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Topology Optimization of Eddy Current Systems by Level-Set and Primal-Dual Methods

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Abstract—**The paper deals with a topology optimization of electromagnetic media described by the eddy current equations. This problem finds many applications in the design of modern high power electronic devices. The construction of a subtle layout of the devices requires to avoid significant losses in the power transmission due to parasitic inductivities. We consider a minimization problem with an objective function related to the energy dissipation given by the Joule-Lenz law. Our purpose is to find the optimal material distribution in the conductive medium with a prescribed fluxes through the ports. The structural boundary of the design domain changes during the optimization process. A level-set method is proposed for an implicit representation of this boundary. The description includes the evolution of the scalar level-set function and thus, the optimal propagation of the design boundary by solving the Hamilton-Jacobi equation. Another approach to find the optimal material design is to consider the electromagnetic potentials as state variables and the conductivity as a design variable. This formulation gives rise to a nonlinear minimization problem which we solve by the primal-dual approach. Some numerical experiments by using the second approach applied to a two-dimensional isotropic electromagnetic model are presented.**

Keywords: eddy current equations, topology optimization, level-set method, primal-dual approach

1 Introduction

During the past two decades, the problem to find the optimal structural design of material systems and devices has attracted a lot of attention (cf., e.g., [1, 2, 4, 7, 8, 14, 17, 18]). Various techniques and efficient numerical methods have been developed and implemented especially in the field of topology optimization of solid structures, see e.g. [5] and the references therein. Typical for this type of structural optimization, is the possible generating of holes on the domain while finding an optimal placement of material in space.

In this paper, we consider an optimal design of converter modules that are used as electric drives for high power electromotors. The operational mode of such devices is strongly dictated by the Maxwell equations. We focus our attention on the design of electromagnetic media described by the eddy current equations as a particular case of Maxwell's equations in the quasistationary approximation. Usually, eddy currents arise and build up inside the bus bars causing parasitic inductivities that lead to a considerable loss in power transmission. Therefore, our research problem is to find a subtle design of the electric devices in such a way that the energy dissipation is minimized.

Up to our knowledges, the topology optimization of electromagnetic systems described by the Maxwell equations has been relatively less discussed in the literature (see, e.g., [8, 12, 18]). The goal of this study is to present two methods, the level-set and the primal-dual approach, for topology optimization of eddy current systems. The optimization problem is formulated both in a level-set and a primal-dual context.

Section 2 is devoted to the potential formulation of the eddy current equations by means of the electric potential and the magnetic potential. Our objective function focuses on minimization of electric energy dissipation given by the Joule-Lenz law. We are interested in finding the optimal material distribution in a design domain and, in particular, to determine the places with material points and respectively, the voids (no material). In Section 3 we consider a simplified model for which the optimization problem is subjected to a partial differential equation for the electric potential and additional mass and box constraints for the conductivity treated as a design variable.

The level-set method, originally proposed in [11], is described in Section 4 for representing the structural boundaries which move during the optimization to find the optimal shape design. In the recent years, this method has found many applications, especially for elastic structures, cf. [2, 4, 17]. In Section 5 we give a brief description of the primal-dual approach implemented to the discrete optimization problem. The nonlinear equation arising from the first-order necessary optimality conditions is solved by inexact Newton method. In the last section we present some numerical experiments and discuss the computational results.

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2 The eddy current equations

In this section, we focus on electromagnetic fields in the low frequency regime which can be described by the quasistationary limit of Maxwell's equations also known as the eddy current equations

$$\frac{\partial \mathbf{B}}{\partial t} + \mathbf{curl}\,\mathbf{E} = \mathbf{0}, \quad \mathrm{div}\,\mathbf{B} = 0, \quad \mathbf{curl}\,\mathbf{H} = \mathbf{J}, \tag{1}$$

$$\mathbf{B} = \mu\,\mathbf{H}, \quad \mathbf{J} = \sigma\,\mathbf{E}. \tag{2}$$

In the system of equations (1)-(2) we have denoted by $\mathbf{E}$ and $\mathbf{H}$ the electric and the magnetic field and, respectively, by $\mathbf{B}$ and $\mathbf{J}$ the magnetic induction and the current density. The scalar parameters μ and σ refer to the magnetic permeability and the electric conductivity (see, e.g., [10]).

