

A Heuristic Procedure for Designing a Distribution Network in a Supply Chain System Using Harmony Search Meta-heuristic and Linear Programming

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Abstract— One of the most important problems in supply chain management is the distribution network design problem system which involves locating production plants and distribution warehouses, and determining the best strategy for distributing the product from the plants to the warehouses and from the warehouses to the customers. Our goal is to select the optimum numbers, locations and capacities of plants and warehouses to open so that all customer demand is satisfied at minimum total costs of the distribution network (including transportation and production costs). Unlike most of past research, our study allows for multiple levels of capacities available to the warehouses and plants. We developed a mixed integer programming model for the problem and solved it by a heuristic procedure which contains 2 sub-procedures. We used harmony-search meta-heuristic as the main procedure and linear programming to solve a transshipment problem as a subroutine at any iteration of the main procedure.

Index Terms— Distribution Network Design, Harmony Search, Mixed Integer Programming, Supply Chain Management, Transshipment Problem.

I. INTRODUCTION

Today supply chain management is one of the most important research areas of interest of researchers who work on applied operational research. The key goal in this field of study is to determine the best strategy to coordinate production, transportation, inventory with the best schedule and minimum possible cost.

Herein we consider the problem of designing a distribution network that involves determining simultaneously the best sites of both plants and warehouses and the best strategy for distributing the product from the plants to the warehouses and from the warehouses to the customers.

The typical objective of such a distribution network may be designing it such that the demands of all customers are satisfied with the minimum of transportation and warehousing cost. The solution must also satisfy the capacity

restrictions of plants and warehouses.

There are a lot of researches about distribution network design, and have been surveyed by

By Francis et al. [5], Aikens [3], Brandeau and Chiu [4], and Avella et al. [2]., Jayaraman [8] studied the capacitated warehouse location problem that involves locating a given number of warehouses to satisfy customer demands for different products. Pirkul and Jayaraman [9] extended the previous problem by considering locating also a given number of plants. They formulated the problem as a mixed integer model and developed a Lagrangean based heuristic solution procedure. The procedure was tested using problem instances with up to 100 customers, 20 potential warehouses and 10 potential plants.

Tragantalermsak et al. [10] considered a two-echelon facility location problem in which the facilities in the first echelon are incapacitated and the facilities in the second echelon are capacitated. The goal is to determine the number and locations of facilities in both echelons in order to satisfy customer demand of the product. They developed a Lagrangean relaxation based branch and bound algorithm to solve the problem and reported results of computational tests with up to 100 customers and 15 facilities. Gourdin et al. [7] studied a particular type of the incapacitated facility location problem where two customers allocated to the same facility are matched. They developed several methods to solve the problem after deriving valid inequalities, and optimality cuts for the problem. One major drawback in most of past research studies like [7–9, 10] is that they limit the number of capacity levels available to each facility to just one. However, as it is the case in practice, there exist usually several capacity levels to choose from for each facility. The use of different capacity levels makes the problem more realistic and, at the same time, more complex to solve. Another major drawback in some previous studies like [8, 9] is that they limit the number of facilities to open to a pre-specified value. Moreover, these studies fail to describe how this value can be determined in advance.

For a more details survey look at M.T. Melo[11].

Our current study has an advantage over past research by presenting a unified model of the problem that includes the numbers, locations, and capacities of both warehouses and plants as decision variables of the model and develops at the same time the optimal distributing strategy from the plants to the warehouses and from the warehouses to the customers.

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The remainder of this paper is organized as follows. In Section 2, a mathematical formulation of the distribution design problem is presented. A two phase heuristic procedure for the problem is proposed in Section 3. It uses harmony search metaheuristic in the first phase and solves a linear program as the next phase. Computational results are reported in Section 4. A summary of the work presented in this paper is given in Section 5.

