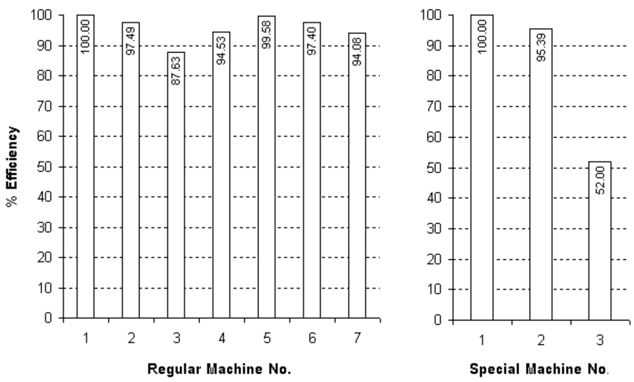


Maximum regular machine time = 28800 sec
Minimum regular machine time = 25236.4 sec
Average regular machine time = 27594.2 sec
Maximum special machine time = 28800 sec
Minimum special machine time = 14977.3 sec
Average special machine time = 23750 sec

Fig.4 Production time of each machine



Maximum regular machine efficiency = 100%
Minimum regular machine efficiency = 87.6%
Average regular machine efficiency = 95.8%
Maximum special machine efficiency = 100%
Minimum special machine efficiency = 52.0%
Average special machine efficiency = 82.4%

Fig.5 Production efficiency

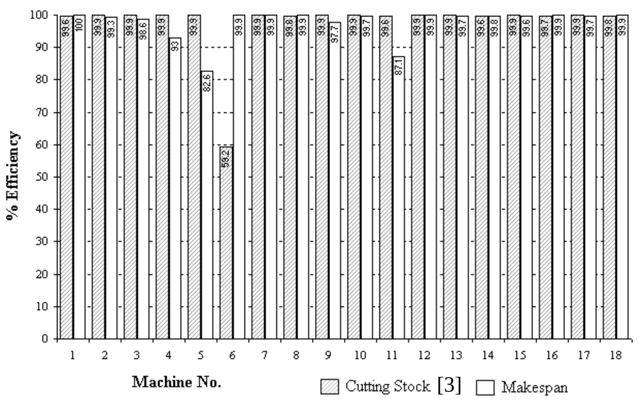


Fig.6 Comparison of production planning model

Dependence and Independence of work might be found within the same or different types of machine. The processes of the product of interest mostly require both dependent and independent work. Some requires completely independent process but rarely for completely dependent one such as shirt collar could be separately sew from shirt waist. From the calculation, the developed model can be applied in completely independent and mixing case. The difference will be the constraints equation. The more the sequences, the more the equations being alike (11) will be considered.

From Tables II and III, it is found that more than one process could be assigned to one machine. The works are distributed in order to achieve the maximum machine utilization. That's why the machine could achieve 100% working efficiency in makespan model. This is the behavior of makespan determination model but not for cutting stock model [3]. Comparing to the method that one process is assigned to one machine mainly, the machine will be idle after finishing the assigned work, then, it has to wait for the residual work from the other machine. The more the distributed work, the more the difficult work preparation might be. In reality, the work distribution could be prepared before the process starts and it is the routine job of an operator for the production of interest. This manner is similar to the production with the small lot size. In this case, the additional benefit is that the production problem could be possibly detected at earlier [17][18].

V. Conclusion and Suggestion

The method used to plan the production of machine, working parallel, is developed. The application of this model could plan the work providing a plan having the efficiency better than the original plan. Even though the planning methods based on makespan and cutting stock could provide high working efficiency plan but the controversy might be that which one would be preferable and when.

The condition of interest is general for the production of interest since the number of order is not too high. The larger the size of enterprise, the larger the order number, the more the number of machines will be. Furthermore, the more the product style and the more the difficult processes will be. The complexity of planning will be increased definitely.

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Table II Production Planning of Product 1 (2.32 minutes)

Machine Number	Process Number (t_k)												
	1	2	3	4	5	6	7	8	9	10	11	12	13
1	198	48	-	63	78	-	-	-	-	-	-	19	19
2	-	-	-	31	-	1	-	-	362	89	-	26	6
3	-	1	-	61	3	-	-	-	22	1	-	-	175
4	1	182	-	11	-	85	-	-	-	58	-	229	6
5	1	9	-	0	53	237	-	-	13	19	78	22	-
6	-	-	-	202	-	213	-	-	-	-	-	-	1
7	138	22	-	2	-	-	-	-	1	-	-	132	117
8	1	15	-	54	-	112	-	-	197	26	29	39	-
9	-	-	-	-	258	-	-	-	2	37	-	-	-
10	-	-	-	1	-	-	-	-	-	-	-	-	230
11	-	-	-	8	32	-	-	-	48	-	154	78	45
12	68	-	-	-	2	-	-	-	-	-	306	-	-
13	206	4	-	-	142	-	-	-	-	-	-	97	1
14	44	-	-	137	28	1	-	-	88	-	-	89	18
15	76	-	-	1	132	-	-	-	-	190	-	-	-
16	1	2	-	163	5	31	-	-	-	116	-	-	-
17	-	450	-	-	-	38	-	-	1	197	-	3	-
18	-	1	-	-	1	16	-	-	-	1	167	-	116

Table III Sequential production planning of product 2 (20.38 minutes)

Regular Machine Number	Process Number (t_k)												
	1	2	3	4	5	6	7	8	9	10	11	12	13
1	102	-	-	102	-	162	-	162	-	14	-	-	-
2	-	-	-	-	147	-	-	-	-	205	-	-	-
3	1	-	-	1	-	278	-	278	-	-	-	-	-
4	82	-	-	82	1	-	-	-	-	258	-	-	-
5	87	-	-	87	170	-	-	-	-	-	-	-	-
6	23	-	-	23	182	59	-	59	-	18	-	-	-
7	205	-	-	205	-	1	-	1	-	5	-	-	-

Special Machine Number	Process Number (t_o)	
	14	15
1	72	322
2	402	-
3	26	178

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