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Interactive Random Fuzzy Two-Level Programming through Possibility-based Fractile Criterion Optimality

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Abstract—**This paper considers two-level linear programming problems where each coefficient of the objective functions is expressed by a random fuzzy variable. A new decision making model is proposed in order to maximize both of possibility and probability with respect to the objective function value. After the original random fuzzy two-level programming problem is reduced to a deterministic one through the proposed model, interactive programming to derive a satisfactory solution for the decision maker at the upper level in consideration of the cooperative relation between decision makers is presented.**

Keywords: random fuzzy variable, cooperative two-level programming, possibility measure, fractile criterion, interactive algorithm

1 Introduction

In the real world, we often encounter situations where there are two decision makers in an organization with a hierarchical structure, and they make decisions in turn or at the same time so as to optimize their objective functions. When we formulate two-level programming problems which closely represents such a real-world decision situation under hierarchical structure, it is often the case that the objective functions and the constraints involve many uncertain parameters.

From a probabilistic point of view, two-level or multilevel programming with random variable coefficients was developed by Nishizaki et al. [13] and Roghanian et al. [15]. Considering the vague nature of the DM's judgments in two-level linear programming, a fuzzy programming approach was first presented by Shih et al. [17] and further studied by Sakawa et al. [16].

Although these studies focused on either fuzziness or randomness included in two-level decision making situations, it is important to realize that simultaneous considerations of both fuzziness and randomness would be required in order to to utilize two-level programming for resolution of conflict in decision making problems in real-world decentralized organizations. For example, when estimating the values of coefficients in problems as random variables, their mean values are estimated as constants using statistical analysis. However, in more realistic cases, the direct use of mean values estimated based on past data may not be appropriate for decision making for future planning. For dealing with such decision making situations, a random fuzzy variable, first defined by Liu [11], draws attention as a new tool for decision making under random fuzzy environments [5, 6, 11, 19].

In general, two-level programming models are classified into two categories; one is a noncooperative model employing the solution concept of Stackelberg equilibrium, and the other is a cooperative model for situations where there exists communication and some cooperative relationship among the decision makers in such as decentralized large firms with divisional independence.

In this paper, we focus on the cooperative case and consider solution methods for decision making problems in hierarchical organizations under random fuzzy environments. A new decision making model is proposed based on the fusion of stochastic programming model and possibilistic programming model in order to maximize both of possibility and probability with respect to the attained objective function value. After showing that the random fuzzy two-level programming problem can be reduced to a deterministic one through the proposed model called possibility-based fractile model, we present an interactive algorithm to derive a satisfactory solution for the decision maker at the upper level in consideration of the cooperative relation between decision makers.

2 Random fuzzy two-level linear programming problems

A random fuzzy variable, first introduced by Liu [11], is defined follows:

Definition 1 (Random fuzzy variable [11]) *A random fuzzy variable is a function ξ from a possibility space $(\Theta, P(\Theta), Pos)$ to collection of random variables R. An*

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n-dimensional random fuzzy vector $n = (\xi_1, \xi_2, \ldots, \xi_n)$ *is an n-tuple ofrandom fuzzy variables* $\boldsymbol{\xi} = (\xi_1, \xi_2, \ldots, \xi_n)$.

Intuitively speaking, a random fuzzy variable is an extended mathematical concept of random variable in the sense that it is defined as a fuzzy set defined on a universal set of random variables. For instance, the random variables with fuzzy mean values are represented with random fuzzy variables. It should be noted here that a random fuzzy variable is not different from a fuzzy random variable [4, 9, 10, 12], which is used to deal with the ambiguity of realized values of random variables, not focusing on the ambiguity of parameters charactezing random variables like random fuzzy variables.

In this paper, we deal with two-level linear programming problems involving random fuzzy variable coefficients in objective functions formulated as:

$$\left.\begin{array}{ll} \underset{\text{for DM1}}{\text{minimize}} & z_1(\boldsymbol{x}_1, \boldsymbol{x}_2) = \bar{\tilde{\boldsymbol{C}}}_{11}\boldsymbol{x}_1 + \bar{\tilde{\boldsymbol{C}}}_{12}\boldsymbol{x}_2 \\ \underset{\text{for DM2}}{\text{minimize}} & z_2(\boldsymbol{x}_1, \boldsymbol{x}_2) = \bar{\tilde{\boldsymbol{C}}}_{21}\boldsymbol{x}_1 + \bar{\tilde{\boldsymbol{C}}}_{22}\boldsymbol{x}_2 \\ \text{subject to} & A_1\boldsymbol{x}_1 + A_2\boldsymbol{x}_2 \leq \boldsymbol{b} \\ & \boldsymbol{x}_1 \geq \boldsymbol{0}, \ \ \boldsymbol{x}_2 \geq \boldsymbol{0} \end{array}\right\}. \quad (1)$$

