Proceedings of the International MultiConference of Engineers and Computer Scientists 2010 Vol I,
IMECS 2010, March 17 - 19, 2010, Hong Kong

Efficient Algorithm for the Paired-Domination Problem in Convex Bipartite Graphs

Ruo-Wei Hung*,† Chi-Hyi Laio, and Chun-Kai Wang

Abstract—**Let $G = (V, E)$ be a graph without isolated vertices. A matching in G is a set of independent edges in G. A perfect matching M in G is a matching such that every vertex of G is incident to an edge of M. A set $S \subseteq V$ is a *paired-dominating set* of G if every vertex not in S is adjacent to a vertex in S, and if the subgraph induced by S contains a perfect matching. The *paired-domination problem* is to find a paired-dominating set of G with minimum cardinality. The paired-domination problem on bipartite graphs has been shown to be NP-complete. A bipartite graph $G = (U, W, E)$ is *convex* if there exists an ordering of the vertices of W such that, for each $u \in U$, the neighbors of u are consecutive in W. In this paper, we present an $O(|U| \log |U|)$-time algorithm to solve the paired-domination problem in convex bipartite graphs.**

Keywords: graph algorithms, paired-domination, convex bipartite graphs

1 Introduction

The problem of placing monitoring devices in a system such that every site in the system (including the monitoring devices themselves) is adjacent to a monitor and every monitor is paired with a backup monitor, can be modeled by paired-domination in graphs. In this paper, we consider the paired-domination problem in convex bipartite graphs.

A set S of vertices of a graph $G = (V, E)$ is a *dominating set* of G if every vertex not in S is adjacent to a vertex in S. The *domination problem* is to find a dominating set of G with minimum cardinality. Variations of the domination problem seek to find a minimum dominating set with some additional properties, e.g., to be independent or to induce a connected graph. These problems arise in a number of distributed network applications, where the problem is to locate the smallest number of centers in networks such that every vertex is nearby at least one center. Domination and its variations in graphs have been thoroughly studied, and the literature on this subject has been surveyed and detailed in two books [8, 9].

*Department of Computer Science and Information Engineering, Chaoyang University of Technology, Wufong, Taichung 413, Taiwan.

†Corresponding author's e-mail: rwhung@cyut.edu.tw

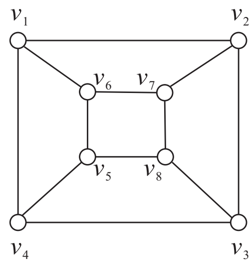
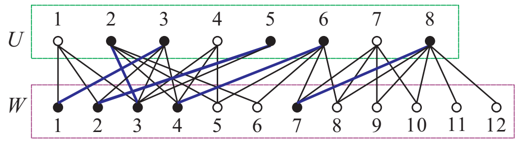


Fig. 1: The tree-cube graph Q_3.

A *matching* in a graph G is a set of independent edges in G. A *perfect matching* M in G is a matching in G such that every vertex of G is incident to an edge of M. A *paired-dominating set* of a graph G is a dominating set S of G such that the subgraph $G[S]$ induced by S contains a perfect matching M. Two vertices joined by an edge of M are said to be *paired*. Every graph without isolated vertices has a paired-dominating set, since the vertices incident to edges of any maximal matching form such a set [10]. The *paired-domination number* of a graph G, denoted by $\gamma_{\mathrm{p}}(G)$, is the minimum cardinality of a paired-dominating set of G. The *paired-domination problem* is to find a paired-dominating set of G with cardinality $\gamma_{\mathrm{p}}(G)$. For example, for the three-cube graph Q_3 shown in Fig. 1, $S = \{v_1, v_2, v_3, v_4\}$ is a paired-dominating set of Q_3 since S is a dominating set and the subgraph induced by S contains a perfect matching $M = \{(v_1, v_4), (v_2, v_3)\}$, and $\gamma_{\mathrm{p}}(Q_3) = 4$.