II. MODEL FORMULATION

The following notation is used in the formulation of the model:

- K index set of customers/customer zones
- W index set of potential warehouse sites
- P index set of potential plant sites
- R index set of capacity levels available to the potential warehouses
- H index set of capacity levels available to the potential plants
- C_{ij} cost of supplying one unit of demand to customer zone i from warehouse at site j
- D_{jk} cost of supplying one unit of demand to warehouse at site j from plant at site k
- F_{rj} fixed cost per unit of time for opening and operating warehouse with capacity level r at site j
- G_{hk} fixed cost per unit of time for opening and operating plant with capacity level h at site k
- a_i demand per unit of time of customer zone i
- b_{rj} capacity with level r for the potential warehouse at site j
- e_{hk} capacity with level h for the potential plant at site k

The decision variables are:

X_{ij} = amount of demand of customer zone i delivered from warehouse at site j

Y_{rjk} = amount of shipment from plant at site k to warehouse at site j with capacity level r

$$U_j^r = \begin{cases} 1, & \text{if a warehouse with capacity level } r \\ & \text{is located at site } j \\ 0, & \text{Otherwise} \end{cases}$$

$$V_k^h = \begin{cases} 1, & \text{if a plant with capacity level } h \\ & \text{is located at site } k \\ 0, & \text{Otherwise} \end{cases}$$

So the mixed integer program would be:

$$\min Z = \sum_{i \in K} \sum_{j \in W} C_{ij} X_{ij} + \sum_{r \in R} \sum_{j \in W} \sum_{k \in P} D_{jk} Y_{rjk} + \sum_{j \in W} \sum_{r \in R} F_j^r U_j^r + \sum_{k \in P} \sum_{h \in H} G_k^h V_k^h$$

Subject to:

$$\sum_{j \in W} X_{ij} = a_i \quad \forall i \in K \quad (1)$$

$$\sum_{i \in K} X_{ij} \leq \sum_{r \in R} b_j^r U_j^r \quad \forall j \in W \quad (2)$$

$$\sum_{r \in R} U_j^r \leq 1 \quad \forall j \in W \quad (3)$$

$$\sum_{i \in K} X_{ij} \leq \sum_{k \in P} \sum_{r \in R} Y_{rjk} \quad \forall j \in W \quad (4)$$

$$\sum_{j \in W} \sum_{r \in R} Y_{rjk} \leq \sum_{h \in H} e_h^k V_h^k \quad \forall k \in P \quad (5)$$

$$\sum_{h \in H} V_k^h \leq 1 \quad \forall k \in P \quad (6)$$

$$X_{ij} \geq 0 \quad (7)$$

$$Y_{rjk} \geq 0 \quad (8)$$

$$U_j^r \in \{0,1\} \quad (9)$$

$$V_k^h \in \{0,1\} \quad (10)$$

The model minimizes total costs made of: the costs to serve the demands of customers from the warehouses, the costs of shipments from the plants to the warehouses, and the costs associated with opening and operating the warehouses and the plants.

The first constraint forces the model to satisfy the demands of all customer zones. Constraint sets (2) and (4) guarantee that the total customer demands satisfied by an open warehouse do not exceed both the capacity of the warehouse and the total shipments to the warehouse from all open plants, respectively. Constraint set (3) and (6) ensures that a warehouse and a plant, respectively, can be assigned at most one capacity level. Constraints in set (5) represent the capacity restrictions of the plants in terms of their total shipments to the warehouses. Finally, constraints in sets (7) and (8) enforce the non-negativity restrictions on the corresponding decision variables and constraints in sets (9) and (10) enforce the integrality restrictions on the binary variables.

III. SOLUTION PROCEDURE

Our proposed algorithm consists of two separate phases. We used harmony search metaheuristic as the main procedure to solve the model. It has been used to find the best possible combination of open and closed plant and warehouses and the level in which they should work. Then at any of the iterations of harmony search, when we determined the value of the binary variables we will solve a simple transshipment problem.