It should be emphasized here that randomness and fuzziness of the coefficients are denoted by the "dash above" and "wave above" i.e., "$^-$" and "$^\sim$", respectively. In this formulation, $\boldsymbol{x}_1$ is an n_1 dimensional decision variable column vector for the DM at the upper level (DM1), $\boldsymbol{x}_2$ is an n_2 dimensional decision variable column vector for the DM at the lower level (DM2), $z_1(\boldsymbol{x}_1, \boldsymbol{x}_2)$ is the objective function for DM1 and $z_2(\boldsymbol{x}_1, \boldsymbol{x}_2)$ is the objective function for DM2. In (1), $\bar{\tilde{\boldsymbol{C}}}_{lj}$, $l = 1, 2$, $j = 1, 2$ are vectors whose elements $\bar{\tilde{C}}_{ljk}$, $k = 1, 2, \ldots, n_j$ are random fuzzy variables which are normal random variables with ambiguous mean values. In particular, we assume that the probability function of $\bar{\tilde{C}}_{ljk}$ is formally represented with

$$f_{ljk}(z) = \frac{1}{\sqrt{2\pi}\sigma_{ljk}} \exp^{-\frac{(z-\tilde{M}_{ljk})^2}{2\sigma_{ljk}^2}}, \quad (2)$$

where $\tilde{M}_{ljk}$ is an L-R fuzzy number characterized by the following membership function (see Figure 1):

$$\mu_{\tilde{M}_{ljk}}(\tau) = \begin{cases} L\left(\dfrac{m_{ljk} - \tau}{\alpha_{ljk}}\right) & (m_{ljk} \geq \tau) \\ R\left(\dfrac{\tau - m_{ljk}}{\beta_{ljk}}\right) & (m_{ljk} < \tau). \end{cases} \quad (3)$$

Functions L and R are called reference functions or shape functions which are nonincreasing upper semi-continuous functions $[0, \infty) \to [0, 1]$.

Random fuzzy two-level linear programming problems formulated as (1) are often seen in actual decision making situations. For example, consider a supply chain planning where the distribution center (DM1) and the production part (DM2) hope to minimize the distribution cost and the production cost, respectively. Since coefficients of these objective functions are often affected by the economic conditions varying at random, they can be regarded as random variables.

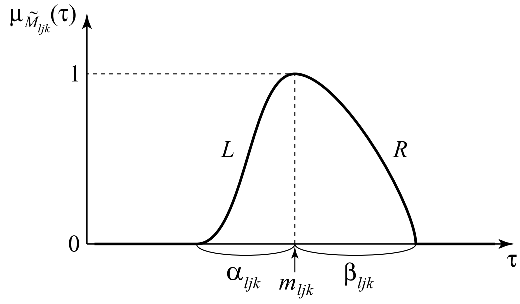

Figure 1: Membership function $\mu_{\tilde{M}_{ljk}}(\cdot)$.

When $\bar{\tilde{C}}_{ljk}$ is a random fuzzy variable characterized by (2) and (3), the membership function of $\bar{\tilde{C}}_j$ is expressed as

$$\mu_{\bar{\tilde{C}}_{ljk}}(\bar{\gamma}_{ljk}) = \sup_{s_{ljk}}\{\mu_{\tilde{M}_{ljk}}(s_{ljk}) | \bar{\gamma}_{ljk} \sim N(s_{ljk}, \sigma_{ljk}^2)\}, \quad (4)$$

where $\bar{\gamma}_{ljk} \in \Gamma$, and Γ is a universal set of normal random variables. Each membership function value $\mu_{\bar{\tilde{C}}_{ljk}}(\bar{\gamma}_j)$ is interpreted as a degree of possibility or compatibility that $\bar{\tilde{C}}_{ljk}$ is equal to $\bar{\gamma}_{ljk}$.