We consider a two-dimensional model with a current density given by

$$\mathbf{J} = (J_1(x_1, x_2, t),\, J_2(x_1, x_2, t),\, 0),$$

which suggests, in particular, the following form of the vector fields $\mathbf{E}$, $\mathbf{H}$, and $\mathbf{B}$

$$\begin{aligned} \mathbf{E} &= (E_1(x_1, x_2, t),\, E_2(x_1, x_2, t),\, 0)\,, \\ \mathbf{H} &= (0,\, 0,\, H(x_1, x_2, t))\,, \\ \mathbf{B} &= (0,\, 0,\, B(x_1, x_2, t)). \end{aligned}$$

We consider now the alternative formulation in terms of the electric (also called scalar) potential φ and the magnetic (also called vector) potential $\mathbf{A}$ according to

$$\mathbf{E} = -\,\mathbf{grad}\,\varphi - \frac{\partial\,\mathbf{A}}{\partial\,t}\,, \qquad \mathbf{B} = \mathbf{curl}\,\mathbf{A}\,.$$

This model can be treated in a given spatial domain $\Omega \subset \mathcal{R}^d$, $d = 2, 3$ with a boundary $\partial\Omega$. Then, introducing the additional condition $\mathrm{div}\,\mathbf{A} = 0$ and taking into account that $\mathrm{div}\,\mathbf{J} = 0$ in the interior of the domain, (1)-(2) give rise to the following coupled system of PDEs for the electric potential φ and the magnetic potential $\mathbf{A}$

$$\mathrm{div}\,(\sigma\,\mathbf{grad}\,\varphi) = 0 \qquad \text{in } \Omega\,, \tag{3}$$

$$\sigma\,\mathbf{n}\cdot\mathbf{grad}\,\varphi = \begin{cases} I_\nu & \text{on } \Gamma_\nu \subset \partial\Omega \\ 0 & \text{elsewhere} \end{cases}\,, \tag{4}$$

$$\sigma\frac{\partial \mathbf{A}}{\partial t} + \mathbf{curl}\,\mu^{-1}\mathbf{curl}\mathbf{A} = \begin{cases} -\sigma\,\mathbf{grad}\,\varphi & \text{in } \Omega \\ 0 & \text{in } \mathbf{R}^3 \setminus \bar{\Omega} \end{cases}. \tag{5}$$

In equation (4), we refer to I_ν as the fluxes associated with the contacts $\Gamma_\nu \subset \partial\Omega$, $1 \le \nu \le N_c$ with a total number of contacts N_c. The fluxes have to satisfy the compatibility condition

$$\sum_{\nu=1}^{N_c} I_\nu = 0. \tag{6}$$

Note that the equation (5) is considered with appropriate initial and boundary conditions.

The electric energy dissipation given by the Joule-Lenz law reads as follows

$$f(\varphi, \sigma, \mathbf{A}) := \int_\Omega \mathbf{J}\cdot\mathbf{E}\,dx\,. \tag{7}$$

In the stationary regime, (7) reduces to

$$f(\varphi, \sigma) = -\int_\Omega \mathbf{J}\cdot\mathbf{grad}\,\varphi\,dx = -\int_\Omega \mathrm{div}(\varphi\,\mathbf{J})\,dx\,. \tag{8}$$

The last equality in (8) follows from

$$\mathrm{div}\,(\varphi\,\mathbf{J}\,) = \mathbf{J}\cdot\mathbf{grad}\,\varphi + \varphi\,\mathrm{div}\,\mathbf{J}$$

with $\mathrm{div}\,\mathbf{J} = 0$. Using the Gauss theorem and the Neumann boundary conditions from (4) we get

$$f(\varphi, \sigma) = -\int_{\partial\Omega} \mathbf{n}\cdot\mathbf{J}\varphi\,ds = \sum_{\nu=1}^{N_c}\int_{\Gamma_\nu} I_\nu\varphi\,ds\,. \tag{9}$$

The last expression is considered as an objective functional in our optimization problem. Our purpose is to minimize the electric energy dissipation.

