Simulated Annealing in Multifactor Equity Portfolio Management

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***Abstract*—Building a multifactor excess return model to select portfolio stocks has become a widely used tool for portfolio management. The decision to include return factors in such a model varies widely from practitioner to practitioner. Factor weightings in most models are commonly determined by linear regression. The linear regression approach ranks stocks by expected future returns and then selects a portfolio from among the highest ranked stocks. The regression model related to portfolio construction produces a model that is a "best" historic fit to the returns of every stock. No measure of portfolio return or risk can be considered in determining the regression coefficients. In addition to portfolio risk, restrictions on factor weights, portfolio structure, transaction costs, or turnover cannot be easily incorporated in determining the regression model. This thesis considers the portfolio construction model as a global optimization problem, rather than a curve fitting problem. No assumptions about convexity of optimization function $F(\theta)$ are made and, therefore, it may have several local maxima. To control risk throughout a portfolio construction process, this work considers the model's objective function to maximize return divided by risk - the Sharpe ratio. The model is constructed using a ten-year "moving window" of stock market information and then tested on the next year. We then examine the performance of a simulated annealing search and arbitrage pricing theorem to solve this global optimization problem.**

***Index Terms*—Portfolio Management, Simulated Annealing, Arbitrage Pricing Theorem, Global Optimization.**

I. Introduction

Equity portfolio return variance may represent investment risk, which is subdivided into two types: System risk that affects an entire financial market or system, and not just specific participants. It is not possible to avoid system risk through diversification; non-system risk refers to the unique risk of individual securities . The market typically decides system risk which cannot change, but non-system risk may disperse the portfolio theory to form the best portfolio [1]. Any investment entails risk, however investing in diversified securities as an essential strategy will guarantee investment return. The securities investor is typically not able to concentrate all funds in some types of securities, but should simultaneously disperse the investment in many kinds of securities. The portfolio used between investment tools to offset risk, will reduce the non-system risk of investment. Traditional portfolio analysis limits investor decisions, but qualitative analyses explains the benefit of multiple investments, with the goal of maximizing investor's expectations. The consumer who chooses market investments will be under some risk, but investor choice causes expectation return for the greatest portfolio. However, modern investment studies use Markowitz's quantification pattern to establish portfolios. The mean value - variance portfolio pattern of the Markowitz pattern obtains the most effective portfolio by mathematics, and this type of multiplication is called Markowitz Diversification. The investment multiplication idea with the guidance of a specialized manager's investment knowledge can create good investment achievements. Invests the manager when the choice investment, anticipated can under the reasonable risk, obtains the highest return, but non-investment the price bad profit which obtains in the sole stock. Portfolios in the current market operate with each industrial sector equipped with several analysts, which select investments according to their experience of business stock. And plan the transaction with the help of the portfolio manager to achieve a dispersible sector risk effect. Fast computer operational capability may reduce analysis time, cost, and increase portfolio achievements using optimization technology. This paper uses simulated annealing to construct a portfolio model and evaluates performance by the Sharpe ratio. The results serve as references for portfolio managers making decisions.

December 3, 2009.

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II. Motive and goal

The portfolio manager who face thousands of securities, must apply some principles to choose securities worth investing in, and combine these securities effectively, to result in portfolio investment achievements that meet investor needs. These principles typically include: (1) under the same expectation return standard, they can choose small securities risk, (2) under the same investment risk, they can choose higher expectation return securities. The manager must use these principles to form an effective portfolio.

Portfolio analysis deals with the smallest risk under the anticipated return – the effective portfolio. Therefore the effective portfolio refers to a slight investment return variability (risk), and the multiplication method constitutes the best efficiency investment method. The typical effective portfolio includes the abscissa axis risk, and the ordinate axis indicates the return curve (Figure 1). This represents the best risk and return choices for the portfolio, called the Efficient Frontier Curve. Portfolio efficiency higher than this line does not exist, but lower than this line,

ISBN: 978-988-18210-5-8
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

the combination will be inferior to an on-line combination. A portfolio that includes risk receives in exchange the highest return, and perhaps undertakes the smallest risk under a fixed return. This study used the financial ratio (e.g. P/E ratio) for the pattern to change the factor, and used simulated annealing for the mathematical model optimization tool. The results were compared to the Russell 2000 high return and the low risk results.