Then, applying the results shown by Liu [11], the objective function $\bar{\tilde{\boldsymbol{C}}}_l\boldsymbol{x}$ is defined as a random fuzzy variable characterized by the following membership function:

$$\begin{aligned} &\mu_{\bar{\tilde{\boldsymbol{C}}}_l\boldsymbol{x}}(\bar{u}_l) \\ &\triangleq \sup_{\bar{\boldsymbol{\gamma}}_l}\left\{ \min_{1\leq k\leq n_j,\ j=1,2} \mu_{\bar{\tilde{C}}_{ljk}}(\bar{\gamma}_{ljk}) \right| \\ &\qquad\qquad \left. \bar{u}_l = \sum_{j=1}^{2}\sum_{k=1}^{n_j} \bar{\gamma}_{ljk}x_{jk} \right\}, \ \forall \bar{u}_l \in Y_l, \quad (5) \end{aligned}$$

where $\bar{\boldsymbol{\gamma}}_l = (\bar{\gamma}_{l11}, \ldots, \bar{\gamma}_{l1n_1}, \bar{\gamma}_{l21}, \ldots, \bar{\gamma}_{l2n_2})$ and Y_l is defined by

$$Y_l = \left\{ \sum_{j=1}^{2}\sum_{k=1}^{n_j} \bar{\gamma}_{ljk}x_{jk} \middle| \bar{\gamma}_{ljk} \in \Gamma, \right\}, \ l = 1, 2.$$

From (4) and (5), we obtain

$$\begin{aligned} &\mu_{\bar{\tilde{\boldsymbol{C}}}_l\boldsymbol{x}}(\bar{u}_l) \\ &= \sup_{\boldsymbol{s}_l}\left\{ \min_{1\leq k\leq n_j,\ j=1,2} \mu_{\tilde{M}_{ljk}}(s_{ljk}) \right| \end{aligned}$$

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$$\left.\bar{\gamma}_{ljk} \sim N(s_{ljk}, \sigma_{ljk}^2),\ \bar{u}_l = \sum_{j=1}^{2}\sum_{k=1}^{n_j} \bar{\gamma}_{ljk} x_{jk} \right\}$$

$$= \sup_{\boldsymbol{s}_l} \left\{ \min_{1\le k\le n_j,\ j=1,2} \mu_{\tilde{M}_{ljk}}(s_{ljk}) \right| \bar{u}_l \sim N\left(\sum_{j=1}^{2}\sum_{k=1}^{n_j} s_{ljk}x_{jk}, \sum_{j=1}^{2}\sum_{k=1}^{n_j} \sigma_{ljk}^2 x_{jk}^2 \right) \Bigg\} \quad (6)$$

where $\boldsymbol{s}_l = (s_{l11}, \ldots, s_{l1n_1}, s_{121}, \ldots, s_{l2n_2})$.

Considering that the mean values of random variables are often ambiguous and estimated as fuzzy numbers using experts' knowledge based on their experiences, the coefficients are expressed by random fuzzy variables. Then, such a supply chain planning problem can be formulated as a two-level linear programming problem involving random fuzzy variable coefficients like (1).

In stochastic programming, basic optimization criterion is to simply optimize the expectation of objective function values or to decrease their fluctuation as little as possible from the viewpoint of stability. In contrast to these types of optimizing approaches, the fractile model or Kataoka's model [8] has been proposed when the decision maker wishes to optimize a permissible level under the guaranteed probability that the objective function value is better than or equal to the permissible level.

On the other hand, in fuzzy programming, possibilistic programming [7] is one of methodologies for decision making under existence of ambiguity of the coefficients in objective functions and constraints.

In this research, we consider a new random fuzzy two-level programming in order to simultaneously optimize both possibility and probability with the attained objective function values of DMs, by extending both viewpoints of stochastic programming and possibilistic programming.