3 The optimization problem

Define the equation for the electric potential (3) in a weak formulation as follows

$$a(\varphi, v) = L(v), \qquad \text{for } v \in U, \tag{10}$$

where the bilinear energy form is

$$a(\varphi, v) = \int_\Omega \mathbf{grad}\,v\,\cdot\sigma\,\mathbf{grad}\,\varphi\,dx,$$

$L(v)$ is the linear load form and U is the space of all admissible solutions of (3).

Then, the problem to minimize the energy dissipation given by the objective functional (9) is to find

$$\inf_{\varphi,\sigma} f(\varphi, \sigma) = \inf_{\varphi,\sigma}\sum_{\nu=1}^{N_c}\int_{\Gamma_\nu} I_\nu\varphi\;ds, \tag{11}$$

subject to the following constraints

$$\begin{aligned} &a(\varphi, v) = L(v) && \text{with BCs from (4)}, \\ &\int_\Omega \sigma\,dx = C && \text{(mass constraint)}, \\ &\sigma_{\min} \le \sigma \le \sigma_{\max} && \text{(conductivity box constraint)}. \end{aligned} \tag{12}$$

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Here, $\sigma_{\min}$ and $\sigma_{\max}$ are a priori given positive limits for the conductivity and C is a fixed given constant. In general formulations of nonlinear programming problems, the objective function f and the inequality constraints are supposed to be twice continuously differentiable. This requirement is obviously satisfied in our case.

Note that we solve the constrained optimization problem with the partial differential equation (3) for φ incorporated as a part of the constraints. By means of this formulation, we arrive at the basic idea of the so called one-shot methods which stand in contrast to many standard optimization approaches considering the optimization process and the solution of the differential equation separately. Our experience shows that this simultaneous optimization approach together with a numerical solution of the partial differential equation reduces essentially the overall computational complexity of the resulting optimization algorithm, see [13].

4 Level-set approach

As mentioned in the introduction, we are interested in optimal distribution of the conductivity in a prescribed structural domain $\Omega \subset \mathcal{R}^d$, $d = 2, 3$ whose topology can be changed. To formulate the level-set approach for finding the optimal design, we consider a fixed reference domain $D \subset \mathcal{R}^d$ and suppose that D always contains the computational design domain Ω which changes within the optimization process, i.e. $\Omega \subseteq D$.

In the level-set framework, one defines a scalar function $\Phi : D \to \mathcal{R}$ (also referred to as a level-set function) which represents the design boundary $\partial\Omega$ implicitly by

$$\partial\Omega = \{\boldsymbol{x} | \boldsymbol{x} \in D,\ \Phi(\boldsymbol{x}) = 0\}.$$

Note that $\Phi(\boldsymbol{x})$ is one-dimensional higher function than the represented boundary $\partial\Omega$. Thus, one can determine each part of the design domain as follows

$$\Phi(\boldsymbol{x}) = 0\,, \qquad \forall \boldsymbol{x} \in \partial\Omega \cup D\,, \tag{13}$$

$$\Phi(\boldsymbol{x}) > 0\,, \qquad \forall \boldsymbol{x} \in \Omega \setminus \partial\Omega\,, \tag{14}$$

$$\Phi(\boldsymbol{x}) < 0\,, \qquad \forall \boldsymbol{x} \in D \setminus \Omega. \tag{15}$$

In particular, as shown in Fig. 1, the boundary is embedded implicitly as the zero level-set (13) of $\Phi(\boldsymbol{x})$, while the interior of the structure is presented by (14) and the exterior by (15). Note that the boundary $\partial\Omega$ is totally manipulated through the zero level-set function. Hence, all admissible shapes have the form $\Omega = \{\boldsymbol{x} | \boldsymbol{x} \in D, \Phi(\boldsymbol{x}) \geq 0\}$, see Fig. 2.

