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A Mathematical Approach for Reducing the Maximum Traction Energy : The Case of Korean MRT Trains

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Abstract—**This paper describes a reduction in the peak traction energy of mass rapid transit (MRT) railways through timetabling. We develop a mixed integer programming (MIP) model that minimizes the maximum traction energy that occurs when trains are running simultaneously. We tried two approaches. In the first approach, we use the commercial MIP solver CPLEX. In the second approach, we propose a heuristic algorithm. We applied both methods to the current daily timetable of the Korea Metropolitan Subway. We determined a feasible solution that results in an improvement of approximately 32% over the current timetable.**

Keywords: Smoothing Peak Energy, Timetabling, Mathematical Model, Heuristic, MRT

Table 1: Monthly electric charges of an MRT railway

Num.	Type	Details	Charges	Ratio
1	Basic Rate	= Peak Power [kW] × Unit Rate [won/kW]	219,355	15.3%
2	Usage Rate	= Power Consumption [kW] × Used Power Unit Rate [won/kW]	219,355	72.6%
3	Allotment	= (Basic Rate + Usage Rate) × Allotment Rate	46,525	3.3%
4	VAT	= (Basic Rate + Usage Rate) × VAT Rate	125,744	8.8%
5	Total	= (1 + 2 + 3 + 4)	1,429,710	100%

1 Introduction

Growing concern about climate change has led to a demand for green energy. As a result, railways are being reevaluated as an environmentally friendly mode of transportation. Mass rapid transit (MRT) railways, which are an important means of public transportation in urban areas, have operational characteristics that include short headways, frequent departures and arrivals, and a shorter powered distance relative to the coasting and braking distance between stations. Therefore, when multiple trains are operating in the same power supply system, the peak power energy is increased. This paper proposes a mathematical approach that can smooth the peak power demand in timetables. Table 1 shows the electricity billing of an MRT railway. The basic rate is about 15% of the monthly power rate, and is related to the maximum power consumption, which is dealt with here. The greater the deviation in the peak power due to operations at a concentrated power consumption, the greater the charge for electricity.

In 2006, the European International Union of Railways (UIC) and 27 institutes began the *Railenergy* Project in order to respond to the rising cost of energy. The goal of the project is to reduce the total energy consumption of the railroad system by 6% by 2020. Of that goal, 2% will be saved in railway operations as a result of energy-efficient driving and timetabling. A number of software packages that can timetable have been developed; however, they are limited to simple calculations of energy consumption.

Many researchers have addressed ways to reduce energy consumption by railroads. In a study of the energy savings with train operations, Albrecht *et al.* [1] studied a way to reduce the peak energy consumption and maximize the regenerative energy by synchronizing braking and powering using the reserve time when running between stations. They proposed a genetic algorithm to do this. Gordon *et al.* [2] presented several strategies for train operation with reduced energy consumption, especially a method that coordinates coasting and the stop and start times of trains. In a study of timetabling, Lindner *et al.* [3] proposed integer programming, which supports cyclic timetabling considering energy costs. Medanic *et al.* [4] developed a discrete-event model that enables the energy-efficient timetabling of freight trains on single track sections. This model supports fast, easy recalculation. In a similar study that adjusts timetables, Kim and Oh [5] presented a mathe-

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matical model that minimizes the number of trains power running at the same time. This study reduced the power running trains by 25% at the peak time, however, it was hard to know how much can save the energy consumption. Chen *et al.* [6] proposed a method that minimizes the peak energy consumption by coordinating the train stop times at each station of MRT railways. Their study classified the stop modes at stations into short (25 s) and long (35 s) dwell time modes, and used genetic algorithms. A simulation of the orange line route of the Kaohsiung MRT, which has 14 stations, showed that there was an approximately 28% energy saving in the maximum traction energy. A one-way journey takes 22.5 minutes, and there is an average headway of 6 to 9 minutes. In the high-density sections where the headway is 2 to 4 minutes, it is difficult to coordinate the dwell time in stations in 5-s steps. In addition, if the journey time differs from the schedule by more than one minute, it is difficult to meet the service requirements, including the train routing plan and crew scheduling. In this paper, we adjust only the starting time by ± 30 s so that the arrival time deviates from the schedule by less than ± 30 s.