To begin with, let us express the probability $\Pr\left(\omega \middle| \tilde{\bar{\boldsymbol{C}}}_l(\omega)\boldsymbol{x} \le f_l\right)$ as a fuzzy set $\tilde{P}_l$ and define the membership function of $\tilde{P}$ as follows:

$$\mu_{\tilde{P}_l}(p_l) = \sup_{\bar{u}_l} \left\{ \mu_{\tilde{\bar{C}}_l\boldsymbol{x}}(\bar{u}_l) \middle| p_l = \Pr(\omega | \bar{u}_l(\omega) \le f_l) \right\}. \quad (7)$$

From (6) and (7), we obtain

$$\mu_{\tilde{P}_l}(p_l)$$
$$= \sup_{\boldsymbol{s}_l} \min_{1\le k\le n_j,\ j=1,2} \left\{ \mu_{\tilde{M}_{ljk}}(s_{ljk}) \middle| p_l = \Pr(\omega|\bar{u}_l(\omega) \le f_l),\ \bar{u}_l \sim N\Big(\sum_{j=1}^{2}\sum_{k=1}^{n_j} s_{ljk}x_{jk}, \sum_{j=1}^{2}\sum_{k=1}^{n_j} \sigma_{ljk}^2 x_{jk}^2 \Big) \right\}.$$

Assuming that each of DMl, $l = 1, 2$ has a goal G_l for the attained probability expressed as "$\tilde{P}_l$ should be greater than or equal to some value $\hat{p}_l$. Then, the possibility that $\tilde{P}_l$ satisfies the goal G_l is expressed as

$$\Pi_{\tilde{P}_l}(G_l) \stackrel{\triangle}{=} \sup_{p_l}\{\mu_{\tilde{P}_l}(p_l) | p_l \ge \hat{p}_l\}.$$

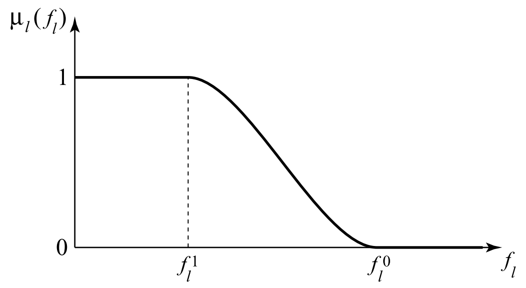

Figure 2: Membership function $\mu_l(\cdot)$.

Then, as a new optimization model in two-level programming problems with random fuzzy variables, we consider the following possibility-based fractile model:

$$\left.\begin{array}{ll} \underset{\text{for DM1}}{\text{maximize}} & \mu_1(f_1) \\ \underset{\text{for DM2}}{\text{maximize}} & \mu_2(f_2) \\ \text{subject to} & \Pi_{\tilde{P}_1}(G_1) \ge h_1 \\ & \Pi_{\tilde{P}_2}(G_2) \ge h_2 \\ & A_1\boldsymbol{x}_1 + A_2\boldsymbol{x}_2 \le \boldsymbol{b} \\ & \boldsymbol{x}_1 \ge \boldsymbol{0},\ \boldsymbol{x}_2 \ge \boldsymbol{0} \end{array}\right\} \quad (8)$$

where h_1 and h_2 are constants. Functions μ_1 and μ_2 are the membership functions of fuzzy goals for the aspiration levels f_1 and f_2, respectively, which reflects the vagueness in human judgments for the aspiration levels (see Figure 2).

Then we can derive the following theorem:

Theorem 1 *Let T denotes the distribution function of $N(0,1)$ and $K_{\hat{p}_l} \stackrel{\triangle}{=} T^{-1}(\hat{p}_l)$. Then, $\Pi_{\tilde{P}_l}(G_l) \ge h_l$ in problem (8) is equivalently transformed into*

$$\sum_{j=1}^{2}\sum_{k=1}^{n_j}\{m_{ljk} - L^*(h_l)\alpha_{ljk}\}x_{jk} + K_{\hat{p}_l}\sqrt{\sum_{j=1}^{2}\sum_{k=1}^{n_j}\sigma_{ljk}^2 x_{jk}^2} \le f_l$$

where $L^*(h_l)$ is a pseudo inverse function defined as $L^*(h_l) = \sup\{t | L(t) \ge h_l\}$.

Proof: The constraint $\Pi_{\tilde{P}_l}(G_l) \ge h_l,\ l = 1, 2$ in problem (8) is equivalently transformed as follows:

$$\begin{aligned} & \Pi_{\tilde{P}_l}(G_l) \ge h_l \\ \Leftrightarrow\ & \exists p_l :\ \mu_{\tilde{P}_l}(p_l) \ge h_l,\ p_l \ge \hat{p}_l \\ \Leftrightarrow\ & \exists p_l :\ \sup_{\boldsymbol{s}_l} \min_{1\le k\le n_j,\ j=1,2} \mu_{\tilde{M}_{ljk}}(s_{ljk}) \ge h_l, \end{aligned}$$