If we suppose that the zero level-set moves in the normal direction to itself, then the level-set function changes dynamically in time, i.e.

$$\Phi(\boldsymbol{x}(t), t) = k \qquad \text{for any } \boldsymbol{x} \in \Omega(t)\,, \tag{16}$$

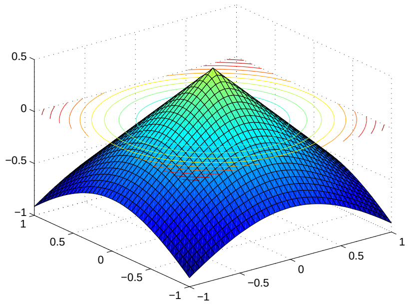

Figure 1: Level-set representation

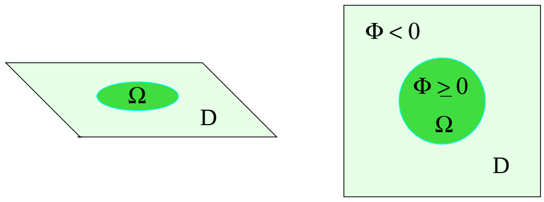

Figure 2: a) Zero level-set: b) Design domain $\Omega \subseteq D$

where $k = 0$ for $\boldsymbol{x} \in \partial\Omega(t)$. By differentiation of (16) with respect to time t and applying the chain rule, one arrives at the so called Hamilton-Jacobi-type equation

$$\frac{\partial\Phi(\boldsymbol{x}, t)}{\partial t} + \nabla\Phi(\boldsymbol{x}, t)\frac{d\boldsymbol{x}}{dt} = 0\,. \tag{17}$$

The latter equation can also be written in the form

$$\frac{\partial\Phi(\boldsymbol{x}, t)}{\partial t} = -\nabla\Phi(\boldsymbol{x}, t)\frac{d\boldsymbol{x}}{dt} \equiv -\boldsymbol{v}\,\nabla\Phi(\boldsymbol{x}, t)\,, \tag{18}$$

where $\boldsymbol{v} := d\boldsymbol{x}/dt$ denotes the speed of the zero level-set, which is related to the optimized objective functional. Thus, the structural optimization process can be treated as the movement of a point on the boundary which is exactly driven by the objective. For recent implementations of this method in structural optimization, we refer to [2, 4, 15, 17].

Our optimization problem described in Section 3 can be formulated by means of level-set model as follows. Find:

$$\inf_{\Phi} f(\varphi, \sigma, \Phi) = \inf_{\Phi} \sum_{\nu=1}^{N_c} \int_D I_\nu \varphi\, \delta(\Phi)\, |\nabla\Phi|\, dx, \tag{19}$$

subject to the following constraints

$$a(\varphi, v, \Phi) = L(v, \Phi) \qquad \text{with BCs from (4)}\,, \tag{20}$$

$$\int_D \sigma \bar{H}(\Phi)\, dx = C \qquad \text{(mass constraint)}\,,$$

$$\sigma_{\min} \leq \sigma \leq \sigma_{\max} \quad \text{(conductivity box constraint)}.$$

The energy form is now defined as

$$a(\varphi, v, \Phi) = \int_D \mathbf{grad}\, v \cdot \sigma\, \mathbf{grad}\, \varphi\, \bar{H}(\Phi)\, dx.$$

In this formulation we use minimization of the objective functional with respect to the level-set function Φ, the Heaviside function $\bar{H}(\Phi)$, and the Dirac function $\delta(\Phi)$, cf., e.g., [17].