Paired-domination was introduced by Haynes and Slater [10] with the following application in mind. If, in a graph G, we consider each vertex as the possible location for a guard capable of protecting every vertex adjacent to it, then "domination" requires every vertex to be protected. In paired-domination, each guard is assigned another adjacent guard, and they are designed to provide a backup for each other. The problem of determining the paired-domination number $\gamma_{\mathrm{p}}(G)$ of an arbitrary graph G has been known to be NP-complete [10]. The paired-domination problem is still NP-complete in some special classes of graphs such as bipartite graphs, chordal graphs, and split graphs [3]. However, the problem admits polynomial-time algorithms when the input is restricted to some special classes of graphs, including trees [12], circular-arc graphs [4], permutation graphs [5], block

ISBN: 978-988-17012-8-2
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)



U	*begin*	*end*
1	1	3
2	3	6
3	1	4
4	3	5
5	2	4
6	4	8
7	7	10
8	7	12

Fig. 2: Compact representation of a convex bipartite graph and a paired-domination set (filled circles incident to bold edges).

graphs, and interval graphs [3].

Let $G = (U, W, E)$ represent an undirected, bipartite graph, where U and W is a partition of the vertices and E is the edge set in which each edge (u, w) is such that $u \in U$ and $w \in W$. The paired-domination problem on bipartite graphs has been shown to be NP-complete [3]. In this paper, we will investigate the time complexity of the paired-domination problem on convex bipartite graphs which form a subclass of bipartite graphs.

Convex bipartite graphs were introduced by Glover [7], motivated by some industrial applications. Since then several algorithms have been developed for problems in this kind of graph [2, 6, 11, 13, 14]. Let $G = (U, W, E)$ be a bipartite graph. The graph G is called *convex* if the vertices in W can be ordered in such a way that, for each $u \in U$, the neighbors of u are consecutive in W. For convenience, we consider that $U = \{1, 2, \cdots, |U|\}$ and $W = \{1, 2, \cdots, |W|\}$, and that the vertices in W are given according to the ordering mentioned above. This ordering can be obtained in a preprocessing step by a linear time algorithm [1]. That is, the vertices of W are represented by integers from 1 to $|W|$, and they are given according to their representing integers in an increasing manner. We say that a vertex $u \in W$ is smaller (larger) than a vertex $v \in W$ if the integer number of u is smaller (larger) than that of v. A convex bipartite graph has a *compact* representation by a set of $|U|$ triples of the form $(i, begin(i), end(i))$, where i is a vertex in U, $begin(i)$ and $end(i)$ are the smallest and largest vertices, respectively, in the consecutive vertices of W connected to i. Fig. 2 shows a convex bipartite graph in its compact representation and a paired-dominating set on it. In this paper, we will present an $O(|U| \log |U|)$-time algorithm to solve the paired-domination problem in convex bipartite graphs.

2 Terminologies

We begin with an elementary observation about paired-dominating sets of a graph. Let G be a graph without isolated vertices. Haynes and Slater [10] observed that a paired-dominating set of G does exist and its paired-domination number $\gamma_{\mathrm{p}}(G)$ is even.

Lemma 2.1. *[10] Let G be a graph without isolated vertices. Then, there exists a paired-dominating set in G and $\gamma_{\mathrm{p}}(G)$ is even.*

Hereafter, let $G = (U, W, E)$ be a convex bipartite graph. We denote by $[i, j]$ the set of consecutive integers $\{i, i+1, \cdots, j\}$. Thus, $U = [1, |U|]$ and $W = [1, |W|]$. We call $[i, j]$ an integer interval starting from i and ending at j. For simplicity, an integer interval is also called an interval. Further, we also let U denote an array representing G in a compact representation. Each element of the array $U[1..|U|]$ has the fields *begin* and *end*. The triple $(i, begin(i), end(i))$ of the compact representation of G is represented here by $(i, U[i].begin, U[i].end)$. For simplicity, we will use $i.begin$ and $i.end$ to represent $U[i].begin$ and $U[i].end$, respectively. We may assume that the input convex bipartite graph has no isolated vertices since isolated vertices can be easily detected. By definition of a convex bipartite graph, the neighbor of a vertex u in U can be represented as an interval $I_u = [u.begin, u.end]$. Then, the neighbors of vertices of U can be represented by a set of intervals which is called the interval representation $I(U)$ of U. For an interval $I_u \in I(U)$, the smallest integer and largest integer in I_u are called the *leftmost* integer and *rightmost* integer of I_u, respectively. Further, interval $I_u = [u.begin, u.end]$ is said to be *dominated* by integer ℓ if $u.begin \leqslant \ell \leqslant u.end$.