This paper is organized as follows. Section 2 defines the timetabling problem to minimize the maximum traction energy. Section 3 formulates the mathematical model and suggests solution approaches. Section 4 presents the results of an experiment examining the current timetable, and Section 5 presents the conclusion and direction of further study.

2 Problem Definition

In this section, to describe a reduction in the peak of traction energy when timetabling, *time slot* and *traction energy* are defined. The *time slot* divides continuous time into discrete 15-second intervals. Time is expressed in discrete units because the existing timetable in Korea is based on a 30-second unit scale and the electric energy consumption over time need not be calculated continuously. Since an analysis of the train speed profile showed that there are sections where the powering time is less than 30 seconds, the unit time interval was set to 15 seconds.

In this paper, the *traction energy* was obtained from a train performance simulator (TPS) developed by the Korea Railroad Research Institute (KRRI). It simulates the speed, distance, and power consumption against time for a single train. Table 2 shows sample TPS results for one train traveling between stations. Generally, the energy consumption of a train can be divided into three phases: the traction phase requires high power (time slot 1); the coast phase requires low or no power (time slot 2, 3); and the deceleration phase may export regenerated brake power (time slot 4). However, any regenerated energy is used mainly for the vehicle cooling-heating system and has a low reuse rate. Therefore, it is reasonable to assume that the regenerated energy is zero.

Table 2: Sample Results of TPS

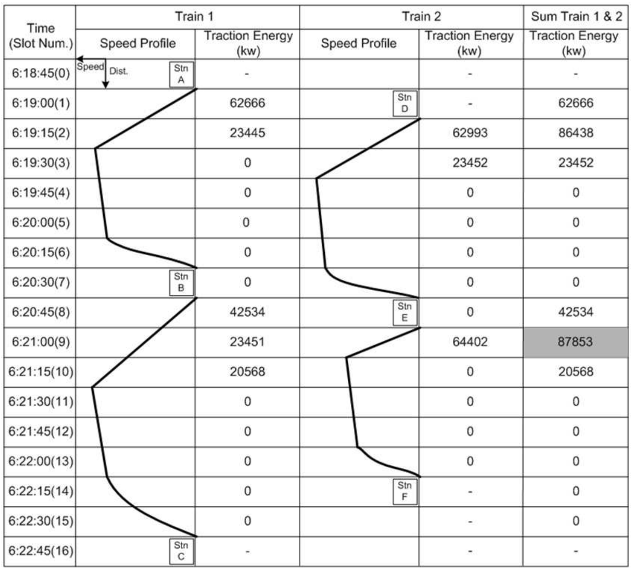
Time Slot	Time(s)	Dist. (m)	Speed(km/h)	Power(kW)
1	0	0	0	0
	1	0.7	4.88	419.5
	2	2.7	9.75	1258.1
	3	6.1	14.61	2096.0
	4	10.8	19.46	2932.9
	5	16.9	24.31	3768.7
	⋮	⋮	⋮	⋮
2	15	143.5	62.04	5943.8
	16	161.0	64.18	5423.4
	17	178.8	63.89	0
	18	196.5	63.54	0
	19	214.1	63.19	0
	20	231.6	62.83	0
	⋮	⋮	⋮	⋮
3	30	401.4	59.41	0
	31	417.8	59.14	0
	32	434.2	58.86	0
	33	450.5	58.59	0
	34	466.8	58.32	0
	35	482.9	58.05	0
	⋮	⋮	⋮	⋮
4	45	638.6	48.81	$-4332.9 \approxeq 0$
	46	651.6	45.31	$-4047.8 \approxeq 0$
	47	663.7	41.81	$-3759.9 \approxeq 0$
	48	674.9	38.31	$-3469.1 \approxeq 0$
	49	685.0	34.81	$-3175.9 \approxeq 0$
	50	694.2	31.31	$-2880.2 \approxeq 0$
	⋮	⋮	⋮	⋮