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$$
\begin{aligned}
& p_l \geq \hat{p}_l,\ p_l = \Pr\left(\omega | \bar{u}_l(\omega) \leq f_l\right), \\
& \bar{u}_l \sim N\Big(\sum_{j=1}^{2}\sum_{k=1}^{n_j} s_{ljk}x_{jk}, \sum_{j=1}^{2}\sum_{k=1}^{n_j}\sigma_{ljk}^2 x_{jk}^2\Big) \\
\Leftrightarrow\ & \exists \boldsymbol{s}_l,\ \exists \bar{u}_l :\ \min_{1\leq k\leq n_j,\ j=1,2} \mu_{\tilde{M}_{ljk}}(s_{ljk}) \geq h_l, \\
& \bar{u}_l \sim N\left(\sum_{j=1}^{2}\sum_{k=1}^{n_j} s_{ljk}x_{jk}, \sum_{j=1}^{2}\sum_{k=1}^{n_j}\sigma_{ljk}^2 x_{jk}^2\right), \\
& \Pr\left(\omega | \bar{u}_l(\omega) \leq f_l\right) \geq \hat{p}_l \\
\Leftrightarrow\ & \exists \boldsymbol{s}_l,\ \exists \bar{u}_l :\ \mu_{\tilde{M}_{ljk}}(s_{ljk}) \geq h_l, \\
& \bar{u}_l \sim N\left(\sum_{j=1}^{2}\sum_{k=1}^{n_j} s_{ljk}x_{jk}, \sum_{j=1}^{2}\sum_{k=1}^{n_j}\sigma_{ljk}^2 x_{jk}^2\right), \\
& \Pr\left(\omega | \bar{u}_l(\omega) \leq f_l\right) \geq \hat{p}_l \\
\Leftrightarrow\ & \exists \bar{u}_l :\ \Pr\left(\omega | \bar{u}_l(\omega) \leq f_l\right) \geq \hat{p}_l, \\
& \bar{u}_l \sim N\left(\sum_{j=1}^{2}\sum_{k=1}^{n_j} \{m_{ljk} - L^*(h_l)\alpha_{ljk}\}x_{jk},\right. \\
& \qquad\qquad \left.\sum_{j=1}^{2}\sum_{k=1}^{n_j}\sigma_{ljk}^2 x_{jk}^2\right) \\
\Leftrightarrow\ & T\left(\frac{f_l - \sum_{j=1}^{2}\sum_{k=1}^{n_j} \{m_{ljk} - L^*(h_l)\alpha_{ljk}\}x_{jk}}{\sqrt{\sum_{j=1}^{2}\sum_{k=1}^{n_j}\sigma_{ljk}^2 x_{jk}^2}}\right) \geq \hat{p}_l \\
\Leftrightarrow\ & \sum_{j=1}^{2}\sum_{k=1}^{n_j} \{m_{ljk} - L^*(h_l)\alpha_{ljk}\}x_{jk} \\
& \qquad + K_{\hat{p}_l}\sqrt{\sum_{j=1}^{2}\sum_{k=1}^{n_j}\sigma_{ljk}^2 x_{jk}^2} \leq f_l
\end{aligned}
$$

where $K_{\hat{p}_l} \stackrel{\triangle}{=} T^{-1}(\hat{p}_l)$ and T denotes the distribution function of $N(0,1)$. In addition, $L^*(h_l)$ is a pseudo inverse function defined as $L^*(h_l) = \sup\{t | L(t) \geq h_l\}$. □

From Theorem 1, problem (8) is transformed into the following problem:

$$
\left.
\begin{array}{ll}
\underset{\text{for DM1}}{\text{maximize}} & \mu_1(f_1) \\
\underset{\text{for DM2}}{\text{maximize}} & \mu_2(f_2) \\
\text{subject to} & \sum_{j=1}^{2}\sum_{k=1}^{n_j} \{m_{ljk} - L^*(h_l)\alpha_{ljk}\}x_{jk} \\
& \quad + K_{\hat{p}_l}\sqrt{\sum_{j=1}^{2}\sum_{k=1}^{n_j}\sigma_{ljk}^2 x_{jk}^2} \leq f_l,\ l = 1,2 \\
& A_1\boldsymbol{x}_1 + A_2\boldsymbol{x}_2 \leq \boldsymbol{b} \\
& \boldsymbol{x}_1 \geq \boldsymbol{0},\ \boldsymbol{x}_2 \geq \boldsymbol{0}
\end{array}
\right\} \quad (9)
$$