In the other words the main problem will be:

$$\min Z = \text{Transshipment cost} + \sum_{j \in W} \sum_{r \in R} F_j^r U_j^r + \sum_{k \in P} \sum_{h \in H} G_k^h V_k^h$$

Subject to:

$$\sum_{r \in R} U_j^r \leq 1 \quad \forall j \in W \quad (3)$$

$$\sum_{j \in W} \sum_{r \in R} Y_{rjk} \leq \sum_{h \in H} e_h^k V_h^k \quad \forall k \in P \quad (5)$$

$$\sum_{h \in H} V_k^h \leq 1 \quad \forall k \in P \quad (6)$$

$$U_j^r \in \{0,1\} \quad (9)$$

$$V_k^h \in \{0,1\} \quad (10)$$

The transshipment cost is calculated by solving the transshipment problem with the capacity levels determined in the first step. The constraints which do not exist on the

harmony search procedure are included in the transshipment problem.

A. Harmony Search Algorithm

Harmony search is a new meta-heuristic which has newly been presented to solve optimization problems. Like other meta-heuristics it uses some rules to generate new solutions.

New solutions are generated from previous results, with some probability or may be chosen completely random from the all possible solutions (like mutation in GA).

The overall HS procedure would be:

Step 1. Initialize the optimization problem and algorithm parameters.

Step 2. Initialize the harmony memory (HM).

Step 3. Improvise a new harmony from the HM.

Step 4. Update the HM.

Step 5. Repeat Steps 3 and 4 until the termination criterion is satisfied.

In step 3 we have 3 rules. The results are obtained from harmony memory with the probability of HMCR (harmony memory consideration rate) and randomly with the probability of 1-HMCR.

Solutions obtained from harmony memory may be adjusted by the probability of PAR (pitch adjustment rate).

A.1 Initialization of Problem

Harmony memory in this problem consists of a number of vectors. So HM would be a 2 dimensional matrix which has N rows, where N is the size of harmony memory.

each row of is a string which shows the open and close warehouses and plants and the level in which they work. For example fig1 shows an arrangement in which plant number 1 is open and works on the 1st level. The other plants are closed. Warehouses number 1 and 3 are open and work at the levels 3 and 1 respectively and warehouses number 2 and 4 are closed.

1	0	0	3	0	1	0
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Fig1- a sample solution

A.2 Initialization of The Harmony Memory

For initializing the harmony memory we generate some random solution (more than harmony memory size), then we will sort them based on their objective function and keep the best N solution in harmony memory.

A.3 Generating New Solution

There are 3 different ways to generate any of the elements of the new solution.

We choose it randomly from its domain with the probability of 1-HMCR; choose it randomly from the corresponding element of harmony memory with the probability of HMCR. With the probability of PAR we should adjust the element which is generated from HM by increasing or decreasing it by 1 unit.

A.4. Updating the harmony memory

When the solution generated, the open warehouses and plant and their capacity levels are known. So we can now

solve a transshipment problem to find the best solution for this combination of open facilities. We will continue this step and the previous one while some stopping criterion is not satisfied.

IV. COMPUTATIONAL RESULTS

The problem has been coded using Matlab r2009a and the results are shown to be comparable with the solutions obtained by exact algorithms which use branch and bound algorithms. Results are shown in Table 1.

The exact algorithm didn't converge. Only the best solution is compared.

K	W	P	Time(sec)	Gap=%difference between exact solutions and proposed method best solution
6	4	3	>1	0.000
100	5	5	65	1.057*
50	10	10	193	0.738*
100	10	10	269	0.831*

V.CONCLUSION

In this work, we developed an integer program for dealing with the unified problem of finding the number and location of both plants and warehouses in a distribution network and at the same time the best strategy for transportation of goods between them. We developed a heuristic algorithm to solve the problem .the solving procedure included harmony search Metaheuristic and transshipment as its own sub procedures. We then solved some test problems to evaluate the method and found the proposed algorithm to be an accurate and fast one. We used this algorithm for a single product and at one time horizon. The dynamic and multiproduct problems would be a suitable subject for future works.

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