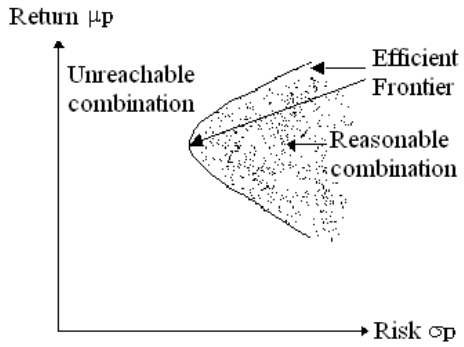

Figure 1. Portfolio efficiency predestined affinity curve

III. RESEARCH SCOPE AND OBJECT

Rolf in 1981 proposed size effect, which explains how small property stock may obtain a high return; gathered a SMALL CAP STOCKS book to explain the small company stock the attraction which invests with a legal person. Russell 2000 specially provided the small property stock investment ratio of the market index for the investor, therefore this research adopted Russell 2000 as a benchmark to conduct research on a 2208 company.

IV. RESEARCH DESIGN

Empirical studies of portfolios may be divided into factor analysis and factor pre-specified methods [2], [3]. The factor analysis method directly takes out the common factor of securities return using the statistical method and simultaneously determines factor sensitivity (weight), to confirm the structure of factors. Because the factors which influence return are numerous, the factor pre-specified method integrates the subset of known financial factors to form the financial model. Compared to the factor analysis method that is unable to include all factors, the factor pre-specified method does not need to collect all factors. This paper used the financial ratios from past scholars to form a mathematic equation.

V. RESEARCH TECHNIQUE

Establishing a multifactor excess return pattern to choose a portfolio is the most popular tool for portfolio management. Many current researches discuss the influence of the return variable. This research used previously obtained variables for this research pattern independent variable, and emphasized using simulated annealing to discover the best pattern, and then adopted the Russell 2000 benchmark as the comparison target, to verify the model's feasibility.

This research used overall stock collection (Universe) as the simulated annealing calculating method study object, elects each stock not necessarily is best, but speaking of the overall stock must be the best choice, emphatically in the whole the return(Overall excess return), also because the individual investor is unable to raise the massive funds to carry on the portfolio, therefore this research pattern quite is suitable provides has the massive fund portfolio manager to do for chooses the stock reference.

Equation 1 shows this research portfolio pattern of a whole optimization question, $f(\theta,\omega)$ is the equation which forms by the complex variable, in which θ is n degree spatial sub-set Θ variable, and is also the direct influence return financial ratio value sub-set, but ω is each variable weight.

$$\max_{\theta\in\Theta} F(\theta) = E(f(\theta,\omega)|\theta\in\Theta), \tag{1}$$

Because we did not have the function pattern geometric figure and the maximum region value number, we also did not know the functions of the smooth degree, the function value, under the function boundary situation. To consider the high return portfolio and give dual attention to the active control risk, we regarded as the excess return mean value dividing variance as the objective function pattern and used simulated annealing as the research technique. The following shows simulated annealing, its principle and how to establish the portfolio pattern.

1. Simulated Annealing

(1) To limit the equation goal,

We use the excess return Sharpe ratio [4],

$$(r_p - r_m)/[\sigma_p^2 + \sigma_m^2 - 2\,\mathrm{cov}(p,m)]^{1/2} \tag{2}$$

And give dual attention to the return and the risk, to reduce investment study error by the manager, in which r_m and σ_m^2 is the constant. To construct a pattern process for effective control risk and coordinate experience principle, we established many limits:

A. all variable weights may depend on user experience to establish the biggest and the minimum value, and the dividend may invest again

B. all variables by the portfolio manager's experience establish its biggest and minimum value

C. each transaction considers the transaction cost

D. any portfolio has the fixed stock number and the investment aggregate amount

E. each sector first buys several stocks and later sells several stocks

(2) Simulated Annealing

Kirkpatrick [5]–[7] and Vecchi proposed simulated annealing in 1983 that currently applies to the industrial world VLSI design, the work research (Operation Research) and the production management (Operational Management)

ISBN: 978-988-18210-5-8
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

and so on in correlation domain optimization, and also calculates this technical application in financial investment.