or equivalently

$$
\left.
\begin{array}{ll}
\underset{\text{for DM1}}{\text{maximize}} & \mu_1\left(\sum_{j=1}^{2}\sum_{k=1}^{n_j} \{m_{1jk} - L^*(h_1)\alpha_{1jk}\}x_{jk} + K_{\hat{p}_1}\sqrt{\sum_{j=1}^{2}\sum_{k=1}^{n_j}\sigma_{1jk}^2 x_{jk}^2}\right) \\
\underset{\text{for DM2}}{\text{maximize}} & \mu_2\left(\sum_{j=1}^{2}\sum_{k=1}^{n_j} \{m_{2jk} - L^*(h_2)\alpha_{2jk}\}x_{jk} + K_{\hat{p}_2}\sqrt{\sum_{j=1}^{2}\sum_{k=1}^{n_j}\sigma_{2jk}^2 x_{jk}^2}\right) \\
\text{subject to} & A_1\boldsymbol{x}_1 + A_2\boldsymbol{x}_2 \leq \boldsymbol{b} \\
& \boldsymbol{x}_1 \geq \boldsymbol{0},\ \boldsymbol{x}_2 \geq \boldsymbol{0}
\end{array}
\right\} \quad (10)
$$

where $K_{\hat{p}_l} \geq 0$ because $\hat{p}_l \geq 1/2$. For simplicity, we express

$$
Z_l^F(\boldsymbol{x}_1, \boldsymbol{x}_2) = \sum_{j=1}^{2}\sum_{k=1}^{n_j} \{m_{ljk} - L^*(h_l)\alpha_{ljk}\}x_{jk} + K_{\hat{p}_l}\sqrt{\sum_{j=1}^{2}\sum_{k=1}^{n_j}\sigma_{ljk}^2 x_{jk}^2},\ l = 1,2 \quad (11)
$$

Now we construct the following interactive algorithm to derive a satisfactory solution for the decision maker at the upper level in consideration of the cooperative relationships between DM1 and DM2.

Interactive random fuzzy two-level programming though the possibility-based fractile model

Step 1 In order to calculate the individual minimum and maximum of f_1 and f_2, solve the following problems:

$$
\left.
\begin{array}{ll}
\text{minimize} & Z_l^F(\boldsymbol{x}_1, \boldsymbol{x}_2) \\
\text{subject to} & A_1\boldsymbol{x}_1 + A_2\boldsymbol{x}_2 \leq \boldsymbol{b} \\
& \boldsymbol{x}_1 \geq \boldsymbol{0},\ \boldsymbol{x}_2 \geq \boldsymbol{0}
\end{array}
\right\},\ l = 1,2 \quad (12)
$$

$$
\left.
\begin{array}{ll}
\text{maximize} & Z_l^F(\boldsymbol{x}_1, \boldsymbol{x}_2) \\
\text{subject to} & A_1\boldsymbol{x}_1 + A_2\boldsymbol{x}_2 \leq \boldsymbol{b} \\
& \boldsymbol{x}_1 \geq \boldsymbol{0},\ \boldsymbol{x}_2 \geq \boldsymbol{0}
\end{array}
\right\},\ l = 1,2. \quad (13)
$$

Let $\boldsymbol{x}_{l,\min}$, $\boldsymbol{x}_{l,\max}$, $Z_{l,\min}^F$ and $Z_{l,\max}^F$ be the optimal solution to (12), that to (13), the minimal objective function value to (12) and the maximal objective function value to (13), respectively. Observing that (12) and (13) are convex programming problems, they can be easily solved by some convex programming technique like sequential quadratic programming methods [3].

Step 2 Ask DMs to specify membership functions $\mu_l(\cdot)$, $l = 1,2$ by considering the obtained values of $z_{l,\min}^F$ and $z_{l,\max}^F$, $l = 1,2$.