5 Primal-dual approach

Finite element discretization of the design domain Ω is used to formulate the following discrete nonlinear programming problem

$$\inf_{\boldsymbol{\varphi},\boldsymbol{\sigma}} f(\boldsymbol{\varphi},\boldsymbol{\sigma}), \tag{21}$$

subject to

$$\begin{array}{rr} A(\boldsymbol{\sigma})\,\boldsymbol{\varphi}-\mathbf{b}=0, & \boldsymbol{\sigma}-\sigma_{\min}\mathbf{e}\geq 0, \\ g(\boldsymbol{\sigma})-\mathrm{C}=0, & \sigma_{\max}\mathbf{e}-\boldsymbol{\sigma}\geq 0, \end{array} \tag{22}$$

where $\mathbf{e}\in\mathcal{R}^N,\ \mathbf{e}=(e_1,\dots,e_N)^T,\ e_i=1,\ 1\leq i\leq N.$

Here, A is the finite element stiffness matrix corresponding to (3), $\mathbf{b}$ is the discrete load vector, and $\boldsymbol{\sigma}=(\sigma_i)_{i=1}^N$ is the discrete conductivity vector. We have denoted by N the number of degrees of freedom. Suppose that the conductivity is a constant on each element, i.e., σ_i is the value of σ on the ith element. Note that the lower bound $\sigma_{\min}$ plays a crucial role keeping the ellipticity of the discrete problem.

The Lagrangian function associated with (21)-(22) is

$$\begin{aligned} \mathcal{L}(\boldsymbol{\varphi},\boldsymbol{\sigma},\boldsymbol{\lambda},\eta,\mathbf{z},\mathbf{w}) \;:=\; & f(\boldsymbol{\varphi},\boldsymbol{\sigma}) \\ & +\boldsymbol{\lambda}^{\mathrm{T}}(\mathrm{A}(\boldsymbol{\sigma})\boldsymbol{\varphi}-\mathbf{b})+\eta\,(g(\boldsymbol{\sigma})-\mathrm{C}) \\ & -\mathbf{z}^{\mathrm{T}}(\boldsymbol{\sigma}-\sigma_{\min}\mathbf{e})-\mathbf{w}^{\mathrm{T}}(\sigma_{\max}\mathbf{e}-\boldsymbol{\sigma}). \end{aligned} \tag{23}$$

Here, $\boldsymbol{\lambda}$, η and $\mathbf{z}\geq 0$, $\mathbf{w}\geq 0$ are the Lagrange multipliers for the equality and inequality constraints in (22), respectively. The necessary first–order Karush–Kuhn–Tucker (KKT) optimality conditions read as follows

$$\begin{aligned} &\nabla_{\boldsymbol{\varphi}}\mathcal{L}=\nabla_{\boldsymbol{\varphi}}f+\mathrm{A}(\boldsymbol{\sigma})^{\mathrm{T}}\boldsymbol{\lambda}=\mathbf{0}, \\ &\nabla_{\boldsymbol{\sigma}}\mathcal{L}=\partial_{\boldsymbol{\sigma}}(\boldsymbol{\lambda}^T A(\boldsymbol{\sigma})\boldsymbol{\varphi})+\eta\nabla \mathrm{g}(\boldsymbol{\sigma})-\mathbf{z}+\mathbf{w}=\mathbf{0}, \\ &\nabla_{\boldsymbol{\lambda}}\mathcal{L}=\mathrm{A}(\boldsymbol{\sigma})\boldsymbol{\varphi}-\mathbf{b}=\mathbf{0}, \\ &\nabla_{\eta}\mathcal{L}=\mathrm{g}(\boldsymbol{\sigma})-\mathrm{C}=0, \\ &D_1\mathbf{z}=\mathbf{0}\quad\text{and}\quad D_2\mathbf{w}=\mathbf{0},\qquad \mathbf{z}\geq 0,\ \mathbf{w}\geq 0, \end{aligned} \tag{24}$$

where $D_1=\mathrm{diag}(\sigma_i-\sigma_{\min})$ and $D_2=\mathrm{diag}(\sigma_{\max}-\sigma_i)$ denote diagonal matrices in the complementarity conditions.