This study used simulated annealing technology to pursue each variable to the best weight, but chooses this technical the reason is lies in it to have the ability bracelet region best solution to arrive the closed region extreme value. This calculating method design logic is the atom forever can face the energy low direction advance, the high temperature may enable the atom to have the high energy, enlivens in possibly produces the closed region extreme value all value territory, the calculating method penetration temperature gradual drop guides the portfolio return to tend to the closed region extreme value, when each variable weight change all must have a thermal equilibrium to express the present condition, but this thermal equilibrium may use a probability assignment to describe it, this probability assignment is everybody knowledge Boltzmann assignment, this research defines the thermal equilibrium probability assignment for equation 3:

$$P(E) = (1/Z(T)) \cdot e^{(-E/(K_B * T))} \quad (3)$$

Among them,
Z(T) : Standardized constant
E : The definition of condition energy
P(E) : Find a condition probability
T : Temperature
K_B : The Boltzmann constant

The following shows that the evolution type maximization migration probability from Figure 2 (left chart) is 1 or 0, so long as the next condition ΔE is positive, the motion probability of 0, will resist. But the right chart of simulated annealing probability assignment graph is constituted by the curve, even if ΔE is positive (e.g. A) also has a small probability to accept.

Therefore, we use following flow establish the portfolio:

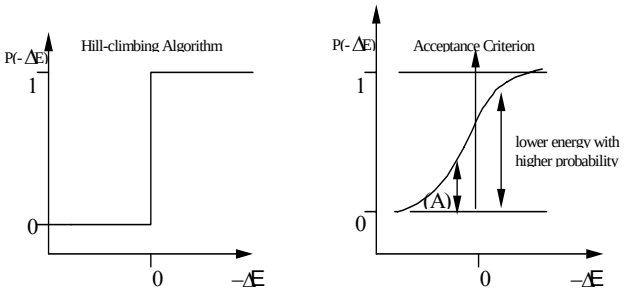

Figure 2. The evolution type maximization calculating method only accepts that ΔE is smaller than or equal to the zero (left chart), but simulated annealing accepts that ΔE is bigger than zero (right chart).

A. New state generation function

To find a new condition, this research used a group of variable initial weights using the simulation formula, aimed at the primitive weight, and the systematized fluctuation each variable weight value. The penetration regional change weight value found the direction of portfolio improvement, and after finding the change direction, further compared the large scale change weight to determine the direction of accuracy. Each change must calculate its portfolio return for this weight combination energy, but during the new change, the old condition of energy disparity will also be the next step to distinguish whether accepts the new condition basis.

B. Accepts a condition

A new weight combination may have a new energy, the new state accepts or defers to the Metropolis calculating method, when the new energy subtracts the old energy content ΔE to be smaller than zero, the motion probability is 1, accepts the new state. If ΔE is bigger than zero, then it must do with the uniform random variable u compares, $e^{-\Delta E/T}$ >u only then may accept the new state, otherwise it resists the new state, and returns to the old state.

C. The cooling procedure

The cooling procedure decides a calculating method potency and convergence rate, and this procedure restrains the pattern gradually in the study. A typical condition temperature must be smaller than the preceding condition temperature, for example: T = T_0/log(1+t) and $T(t) = \frac{T_0}{(1+t)}$ may do for this procedure cooling rule.

D. Pattern execution and confirm

Rolf [8] discovered that no matter whether considering risk, the small company has quite a high excess return, but the company size weight it circulates takes the company in the outside stock market value. Rolf further discovered that for investments in the New York stock exchange stock index (NYSE), the small stock compared to the large-scale stock yielded a 19.8% average return. This research used small stock as the main investment, and used the Russell 2000 stocks as the space collection, to discover the confirmation pattern with the Russell 2000 comparison fit and unfit quality.

E. The portfolio simulation

Company growth speed or the market/book ratio, which grows according to the company, may be divided into value stock and growth stock. Value stock has a low market/book ratio, and belongs to the slow growth industry, for example public utilities and financial organization. But growth stock has the opposite characteristic of high market/book ratio, high P/E ratio, low dividend, high growth and so on. This kind is the fast changing industry, for example the computer company, software company or medicine entrepreneur.