We first partition U into k disjoint clusters $U_1, U_2, \cdots, U_k$ such that $u.begin = v.begin$ if u and v are in the same cluster, and $a.begin < b.begin$ if $a \in U_i$ and $b \in U_j$ for $i < j$. We then sort the vertices of U_i, $1 \leqslant i \leqslant k$, such that a precedes b for $a, b \in U_i$ and $a.end \leqslant b.end$, i.e., the rightmost integer of interval I_a is not larger than the rightmost integer of interval I_b in the interval representation. For example, Fig. 3 shows the clusters and the interval representation $I(U)$ of U for the convex bipartite graph shown in Fig. 2. In addition, intervals I_1, I_3, I_5, I_4, I_2 in $I(U)$ are dominated by integer 3. The above clustering process can be easily done in $O(|U| \log |U|)$ time. In the following, it is assumed that the clustering process has been done, i.e., the sorted clusters of U are given. The following lemma gives the upper bound of $\gamma_{\mathrm{p}}(G)$.

Lemma 2.2. *Let $G = (U, W, E)$ be a convex bipartite graph without isolated vertices, and let U be partitioned into k sorted clusters $U_1, U_2, \cdots, U_k$. Then, $\gamma_{\mathrm{p}}(G) \leqslant 2k$.*

Proof. Let u_i be the vertex in U_i, $1 \leqslant i \leqslant k$, such that $a.end \leqslant u_i.end$ for $a \in U_i$, and let $w_i \in W$ such that $w_i = u_i.begin$. By pairing u_i with w_i for $1 \leqslant i \leqslant k$, we obtain a paired-dominating set PD of G with size $2k$, where $PD = \cup_{1 \leqslant i \leqslant k}\{u_i, w_i\}$. Thus, $\gamma_{\mathrm{p}}(G) \leqslant 2k$. □

Let U_i be a cluster of U. Define $\min(U_i)$ and $\max(U_i)$ to be two vertices in U_i such that $\min(U_i).end \leqslant a.end \leqslant \max(U_i).end$ for $a \in U_i$. Further,

ISBN: 978-988-17012-8-2
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

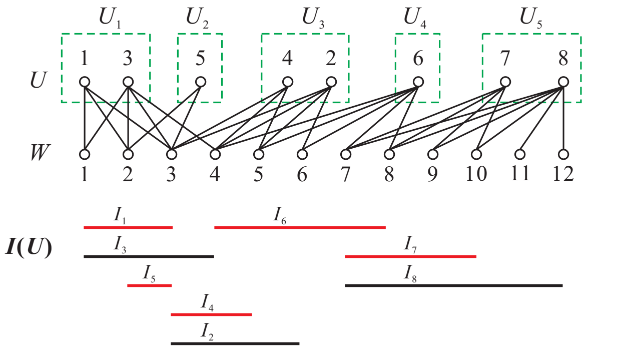


Fig. 3: Clusters and interval representation $I(U)$ of U for the convex bipartite graph shown in Fig. 2.

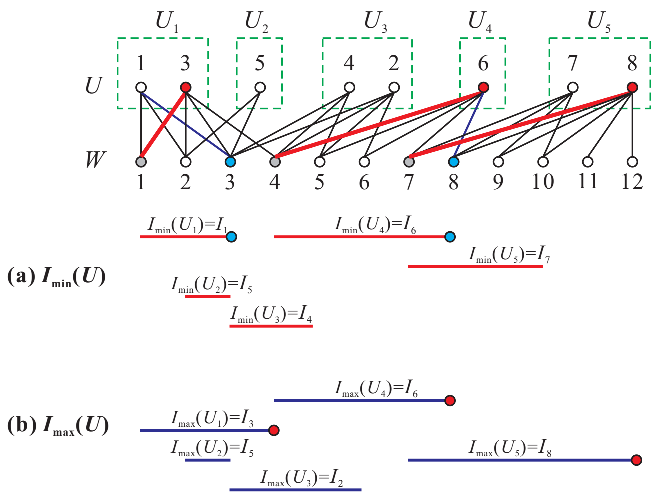


Fig. 4: (a) $I_{\min}(U)$ and (b) $I_{\max}(U)$ for the convex bipartite graph shown in Fig. 3, where the pairs in a minimum paired-dominating set of the graph are highlighted in bold.