Table 3: Sample trains

Train	Dpt. Stn.	Arr. Stn.	Dpt. Time(slot)	Arr. Time(slot)
1	A	B	6:19:00 (1)	6:20:15 (6)
	B	C	6:20:45 (8)	6:23:30 (15)
2	D	E	6:19:15 (2)	6:20:30 (7)
	E	F	6:21:00 (9)	6:22:00 (13)

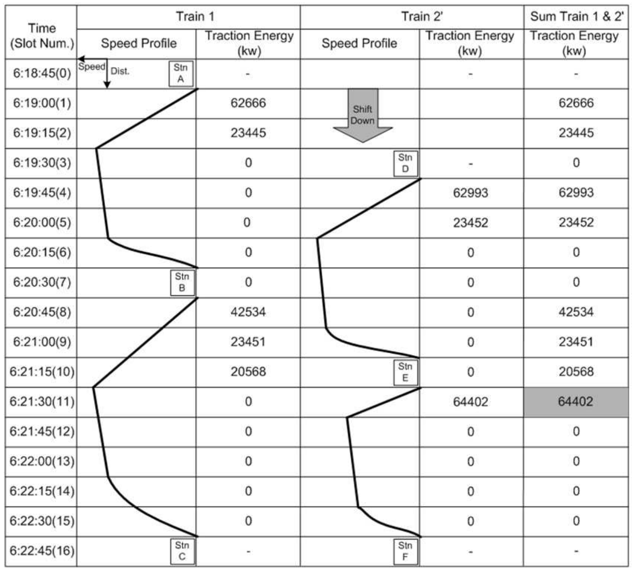
Table 3 shows sample trains in the existing timetable. Two trains use the same electrical power supply simultaneously. For this example, the *traction energy* of the trains in the *time slot* defined above is presented in Fig. 1 for Trains 1 and 2. The maximum traction energy, the sum for the two trains, occurs in *time slot 9*, and it equals $87,353$ kW. In this case, if the starting time of Train 2 from Station D is delayed by 30 seconds, the tps of Train 2 is changed to Train 2', and the maximum traction energy deceases to $64,402$ kW at *time slot 11* (Fig. 2)

The energy-efficient timetabling method proposed here maintains the planned dwell time at a station and the running time between stations, but coordinates the train departure times at the starting station to within ± 30 s



Time (Slot Num.)	Train 1 Speed Profile	Train 1 Traction Energy (kw)	Train 2 Speed Profile	Train 2 Traction Energy (kw)	Sum Train 1 & 2 Traction Energy (kw)
6:18:45(0)	Speed Dist. Stn A	-		-	-
6:19:00(1)		62666	Stn D	-	62666
6:19:15(2)		23445		62993	86438
6:19:30(3)		0		23452	23452
6:19:45(4)		0		0	0
6:20:00(5)		0		0	0
6:20:15(6)		0		0	0
6:20:30(7)	Stn B	0		0	0
6:20:45(8)		42534	Stn E	0	42534
6:21:00(9)		23451		64402	87853
6:21:15(10)		20568		0	20568
6:21:30(11)		0		0	0
6:21:45(12)		0		0	0
6:22:00(13)		0		0	0
6:22:15(14)		0	Stn F	-	0
6:22:30(15)		0		-	0
6:22:45(16)	Stn C	-		-	-

Figure 1: Traction energy from Table 3



Time (Slot Num.)	Train 1 Speed Profile	Train 1 Traction Energy (kw)	Train 2' Speed Profile	Train 2' Traction Energy (kw)	Sum Train 1 & 2' Traction Energy (kw)
6:18:45(0)	Speed Dist. Stn A	-		-	-
6:19:00(1)		62666	Shift Down		62666
6:19:15(2)		23445			23445
6:19:30(3)		0	Stn D	-	0
6:19:45(4)		0		62993	62993
6:20:00(5)		0		23452	23452
6:20:15(6)		0		0	0
6:20:30(7)	Stn B	0		0	0
6:20:45(8)		42534		0	42534
6:21:00(9)		23451		0	23451
6:21:15(10)		20568	Stn E	0	20568
6:21:30(11)		0		64402	64402
6:21:45(12)		0		0	0
6:22:00(13)		0		0	0
6:22:15(14)		0		0	0
6:22:30(15)		0		0	0
6:22:45(16)	Stn C	-	Stn F	-	-