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Step 3 The following maximin problem is solved for obtaining a solution which maximizes the smaller degree of satisfaction between those of the two decision makers:

$$\left.\begin{array}{rl} \text{maximize} & \min\left\{\mu_1\left(Z_1^F(\boldsymbol{x})\right), \mu_2\left(Z_2^F(\boldsymbol{x})\right)\right\} \\ \text{subject to} & A_1\boldsymbol{x}_1 + A_2\boldsymbol{x}_2 \le \boldsymbol{b} \\ & \boldsymbol{x}_1 \ge \boldsymbol{0},\ \boldsymbol{x}_2 \ge \boldsymbol{0} \end{array}\right\}$$

or equivalently,

$$\left.\begin{array}{rl} \text{maximize} & v \\ \text{subject to} & \mu_1\left(Z_1^F(\boldsymbol{x})\right) \ge v \\ & \mu_2\left(Z_2^F(\boldsymbol{x})\right) \ge v \\ & A_1\boldsymbol{x}_1 + A_2\boldsymbol{x}_2 \le \boldsymbol{b} \\ & \boldsymbol{x}_1 \ge \boldsymbol{0},\ \boldsymbol{x}_2 \ge \boldsymbol{0} \end{array}\right\}. \qquad (14)$$

In view of (11), problem (14) is rewritten as:

$$\left.\begin{array}{rl} \text{maximize} & v \\ \text{subject to} & \displaystyle\sum_{j=1}^{2}\sum_{k=1}^{n_j}\{m_{1jk} - L^*(h_1)\alpha_{1jk}\}x_{jk} \\ & \displaystyle\qquad +K_{\hat{p}_1}\sqrt{\sum_{j=1}^{2}\sum_{k=1}^{n_j}\sigma_{1jk}^2 x_{jk}^2} \le \mu_1^*(v) \\ & \displaystyle\sum_{j=1}^{2}\sum_{k=1}^{n_j}\{m_{2jk} - L^*(h_2)\alpha_{2jk}\}x_{jk} \\ & \displaystyle\qquad +K_{\hat{p}_2}\sqrt{\sum_{j=1}^{2}\sum_{k=1}^{n_j}\sigma_{2jk}^2 x_{jk}^2} \le \mu_2^*(v) \\ & A_1\boldsymbol{x}_1 + A_2\boldsymbol{x}_2 \le \boldsymbol{b} \\ & \boldsymbol{x}_1 \ge \boldsymbol{0},\ \boldsymbol{x}_2 \ge \boldsymbol{0} \end{array}\right\}. \qquad (15)$$

Obtaining the optimal value of v to this problem is equivalent to finding the maximum of v so that the set of feasible solutions to (15) is not empty. Although this problem is a nonlinear nonconvex programming problem, we can find the maximum of v by the solution algorithm for convex feasible problems [1] since the constraints of (15) are convex if v is fixed.

Step 4 DM1 is supplied with the current values of $\mu_1\left(Z_1^F(\boldsymbol{x}^*)\right)$and $\mu_2\left(Z_2^F(\boldsymbol{x}^*)\right)$ for the optimal solution $\boldsymbol{x}^*$ calculated in step 3. If DM1 is satisfied with the current membership function values, the interaction process is terminated. If DM1 is not satisfied and desires to update h_l, $l = 1, 2$, ask DM1 to update h_l and return to step 3. Otherwise, ask DM1 to specify the minimal satisfactory level $\hat{\delta}$ for $\mu_1\left(Z_1^F(\boldsymbol{x})\right)$ and the permissible range $[\Delta_{\min}, \Delta_{\max}]$ of the ratio of membership functions $\Delta = \mu_2\left(Z_2^F(\boldsymbol{x})\right) / \mu_1\left(Z_1^F(\boldsymbol{x})\right)$.

Observe that the larger the minimal satisfactory level is assessed, the smaller the DM2's satisfactory degree becomes. Consequently, in order to take account of the overall satisfactory balance between both decision makers, DM1 needs to compromise with DM2 on DM1's own minimal satisfactory level. To do so, the permissible range of the ratio of the satisfactory degree of DM2 to that of DM1 is helpful.