let $I_{\min}(U_i) = [\min(U_i).begin, \min(U_i).end]$ and let $I_{\max}(U_i) = [\max(U_i).begin, \max(U_i).end]$. We can see that for $a \in U_i$, $I_{\min}(U_i) \subseteq I_a \subseteq I_{\max}(U_i)$, where $I_a = [a.begin, a.end]$. In addition, every vertex of U_i is adjacent to all vertices of $I_{\min}(U_i)$ in W. The vertices of W in $I_{\min}(U_i)$ are called *common neighbors* of vertices of U_i. For example, let U_1 be a cluster of U shown in Fig. 3. Then, $\min(U_1) = 1$, $I_{\min}(U_1) = [1, 3]$, $\max(U_1) = 3$, and $I_{\max}(U_1) = [1, 4]$. Let $U_1, U_2, \cdots, U_k$ be the disjoint sorted clusters of U. We define $I_{\min}(U) = \cup_{1 \leqslant i \leqslant k} I_{\min}(U_i)$ and $I_{\max}(U) = \cup_{1 \leqslant i \leqslant k} I_{\max}(U_i)$. Note that the number of intervals in $I_{\min}(U)$ or $I_{\max}(U)$ equals the number of clusters of U. For example, Fig. 4 shows $I_{\min}(U)$ and $I_{\max}(U)$ for the convex bipartite graph shown in Fig. 3.

3 The Algorithm

In this section, we will present an $O(|U| \log |U|)$-time algorithm to solve the paired-domination problem on a convex bipartite graph $G = (U, W, E)$. Let $\widehat{W}_D$ and $\widehat{U}_D$ be the subsets of W and U, respectively. U (resp. W) is said to be dominated by $\widehat{W}_D$ (resp. $\widehat{U}_D$) if every vertex of U (resp. W) is adjacent to at least one vertex of $\widehat{W}_D$ (resp. $\widehat{U}_D$). Let $U_1, U_2, \cdots, U_k$ be the disjoint sorted clusters of U. By Lemma 2.2, $\gamma_{\mathrm{p}}(G) \leqslant 2k$. In the following, we will obtain the lower bound of $\gamma_{\mathrm{p}}(G)$. Our basic idea is sketched as follows. We first find a minimum subset $\widetilde{W}_D$ of W such that $\widetilde{W}_D$ dominates U, and find a minimum subset $\widetilde{U}_D$ of U such that $\widetilde{U}_D$ dominates W. Note that every vertex of W or U is represented by an integer. Since each edge (u, w) in G is such that $u \in U$ and $w \in W$, it is easy to see that $\gamma_{\mathrm{p}}(G) \geqslant 2 \cdot \max\{|\widetilde{W}_D|, |\widetilde{U}_D|\}$. Finally, we construct a paired-dominating set of G with size $2 \cdot \max\{|\widetilde{W}_D|, |\widetilde{U}_D|\}$. In the following, we will show how to construct such two sets $\widetilde{W}_D$ and $\widetilde{U}_D$.