Figure 2: Reducing the maximum traction energy by shifting the train stating time down

(*time slot 2*) to avoid powering in the same time slot. This is because if the times for dwell and running are increased to reduce the peak energy for a high-density traffic line, such as an urban MRT, the quality of the timetable is reduced, reducing the transport capacity and increasing the journey time. Another goal is to follow the current timetable as much as possible, while considering the feasibility (vehicle routing plan, crew scheduling, transport demand, *etc.*) of an energy-efficient timetable. In this method, the state of the train starting time will be one shift up by -30 s, one shift down by +30 s, or maintaining the current timetable, and the traction energy in each time slot will be determined accordingly. Thus, with the current timetable, we determine the *traction energy* ($E_{i,t}$) of train i in *time slot* t, and shift the starting times up ($E^+_{i,t}$) and down ($E^-_{i,t}$).

This is a typical combinatorial optimization problem in which finding the optimum solution becomes much more difficult as the number of trains increases.

3 Mathematical Model and Approaches

3.1 Mathematical model

This section formulates the smoothing of the peak traction energy model in mathematical form. The formulation follows the mixed integer programming model. The notation is defined below.

Sets
- I : train set
- T : time slot set

Parameters

- $E_{i,t}$: traction energy of train i at time t in the current timetable
- $E^+_{i,t}$: traction energy of train i at time t after shifting up
- $E^-_{i,t}$: traction energy of train i at time t after shifting down

Variables
- $y_{i,t} \in R^+$: traction energy of train i in time slot t
- $a_i \in \{0,1\}$: $a_i = 1$ if the starting time of train i is unchanged; otherwise, $a_i = 0$
- $b_i \in \{0,1\}$: $b_i = 1$ if train i is shifted up; otherwise, $b_i = 0$
- $c_i \in \{0,1\}$: $c_i = 1$ if train i is shifted down; otherwise, $c_i = 0$
- $k \in R^+$: maximum traction energy in a time slot

Model

$$\textit{Minimize} \quad k \tag{1}$$

Subject to

$$k \geq \sum_i y_{i,t}, \quad \forall t \in T \tag{2}$$

$$y_{i,t} = E_{i,t} \times a_i + E^+_{i,t} \times b_i + E^-_{i,t} \times c_i, \ \forall t \in T, \ \forall i \in I \tag{3}$$

$$a_i + b_i + c_i \leq 1, \forall i \in I \tag{4}$$

Objective function (1) minimizes the maximum traction energy in the time slot. Constraint (2) calculates the maximum traction energy in a time slot. Constraint (3) determines the traction energy of each train after shifting its starting time up or down, or leaving it in its current slot. Constraint (4) indicates that a train has one of the starting time states.

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3.2 Solution approaches

We propose two approaches for smoothing the peak traction energy. The first approach obtains the optimal solution using CPLEX, which is a commercial optimization software package. Generally, commercial optimization software uses the branch-and-bound method [7]. However, it is difficult to obtain an optimum solution for the mathematical model presented here when the numbers of trains and time slots increase. Therefore, we also consider an approach that uses a heuristic algorithm. The basic idea of the algorithm is to select a train that is powering in a time slot when the maximum traction energy occurs. Next, the sum of the traction energy within ±30 s of the time slot is compared, and the selected train is shifted up or down in the direction where there is less power consumption. Defining the train set as I and the time slot set as T, the notation used in the algorithm is as follows:

- $E_{(i,t)} \in R^+$: traction energy of train i at time t
- $MAXVAL \in R^+$: maximum traction energy before adjusting
- $NEWVAL \in R^+$: maximum traction energy after adjusting
- $MAXTIME \in T$: time slot in which the maximum traction energy occurs
- $UPVAL \in R^+$: sum of the traction energy in the $MAXTIME - 1$ time slot
- $DNVAL \in R^+$: sum of traction energy in the $MAXTIME + 1$ time slot
- $SHIFT_i \in \{0, 1\}$: if train i cannot be shifted, $SHIFT_i = 1$; otherwise, $SHIFT_i = 0$