We use further the idea behind the interior-point methods, see e.g. [19]. The inequality constraints are treated by using the logarithmic barrier functions. Thus, we get the following sequence of minimization subproblems

$$\inf_{\boldsymbol{\varphi},\boldsymbol{\sigma}} \beta(\boldsymbol{\varphi},\boldsymbol{\sigma},\rho) \tag{25}$$

with a parameter-dependent objective function defined as

$$\begin{aligned} \beta(\boldsymbol{\varphi},\boldsymbol{\sigma},\rho):\;=\;& f(\boldsymbol{\varphi},\boldsymbol{\sigma}) \\ -\;& \rho\left(\log(\boldsymbol{\sigma}-\sigma_{\min}\mathbf{e})+\log(\sigma_{\max}\mathbf{e}-\boldsymbol{\sigma})\right), \end{aligned}$$

subject to the equality constraints

$$A(\boldsymbol{\sigma})\,\boldsymbol{\varphi}-\mathbf{b}=\mathbf{0}\qquad\text{and}\qquad g(\boldsymbol{\sigma})-\mathrm{C}=0, \tag{26}$$

where $\beta(\boldsymbol{\varphi},\boldsymbol{\sigma},\rho)$ is the barrier function and $\rho>0$ is the barrier parameter. We suppose here that $\boldsymbol{\sigma}>\sigma_{\min}\mathbf{e}$ and $\sigma_{\max}\mathbf{e}>\boldsymbol{\sigma}$, which is the idea behind the interior-point methods.

Denote by $\boldsymbol{\Psi}:=(\boldsymbol{\varphi},\boldsymbol{\sigma},\boldsymbol{\lambda},\eta,\mathbf{z},\mathbf{w})$ the solution of the optimization subproblem (25)-(26). The KKT conditions (24) lead to the following nonlinear equation

$$\boldsymbol{F}_\rho(\boldsymbol{\Psi}):=\begin{pmatrix} \nabla_{\boldsymbol{\varphi}}f+\mathrm{A}(\boldsymbol{\sigma})^{\mathrm{T}}\boldsymbol{\lambda} \\ \partial_{\boldsymbol{\sigma}}(\boldsymbol{\lambda}^T A(\boldsymbol{\sigma})\boldsymbol{\varphi})+\eta\nabla g(\boldsymbol{\sigma})-\mathbf{z}+\mathbf{w} \\ A(\boldsymbol{\sigma})\boldsymbol{\varphi}-\mathbf{b} \\ g(\boldsymbol{\sigma})-\mathrm{C} \\ D_1\mathbf{z}-\rho\,\mathbf{e} \\ D_2\mathbf{w}-\rho\,\mathbf{e} \end{pmatrix}=\mathbf{0}, \tag{27}$$

where $\nabla_{\mathbf{z}}\mathcal{L}=D_1\mathbf{z}-\rho\,\mathbf{e}$, $\nabla_{\mathbf{w}}\mathcal{L}=D_2\mathbf{w}-\rho\,\mathbf{e}$.

For the nonlinear equation we apply an inexact Newton method and an appropriate modification of the primal-dual matrix. For more details of the iterative procedure by the step-size approach, see [7].

6 Numerical experiments

In this section we present some computational results. The design domain Ω is chosen as a rectangle occupied by an isotropic conductive medium. The domain is decomposed by uniform quadrilateral finite elements. The rotated bilinear (also referred to Rannacher and Turek) basis functions are used for the discrete model. Comparable finite element formulations of eddy current systems can be found in, cf. [3, 6, 9].