We first construct a minimum subset $\widetilde{W}_D$ of W that dominates U. Observe that if there exists a vertex j in $\widetilde{W}_D$ such that it is not in any $I_{\min}(U_i)$, $1 \leqslant i \leqslant k$, then U is clearly not dominated by $\widetilde{W}_D$. Thus, we only consider the vertices of W in $I_{\min}(U_i)$, $1 \leqslant i \leqslant k$. Then, we are given by $I_{\min}(U)$. The problem of finding a minimum subset of W dominating U is equivalent to seek a minimum set of integers in $[1, |W|]$ such that they together dominate intervals of $I_{\min}(U)$. We introduce Procedure **GD-W** to compute such a minimum set of integers that dominates all intervals of $I_{\min}(U)$. Given a set $I_{\min}(U)$ of intervals, Procedure **GD-W** uses a greedy principle to obtain a subset $\widetilde{W}_D$ of W as follows. Initially, let $\widetilde{W}_D = \emptyset$, let $I_{\min} = I_{\min}(U)$, and let $s(I_{\min})$ be the interval in $I_{\min}$ with the least rightmost integer. Let z be the rightmost integer of $s(I_{\min})$ and let I^z be the set of intervals in $I_{\min}$ dominated by z. Let $\widetilde{W}_D = \widetilde{W}_D \cup \{z\}$ and let $I_{\min} = I_{\min} - I^z$. Repeat the above process until $I_{\min} = \emptyset$. Then it outputs $\widetilde{W}_D$ and stops. For example, given a set $I_{\min}(U)$ of intervals shown in Fig. 4(a), Procedure **GD-W** outputs $\widetilde{W}_D = \{3, 8\}$.

By similar strategy in computing $\widetilde{W}_D$, we can find a minimum subset $\widetilde{U}_D$ of U that dominates W. Observe that if there exists a vertex j in $\widetilde{U}_D$ such that $I_j \notin I_{\max}(U)$, then j can be replaced by one vertex i, where $I_i \in I_{\max}(U)$ and $I_j \subseteq I_i$. That is, $\widetilde{U}_D - \{j\} \cup \{i\}$ is still a minimum subset of U such that it dominates W. Thus, we can only consider the vertices whose representing intervals are in $I_{\max}(U_i)$, $1 \leqslant i \leqslant k$. Then, we are given by $I_{\max}(U)$. Our strategy for finding a subset $\widetilde{U}_D$ of U dominating W uses a greedy principle. Initially, let $\widetilde{U}_D = \emptyset$, let $I_{\max} = I_{\max}(U)$, and let $s(I_{\max})$ be the interval in $I_{\max}$ with the least rightmost integer s. Let $I' = \{I_i \in I_{\max} | s(I_{\max}) \subset I_i\}$. If $I' \neq \emptyset$, then let z

ISBN: 978-988-17012-8-2
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

be a vertex of U such that its representing interval I_z is the interval with the largest rightmost integer among I'; otherwise, let $z = s$. Let $\widetilde{U}_D = \widetilde{U}_D \cup \{z\}$. Let $\widehat{I} = \{I_i \in I_{\max} | I_i$ is dominated by integer z and the rightmost integer of I_i is larger than the rightmost integer of $I_z\}$. For $I_i \in \widehat{I}$, let $I_{\widetilde{i}} = [z+1, i.end]$, i.e., $I_{\widetilde{i}}$ is obtained from I_i by removing $[i.begin, z]$. Let $\widetilde{I} = \cup_{I_i \in \widehat{I}} \{I_{\widetilde{i}}\}$. Then, let $I_{\max} = I_{\max} - \{s(I_{\max})\} - I' - \widehat{I} \cup \widetilde{I}$. Repeat the above process until $I_{\max} = \emptyset$. Then it outputs $\widetilde{U}_D$ and stops. We call the above process as Procedure **GD-U**. For example, given a set $I_{\max}(U)$ of intervals shown in Fig. 4(b), Procedure **GD-U** outputs $\widetilde{U}_D = \{3, 6, 8\}$. The following two lemmas show the optimality of Procedures **GD-W** and **GD-U**. Due to the space limitation, we omit the proofs of these lemmas.

Lemma 3.1. *Given a set $I_{\min}(U)$ of intervals, Procedure **GD-W** finds a minimum subset $\widetilde{W}_D$ of W such that $\widetilde{W}_D$ dominates U.*

Lemma 3.2. *Given a set $I_{\max}(U)$ of intervals, Procedure **GD-U** finds a minimum subset $\widetilde{U}_D$ of U such that $\widetilde{U}_D$ dominates W.*

Let $\widetilde{W}_D$ and $\widetilde{U}_D$ be the minimum subsets of W and U output by Procedure **GD-W** and Procedure **GD-U**, respectively. By definition of a convex bipartite graph G, no edge of G can join two vertices of W or U. Let PD be any paired-dominating set of G, and let M be a perfect matching in the subgraph induced by PD. Then, the number of edges of M is at least $\max\{|\widetilde{W}_D|, |\widetilde{U}_D|\}$. Thus, $\gamma_p(G) \geqslant 2 \cdot \max\{|\widetilde{W}_D|, |\widetilde{U}_D|\}$, and, hence, we have the following lemma.