Details of the algorithm are as follows:
Step 0 : initialize the train shift ($SHIFT_i = 0$, $\forall i \in I$) and calculate $MAXVAL$ and $MAXTIME$
Step 1 : go to Step 2 if there is a train i of $SHIFT_i = 0$. If $SHIFT_i = 1$ and $\forall i \in I$, exit the algorithm.
Step 2 : select the i^* of the trains that are $E_{(i,MAXTIME)} > 0$ and $SHIFT_i = 0$
Step 3 : calculate $UPVAL$ and $DNVAL$. If $UPVAL > DNVAL$, shift down train i^*; otherwise, shift up i^*
Step 4 : calculate $NEWVAL$. If $NEWVAL < MAXVAL$, go to Step 0; otherwise, cancel the shift of i^*, $SHIFT_{i^*} = 1$, and go to Step 1.

4 Numerical Results and Case Study

This section presents the results of numerical experiments for the two approaches described in Section 3. The data for these experiments are derived from an actual Seoul Metro MRT train line. The experiment instance includes 23 stations and 504 trains a day, from 06:08:00 to 01:01:00 the next day, with 4,773 time slots. The model and algorithm are implemented using ILOG CPLEX 11.1 and Visual Studio 2008 on a 2.50-GHz Core2 Quad CPU with 3.50 GB of RAM. Table 4 shows the reduction in the maximum traction energy. CPLEX finds a feasible solution within a 12% optimality gap. The maximum traction energy is 25% and 32% less than with the current timetable for the respective approaches. The average traction energy in a time slot is $200,813$ kW. Both methods reduced the standard deviation of the traction energy more than in the current timetable. This confirms that our idea can smooth the energy peaks effectively.

Table 4: Computational Results

Measure	Current Timetable	Heuristic	CPLEX (opt)
Maximum Traction Energy (kW)	950,360	707,270 (-25.5%)	644,042 (-32.2%)
Std. Num. Traction Energy (kW)	178,085.7	171,263.1	167,101.7
Shifted Trains	-	72	331

Figure 3 compares the traction energy between the current timetable and a new timetable developed with the mathematical model. The analysis shows that our model is effective for both non-peak and peak times. Using CPLEX, 331 trains were shifted and the computation time was 600.86 s, while with the heuristic model, only 72 trains were shifted and the computation time was 253.37 s. These results indicate that our heuristic algorithm is slightly better than commercial software in terms of computation time. For the case in Table 1, the smoothing of the power peaks based on our study of an MRT train would save about $60,000 per month in electricity charges, which is about 5% of the total charge.

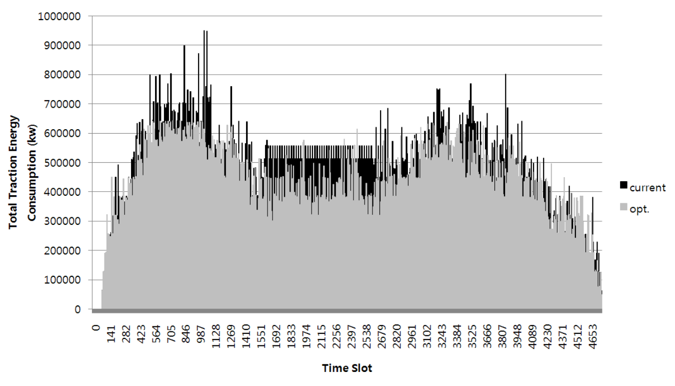

Figure 3: Comparison of the current timetable and the peak-smoothing timetable

5 Conclusion

We propose a mathematical model and a heuristic approach to timetabling to minimize the maximum trac-

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tion energy of trains. We determined a feasible solution that resulted in an improvement of approximately 32% over the current timetable. We demonstrated that our methodology can be applied successfully to energy-efficient timetabling, particularly for a high-density MRT line. Energy-efficient train timetabling can help the MRT company reduce power costs, decreasing the investment required in power facilities.

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