A quadrature rule (exact for polynomials of degree three) has been used to compute the global stiffness matrix corresponding to the elliptic differential equation as a first equality constraint. All the computations have been done with preliminary given values for the range of the conductivity. We have chosen $\sigma_{\min}=0.01$, $\sigma_{\max}=1$, and a constant C computed according to an initial homogeneous distribution with a conductivity $\sigma=0.45$. The preconditioned conjugate gradient method (PCG) has been applied to solve systems with the stiffness matrix.

The primal-dual interior-point method with logarithmic barrier functions is implemented in the numerical experiments. The solutions of the nonlinear equation (27) represent the so called central path

$$P:\{\boldsymbol{\Psi}\,|\,\boldsymbol{F}_\rho(\boldsymbol{\Psi})=\mathbf{0},\ \rho>0\}.$$

Some computational approaches how to choose the barrier parameter ρ are discussed in [7]. The optimization process is computationally unstable when ρ is made too small.

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We have computed the material distribution for various number of contacts N_c. Due to the compatibility condition (6), we always have that the current inflow at the lower port(s) is equal to the current outflow at the upper port(s).

The final optimal design is visualized in Fig. 3 for $N_c = 2$, in Fig. 4 for $N_c = 3$, and in Fig. 5 for $N_c = 6$. In the plane Oxy one can see the contour of the final design domain. On the axis Oz we observe the computed values of the conductivity running from $\sigma_{\min}$ up to $\sigma_{\max}$. For comparable applications of topology optimization of eddy current systems with different objectives, design parameters, and constraints we refer the reader to [8, 12, 16].

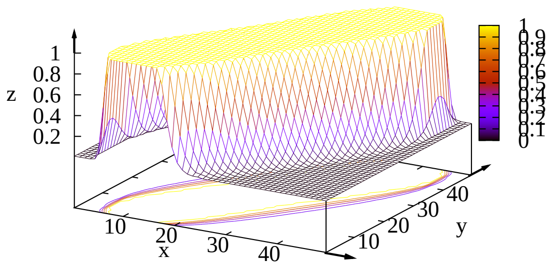

Figure 3: Material distribution (50 × 50 mesh, 2 contacts)

7 Conclusions

In this paper, the level-set method and the primal-dual approach are proposed for the topology optimization of high power electric devices. The conductive electromagnetic system is described by the particular eddy current formulation of Maxwell's equations considered in a quasistationary approximation. The objective functional is to minimize power losses due to parasitic inductivities and thus, to find the optimal distribution of the material. The optimal layout of the domain can be implicitly represented by a successive moving boundary embedded in a scalar (level-set) function of one dimension higher than the represented boundary. The optimization problem is formulated by means of a level-set model and the minimization is considered with respect to the level-set function. Another approach is directly solving the minimization problem with respect to the electric potential as a state variable and the conductivity as a design variable. Numerical results by using finite element discretization of a two-dimensional domain and applications of the primal-dual method are reported and discussed.

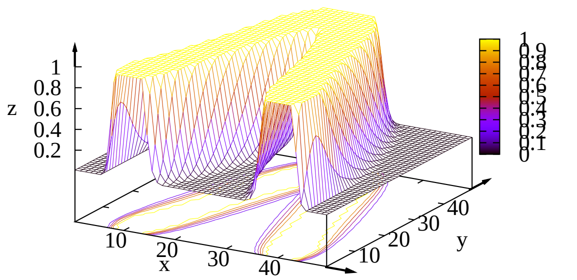

Figure 4: Material distribution (50 × 50 mesh, 3 contacts)

References

[1] Allaire, G., "Shape Optimization by the Homogenization Method", Springer, New York, 2001.

[2] Allaire, G., Jouve, F., and Toader, A.-M., "Structural Optimization Using Sensitivity Analysis and a Level-Set Method", J. Comput. Phys., V194, pp. 363-393, 2004.

[3] Ammari, H., Buffa, A., and Nédélec, J.-C., "A Justification of Eddy Currents Model for the Maxwell Equations," *SIAM Journal on Applied Mathematics*, V60, pp. 1805-1823, 2000.