Lemma 3.3. *Let $\widetilde{W}_D$ and $\widetilde{U}_D$ be the minimum subsets of W and U output by Procedure **GD-W** and Procedure **GD-U**, respectively. Then, $\gamma_p(G) \geqslant 2 \cdot \max\{|\widetilde{W}_D|, |\widetilde{U}_D|\}$.*

Based upon the above three lemmas, our algorithm is given by a convex bipartite graph $G = (U, V, E)$ and contains the following four stages.

Stage 1: Partition U into k disjoint sorted clusters U_1, $U_2, \cdots, U_k$;

Stage 2: Compute the interval representation $I(U)$ of U, and construct $I_{\min}(U)$ and $I_{\max}(U)$ from $I(U)$;

Stage 3: Call Procedure **GD-W** on $I_{\min}(U)$ to find $\widetilde{W}_D$, and call Procedure **GD-U** on $I_{\max}(U)$ to find $\widetilde{U}_D$;

Stage 4: Compute $\gamma_p(G) = 2 \cdot \max\{|\widetilde{W}_D|, |\widetilde{U}_D|\}$, construct a minimum paired-dominating set MPD of G of size $\gamma_p(G)$, and output MPD.

In Stage 4, we construct a minimum paired-dominating set MPD of G of size $\gamma_p(G)$ as follows. Suppose that $|\widetilde{W}_D| = \max\{|\widetilde{W}_D|, |\widetilde{U}_D|\}$. Let $\widetilde{W}_D = \{w_1, w_2, \cdots, w_{|\widetilde{W}_D|}\}$, and let w_i, $1 \leqslant i \leqslant |\widetilde{W}_D|$, be the rightmost integer of interval I_{u_i} in $I_{\min}(U)$, where u_i is a vertex of U. By the construction of Procedure **GD-W**, all vertices of $\{u_1, u_2, \cdots, u_{|\widetilde{W}_D|}\}$ are distinct. By pairing w_i with u_i for $1 \leqslant i \leqslant |\widetilde{W}_D|$, we obtain a minimum paired-dominating set MPD of G of size $2 \cdot |\widetilde{W}_D|$, where $MPD = \cup_{1 \leqslant i \leqslant |\widetilde{W}_D|} \{w_i, u_i\}$. On the other hand, suppose that $|\widetilde{U}_D| = \max\{|\widetilde{W}_D|, |\widetilde{U}_D|\}$. Let $\widetilde{U}_D = \{u_1, u_2, \cdots, u_{|\widetilde{U}_D|}\}$, and let $w_i = u_i.begin$ for $1 \leqslant i \leqslant |\widetilde{W}_D|$, where w_i is a vertex of W. By the construction of Procedure **GD-U**, no two vertices of $\widetilde{U}_D$ are in the same cluster of U. Thus, all vertices of $\{w_1, w_2, \cdots, w_{|\widetilde{U}_D|}\}$ are distinct. By pairing u_i with w_i for $1 \leqslant i \leqslant |\widetilde{U}_D|$, we obtain a minimum paired-dominating set MPD of G of size $2 \cdot |\widetilde{U}_D|$, where $MPD = \cup_{1 \leqslant i \leqslant |\widetilde{U}_D|} \{u_i, w_i\}$. For example, given $I_{\min}(U)$ and $I_{\max}(U)$ shown in Fig. 4, Procedure **GD-W** outputs $\widetilde{W}_D = \{3, 8\}$, and Procedure **GD-U** outputs $\widetilde{U}_D = \{3, 6, 8\}$. Then, $\max\{|\widetilde{W}_D|, |\widetilde{U}_D|\} = |\widetilde{U}_D| = 3$. By the above construction, we obtain a set of pairs $(3, 1), (6, 4), (8, 7)$ and a minimum paired-dominating set MPD of size 6. Let k be the number of disjoint clusters of U. By Lemmas 2.2 and 3.3, $2k \geqslant \gamma_p(G) \geqslant 2 \cdot \max\{|\widetilde{W}_D|, |\widetilde{U}_D|\}$. Then, $|U| \geqslant k \geqslant \max\{|\widetilde{W}_D|, |\widetilde{U}_D|\}$. Thus, the above process for constructing a minimum paired-dominating set of G runs in $O(|U|)$ time, and, hence, Stage 4 can be done in $O(|U|)$ time.