Purchasing value stock has the following advantage: (A) a long time high return, (B) avoids market fluctuation impact, reduces risk. The company with this kind of stock also has the potential of a degenerating or failing industry [9].

The portfolio manager discovers that high growth stock companies typically have more value stock that quickly expands compared to other companies, in the short-term also may provide the high return. The company with a high stock share price also has high risk.

According to above analysis, this study used these financial ratio for the independent variable to establish the

ISBN: 978-988-18210-5-8
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

portfolio model.

F. Russell 2000 as a benchmark

Russell 2000 gives investor's information of the percentage each month each sector should invest. The investor may buy and sell stocks according to each month's announcement suggestion, to obtain a certain standard return. This research used the ratio announced according to Russell 2000 assigned to the fund, but also used simulated annealing to choose the stock. If, after considering risk, this research obtains an overall higher return than benchmark, this may explain simulated annealing technology applied in establishing optimization portfolio validity.

G. Learning and testing method

Time series processing uses the "Moving Window" method to carry on. Figure 3 shows the test time process. Suppose an investment period is half a year; if period one is the learning time then period two is the testing time, if period two is the learning time then period three is the testing time. Each time in this algorithm is continual, to avoiding test interruptions, which creates the tendency to be unable to link up.

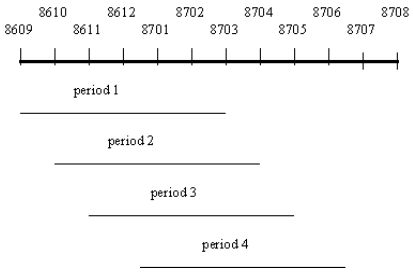

Figure 3. Half year issue of the Moving Window

2. The equity portfolio achievements appraised

Which targets will meet down discusses has to be possible to use for to appraise the portfolio achievements the subject, "the return" will be this important target. If the fund manager only invests the time sector (one-period) in the portfolio, the computation return will be an extremely easy matter, so long as it gains an amount by opening a specified investment amount. But if an investment passes through several time sectors (multi-period), The achievement appraisal must consider the mean average, geometry average return, risk variation and turnover rate. The following aims to obtain what the return represents at each different computing mode.

A. Mean average and geometry average

When the portfolio involves too many investment periods, we can use the average return to examine the achievements of this investment., among "the average" may divide into keeps a promise the average and the geometry average, the usual geometry can be smaller than equally keeps a promise the average, but because the geometry may consider the entire investment period continual return equally, therefore uses it to make the achievements weight quite to be objective, this research various period return computation uses the geometry average return.

B. Time-Weighted Returns and Dollar-Weighted Returns

When we carry on an portfolio, may invest as necessary with will propose the fund, relative will cause this investment the overall return not good to calculate, will have to consider the period question, the break point different will calculate on time the return will also have a difference.

After the time proportion return calculates various return times (various times to gain the first amount again which divides successively each initially investment), it calculates the whole to obtain the average. After the money proportion return is computed, the entire period cash flows the commutation the overall return. The usual money proportion return can the being smaller than time proportion return , that be because the time proportion return has not considered the commutation question, therefore on entire investment period says, the money proportion return is realistic [10].

C. Risk and return computation

General appraisal achievements include three types: (i) the Sharpe pattern, (ii) the Treynor pattern, and (iii)the Jensen pattern. The Jensen achievements only consider the portfolio or the individual securities and the securities city field pattern the vertical range does not integrate the risk factor. But the Sharpe and Treynor targets simultaneously consider the return and risk factors, and may therefore be used to compare fit and unfit quality of the investment achievements. This study therefore used both excess return and risk and the Sharpe Ratio to appraise investment achievements, in which rp and rm represent the portfolio return and the market return (Benchmark return), but σp2、σm2、cov(p,m) are the portfolio risk, the market risk (Benchmark risk) and the covariant.

Sharpe ratio =

$$(r_p - r_m) / [\sigma_p^2 + \sigma_m^2 - 2\,\mathrm{cov}(p,m)]^{1/2}$$

D. Turnover rate

In establishment portfolio process, neighboring two period, sells stock amount dividing retains the stock amount is equal to these two neighboring period cycling rate. The cycling rate may use for to weigh the investment process change the scope, its value higher expression business number of times frequent, short line transaction, increase transaction cost; Opposite, if some manager prepares to pick the long-term investment strategy, then possibly must limit its cycling rate, cannot excessively be high.