Step 5 For the specified value of $\hat{\delta}$, solve the following convex programming problem to maximize the membership function of DM2, $\mu_2\left(Z_2^F(\boldsymbol{x})\right)$ under the constraint that the membership function of DM1, $\mu_1\left(Z_1^F(\boldsymbol{x})\right)$, must be greater than or equal to $\hat{\delta}$.

$$\left.\begin{array}{rl} \text{minimize} & \displaystyle\sum_{j=1}^{2}\sum_{k=1}^{n_j}\{m_{2jk} - L^*(h_2)\alpha_{2jk}\}x_{jk} \\ & \displaystyle\qquad +K_{\hat{p}_2}\sqrt{\sum_{j=1}^{2}\sum_{k=1}^{n_j}\sigma_{2jk}^2 x_{jk}^2} \\ \text{subject to} & \displaystyle\sum_{j=1}^{2}\sum_{k=1}^{n_j}\{m_{1jk} - L^*(h_1)\alpha_{1jk}\}x_{jk} \\ & \displaystyle\qquad +K_{\hat{p}_1}\sqrt{\sum_{j=1}^{2}\sum_{k=1}^{n_j}\sigma_{1jk}^2 x_{jk}^2} \le \mu_1^{-1}(\hat{\delta}) \\ & \boldsymbol{x} \in X \end{array}\right\} \qquad (16)$$

For the optimal solution $\boldsymbol{x}^*$ to (16), calculate $\mu_1\left(Z_1^F(\boldsymbol{x}^*)\right)$ (the satisfactory degree of DM1), $\mu_2\left(Z_2^F(\boldsymbol{x}^*)\right)$ (the satisfactory degree of DM2) and Δ.

Step 6 DM1 is supplied with the current values of $\mu_1\left(Z_1^F(\boldsymbol{x}^*)\right)$, $\mu_2\left(Z_2^F(\boldsymbol{x}^*)\right)$ and Δ calculated in step 5. If $\Delta \in [\Delta_{\min}, \Delta_{\max}]$ and DM1 is satisfied with the current membership function values for the optimal solution $\boldsymbol{x}^*$, the interaction process is terminated. Otherwise, ask DM1 to update the possibility level h_l, $l = 1, 2$ or the minimal satisfactory level $\hat{\delta}$, and return to step 5.

In the proposed algorithm, $\Delta_{\min}$ and $\Delta_{\max}$ are usually set to be less than 1 since $\mu_1\left(Z_1^F(\boldsymbol{x}^*)\right)$ should be greater than $\mu_2\left(Z_2^F(\boldsymbol{x}^*)\right)$ because of the priority of DM1. In step 5, if $\Delta < \Delta_{\min}$, i.e., $\mu_1\left(Z_{1\alpha}^F(\boldsymbol{x}^*)\right)$ is much greater than $\mu_2\left(Z_2^F(\boldsymbol{x}^*)\right)$, DM1 will decrease $\hat{\delta}$ to improve $\mu_2\left(Z_2^F(\boldsymbol{x}^*)\right)$ and increase Δ. Otherwise, if $\Delta_{\max} < \Delta$, i.e., $\mu_1\left(Z_1^F(\boldsymbol{x}^*)\right)$ is slightly greater or less than $\mu_2\left(Z_2^F(\boldsymbol{x}^*)\right)$, DM1 will increase $\hat{\delta}$ to improve $\mu_1\left(Z_1^F(\boldsymbol{x}^*)\right)$ and decrease Δ. On the other hand, if DM1 decreases (increases) h_l, $l = 1, 2$, both $\mu_1\left(Z_1^F(\boldsymbol{x}^*)\right)$ and $\mu_2\left(Z_2^F(\boldsymbol{x}^*)\right)$ would increase (decrease). With this observation, it can be expected that desirable values of $\mu_1\left(Z_1^F(\boldsymbol{x}^*)\right)$, $\mu_2\left(Z_2^F(\boldsymbol{x}^*)\right)$ and Δ will be obtained through a series of update procedures of $\hat{\delta}$ and/or h_l, $l = 1, 2$ with DM1.

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3 Conclusion

In this paper, assuming cooperative behavior of the decision makers, interactive decision making methods in hierarchical organizations under random fuzzy environments have been considered. For the formulated random fuzzy two-level linear programming problems, through the introduction of the possibility-based fractile model as a new decision making model, we has shown that the original random fuzzy two-level programming problem is reduced to a deterministic one. In order to obtain a satisfactory solution for the decision maker at the upper level in consideration of the cooperative relation between decision makers, we have presented an interactive algorithm in which each of optimal solutions of all problems can be analytically obtained by some techniques for solving convex programming problems or convex feasibility problems.

We will consider applications of the proposed method to real-world decision making problems in decentralized organizations together with extensions to other stochastic programming models in the near future. Furthermore, we will extend the proposed models and concepts to noncooperative random fuzzy two-level linear programming problems.

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