Further, Stages 2–3 of the algorithm can be done in $O(|U|)$ time. In addition, Stage 1 runs in $O(|U| \log |U|)$ time. Thus, the algorithm runs in $O(|U| \log |U|)$ time and we conclude with the following theorem.

Theorem 3.4. *The paired-domination problem on a convex bipartite graph $G = (U, W, E)$ can be solved in $O(|U| \log |U|)$ time.*

4 Concluding Remarks

The pair-domination problem can be applied to allocate guards on vertices such that these guards protect every vertex, each guard is assigned another adjacent one, and they are designed as backup for each other. The paired-domination problem on bipartite graphs has been shown to be NP-complete. In this paper, we investigate the complexity of the problem on convex bipartite graphs, which form a subclass of bipartite graphs. We show that the paired-domination problem on a convex bipartite graph $G = (U, W, E)$ can be solved in $O(|U| \log |U|)$ time.

References

[1] K. Booth and G. Lueker, Testing for the consecutive ones property, interval graphs, and graph planarity using PQ-tree algorithms, J. Comput. Syst. Sci. 13 (1976) 335–379.

[2] P. Bose, A. Chan, F. Dehne, and Latzel, M., Coarse grained parallel maximum matching in convex bipar-

ISBN: 978-988-17012-8-2
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

tite graphs, In: 13th International Parallel Processing Symposium (IPPS'99), 1999, pp. 125–129.

[3] L. Chen, C. Lu, and Z. Zeng, Labelling algorithms for paired-domination problems in block and interval graphs, to appear in: J. Comb. Optim.

[4] T.C.E. Cheng, L. Kang, and C.T. Ng, Paired domination on interval and circular-arc graphs, Discrete Appl. Math. 155 (2007) 2077–2086.

[5] T.C.E. Cheng, L. Kang, and E. Shan, A polynomial-time algorithm for the paired-domination problem on permutation graphs, Discrete Appl. Math. 157 (2009) 262–271.

[6] E. Dekel and S. Sahni, A parallel matching for convex bipartite graphs and applications to scheduling, J. Parallel Distrib. Comput. 1 (1984) 185–205.

[7] F. Glover, Maximum matching in a convex bipartite graph, Nav. Res. Logist. Q. 14 (1967) 313–316.

[8] T.W. Haynes, S.T. Hedetniemi, and P.J. Slater, Fundamentals of Domination in Graphs, Marcel Dekker, New York, 1998.

[9] T.W. Haynes, S.T. Hedetniemi, and P.J. Slater, Domination in Graphs: Advanced Topics, Marcel Dekker, New York, 1998.

[10] T.W. Haynes and P.J. Slater, Paired-domination in graphs, Networks 32 (1998) 199–206.

[11] W. Lipski and F. Preparata, Efficient algorithms for finding maximum matchings in convex bipartite graphs and related problems, Acta Inform. 15 (1981) 329–346.

[12] H. Qiao, L. Kang, M. Cardei, and D.Z. Du, Paired-domination of trees, J. Global Optim. 25 (2003) 43–54.

[13] J. Soares and M.A. Stefanes, Algorithms for maximum independent set in convex bipartite graphs, Algorithmica 53 (2009) 35–49.

[14] G. Steiner and J. Yeoman, A linear time algorithm for maximum matchings in convex, bipartite graphs, Comput. Math. Appl. 31(12) (1996) 91–96.

ISBN: 978-988-17012-8-2
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)