VI. DIAGNOSIS AND DISCUSSION

According to Russell (2000) and the simulated annealing, this study makes a diagnosis of the research model, but time series processing uses the Moving Window method , and takes the appraisal pattern fit and unfit quality by the Sharpe Ratio and Turnover.

Table 1 shows that the simulated annealing overall return is higher than Russell 2000. explained this calculating method use in the stock investment feasibility, as for in the identical temperature execution different multiplicity achievements comparison aspect, Epoch

ISBN: 978-988-18210-5-8
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

number of times more experiments by Table to be possible to obtain good Sharpe Ratio and Turnover rate.

The portfolio manager must pay great attention to all stock returns, but we advise that they give attention to less risk and more return. Therefore the portfolio manager can use the portfolio theory to disperse risk. We use factor pre-specified method and simulated annealing under the fixed variable to find the approximate optimization model. According to the diagnosis result, it surpasses the market benchmark in return and risk. The portfolio manager may prefer to maximize the Sharpe ratio rather than the expenditure time of simulated annealing cooling process. We advise building an optimized portfolio model by simulated annealing to obtain more excess return and less risk.

Table 1 Using simulated annealing to establish a portfolio

			Portfolio	Bench-mark	Excess Return	Sharpe Ratio	Turnover
Use Simulated Annealing	EPOCH = 10	one year time	5.00 %	4.41%	0.59 %	N/A	6.65%
		two years time	14.68 %	11.97%	2.71 %	8.725	7.21%
		three years time	19.93 %	15.81%	4.11 %	2.365	5.69%
	EPOCH = 100	one year time	9.49 %	4.41%	5.07 %	N/A	5.17%
		two years time	21.41 %	11.97%	9.44 %	17.784	4.97%
		three years time	18.83 %	15.81%	3.02 %	11.791	4.58%

VII. Conclusion

Section VI of the diagnosis obtains the following conclusions.

1. No matter the investment period length, simulated annealing obtains good achievements. Therefore we suggest that portfolio managers use simulated annealing to form an optimal portfolio.
2. In view of the risk management benefit aspect, the US has many male letter strength also was considered neutral the association can propose the appraisal risk management model benefit method, for example the BARRA association thought an outstanding portfolio manager or the fund manager at present should meet some requirements, following will enumerate the partial condition which this research will have to discuss, takes advantage of this explained this research might enhance invests manager's benefit, and will operate actually in the market, does for this research conclusion.
   A. But the manager undertakes is the expected risk is not the random risk. To obtain the return to have to undertake the risk, however degree of the load must regard should invest research ability and the specialty the manager decides. If the complete risk with will invest policy of the manager to be consistent, but the major part risk comes from the manager not familiar or lacks the reference the domain, the minority risk comes from the manager specialized domain, but then the portfolio achievements performance will depend upon the luck and the non-information, will have like the manager to be ordinary at gambling. But the manager undertakes is the expected risk is not the random risk, but the effective risk management guarantees the risk to be equal to manager's knowledge and ability. We use variance idea, may instantly figure out the investment risk, if too high, may use the simulated annealing and adjust it, invest different company's stock can effective control risk.
   B. The definite idea is consistent with the method. Basically the portfolio manager is supposed to understand clearly the portfolio plan, and then move in order. However his actual behavior often differs from the plan. A good investment pattern should reflect the actual portfolio situation. Investments in a certain time may reflect the portfolio manager's carelessness and errors. This research used a simulation program to suggest which stocks the manager should buy and sell each month, the risk degree and actual figures out which may obtain the return and risk. The computer may reduce investment loss which the artificial mistake creates.
   C. Enhance portfolio achievements. A month of average achievement performance does not represent a good portfolio. Good achievements are produced by continuous high return and low risk. This research emphasizes overall geometry average return, to achieve this goal.
3. We used simulated annealing and the financial ratio to establish a return and risk portfolio pattern, and after comparing with the benchmark, confirmed its feasibility. This study will hopefully be a stock reference for the portfolio manager or the fund manager.

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ISBN: 978-988-18210-5-8
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ISBN: 978-988-18210-5-8
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)