[4] Amstutz, S. and Andrä, H., "A New Algorithm for Topology Optimization Using a Level-Set Method", J. Comput. Phys., V1216, pp. 573-5889, 2006.

[5] Bendsøe, M.P. and Sigmund, O., "Topology Optimization, Theory, Methods, and Applications", Springer, New York, 2003.

[6] Biro, O. and Preis, K., "Various FEM Formulations for the Calculation of Transient 3D Eddy Currents

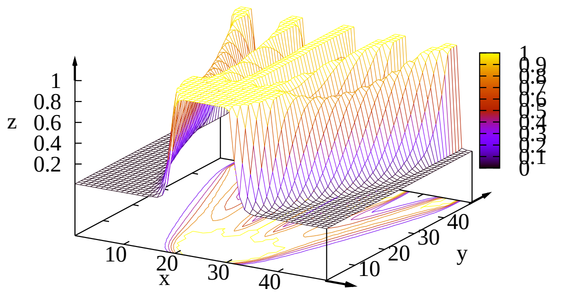

Figure 5: Material distribution (50 × 50 mesh, 6 contacts)

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in Nonlinear Media," *IEEE Transactions on Magnetics*, V31, pp. 1307-1312, 1995.

[7] Hoppe, R.H. and Petrova, S.I., "Applications of Primal-Dual Interior Methods in Structural Optimization", Comput. Methods Appl. Math., V3, N1, pp. 159-176, 2003.

[8] Kang, J., Kim, C., and Wang, S., "Reliability-Based Topology Optimization for Electromagnetic Systems", COMPEL, V23, N3, pp. 715-723, 2004.

[9] Krawczyk, A. and Tegopoulos, J.A., "Numerical Modelling of Eddy Currents", Monographs in Electrical and Electronic Engineering, V32, Oxford University Press, 1993.

[10] Landau, L.D., Lifshitz, E.M., and Pitaevskii, L.P., "Electrodynamics of Continuous Media", V8, 2nd Edition, 1984.

[11] Osher, S. and Sethian, J.A., "Front Propagation with Curvature Dependent Speed: Algorithms Based on Hamilton-Jacobi Formulations", J. Comp. Phys., V78, pp. 12-49, 1988.

[12] Park, I.-H., Kwak, I.-G.,Lee, H.-B., Hahn, S.-Y., and Lee, K.-S., "Design Sensitivity Analysis for Transient Eddy Current Problems Using Finite Element Discretization and Adjoint Variable Method", IEEE Trans. Magn., V32, N3, pp. 1242-1245, 1996.

[13] Petrova, S.I., "Applications of One-Shot Methods in PDEs Constrained Optimization", Math. Comput. Simul., V80, N3, pp. 581-597, 2009.

[14] Rozvany, G.I.N., "Structural Design via Optimality Criteria", Kluwer, Dordrecht, 1988.

[15] Sethian, J.A., "Level Set Methods and Fast Marching Methods: Evolving Interfaces in Computational Geometry, Fluid Mechanics, Computer Vision, and Material Science", Cambridge University Press, 1999.

[16] Shim, H., Wang, S., and Hameyer, K., "Topology Optimization of Magnetothermal Systems Considering Eddy Current as Joule Heat", IEEE Trans. Magn., V43, N4, pp. 1617-1620, 2007.

[17] Wang, M.Y., Wang, X., and Guo, D., "A Level-Set Method for Structural Topology Optimization", Comput. Meth. Appl. Mech. Eng., V192, N1-2, pp. 227-246, 2003.

[18] Wang, S. and Kim, Y., "A Study on the Topology Optimization of Electromagnetic Systems", in CEFC'2000, p.336, 2000.

[19] Wright, M.H., "The Interior-Point Revolution in Optimization: History, Recent Developments, and Lasting Consequences", Bull. Am. Math. Soc., New Ser., V42, N1, pp. 39-56, 2005.

ISBN: 978-988-17012-8-2
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