Document Clustering Through Non-Negative Matrix Factorization: A Case Study of Hadoop for Computational Time Reduction of Large Scale Documents

Bishnu Prasad Gautam, Dipesh Shrestha, Members IAENG¹

Abstract— In this paper we discuss a new model for document clustering which has been adapted using non-negative matrix factorization method. The key idea is to cluster the documents after measuring the proximity of the documents with the extracted features. The extracted features are considered as the final cluster labels and clustering is done using cosine similarity which is equivalent to k-means with a single turn. This model was implemented using apache lucene project for indexing documents and mapreduce framework of apache hadoop project for parallel implementation of k-means algorithm. Since experiments were carried only in one cluster of Hadoop, the significant reduction in time was obtained by mapreduce implementation when clusters size exceeded 9 i.e. 40 documents averaging 1.5 kilobytes. Thus it is concluded that the feature extracted using NMF can be used to cluster documents considering them to be final cluster labels as in k-means, and for large scale documents, the parallel implementation using mapreduce can lead to reduction of computational time. We have termed this model as KNMF (K-means with NMF algorithm).

Index Terms— Document Clustering, KNMF, MapReduce

1. INTRODUCTION

The need for the organisation of data is a must for a quick and efficient retrieval of information. A robust means for organisation of data in any organisation has been the use of databases. Databases like the relational, object-oriented or object-relational databases, all have well structured format to keep data. Not all information that an organisation generates is kept or can be kept in databases. Information is stored in huge amount in form of unstructured or semi-structured documents. Organising these documents into meaningful groups is a typical sub-problem of Information Retrieval, in which there is need to learn about the general content of data, Cutting D et al. [1].

1.1 Document clustering

Dipesh Shrestha was affiliated with Wakkanai Hokusei Gakuen University, Wakkanani, Hokkaido, Japan. His research interests are data mining, cloud computing and system engineering. He is now working as a System Engineer with DynaSystem Co., Ltd., 1-14, North 6, West 6, Kita-ku, Sapporo, Hokkaido, Japan (Contact phone: +81-11-708-6786, Fax: +81-11-708-6787, e-mail: d_shre@dynasystem.co.jp)

Document clustering can loosely be defined as "clustering of documents". Clustering is a process of recognizing the similarity and/or dissimilarity between the given objects and thus, dividing them into meaningful subgroups sharing common characteristics. Good clusters are those in which the members inside the cluster have quite a deal of similar characteristics. Since clustering falls under unsupervised learning, predicting the documents to fall into certain class or group isn't carried out. Methods under document clustering can be categorized into two groups as follows:

a. Document partitioning (Flat Clustering)

This approach divides the documents into disjoint clusters. The various methods in this category are : k-means clustering, probabilistic clustering using the Naive Bayes or Gaussian model, latent semantic indexing (LSI), spectral clustering, non-negative matrix factorization (NMF).

b. Hierarchical clustering

This approach finds successive clusters of document from obtained clusters either using bottom-up (agglomerate) or top-bottom (divisive) approach.

1.2 Feature extraction

Traditional methods in document clustering use words as measure to find similarity between documents. These words are assumed to be mutually independent which in real application may not be the case. Traditional Vector Space Information Retrieval model uses words to describe the documents but in reality the concepts, semantics, features, and topics are what describe the documents. The extraction of these features from the documents is called Feature Extraction. The extracted features hold the most important idea and concept pertaining to the documents. Feature extraction has been successfully used in text mining with unsupervised algorithms like Principal Components Analysis (PCA), Singular Value Decomposition (SVD), and Non-Negative Matrix Factorization (NMF) involving factoring the document-word matrix [5].

1.3 Latent Semantic Indexing (LSI)

Latent Semantic Indexing (LSI) is a novel Information Retrieval technique that was designed to address the deficiencies of the classic VSM model. In order to overcome the shortcomings of VSM model, LSI estimates the structure in word usage through truncated Singular Value

¹ Bishnu Prasad Gautam is affiliated with Wakkanai Hokusei Gakuen University, Wakabadai 1 chome 2290-28, Wakkanai, Hokkaido, Japan. Currently doing research in cloud computing, networking and distributed computing(Contact phone: +81-162-32-7511, Fax: +81-162-32-7500, e-mail: gautam@wakhok.ac.jp)

Decomposition (SVD). Retrieval is then performed using a database of singular values and vectors obtained from the truncated SVD. Application of LSI with results can be found in Berry et al. [14] and Landauer et al. [15].

1.4 Non-negative matrix factorization (NMF)

Non-negative matrix factorization is a special type of matrix factorization where the constraint of non-negativity is on the lower ranked matrices. It decomposes a matrix V_{mn} into the product of two lower rank matrices W_{mk} and H_{kn} , such that V_{mn} is approximately equal to W_{mk} times H_{kn} .

$$V_{mn} \approx W_{mk} \cdot H_{kn} \tag{1}$$

Where, $k \ll \min(m,n)$ and optimum value of k depends on the application and is also influenced by the nature of the collection itself [13]. In the application of document clustering, k is the number of features to be extracted or it may be called the number of clusters required. V contains column as document vectors and rows as term vectors, the components of document vectors represent the relationship between the documents and the terms. W contains columns as feature vectors or the basis vectors which may not always be orthogonal (for example, when the features are not independent and have some have overlaps). H contains columns with weights associated with each basis vectors in W.

Thus, each document vector from the document-term matrix can be approximately composed by the linear combination of the basis vectors from W weighted by the corresponding columns from H. Let v_i be any document vector from matrix V, column vectors of W be $\{W_1, W_{2,...,W_k}\}$ and the corresponding components from column of matrix H be $\{h_{i1}, h_{i2,...,h_{ik}}\}$ then,

$$w_i \approx W_1 \cdot h_{i1} + W_2 \cdot h_{i2} + \dots + W_k \cdot h_{ik}$$
⁽²⁾

NMF uses an iterative procedure to modify the initial values of W_{mk} and H_{kn} so that the product approaches V_{mn} The procedure terminates when the approximation error converges or the specified number of iterations is reached. The NMF decomposition is non-unique; the matrices W and H depend on the NMF algorithm employed and the error measure used to check convergence. Some of the NMF algorithm types are, multiplicative update algorithm by Lee and Seung [2], sparse encoding by Hoyer [10], gradient descent with constrained least squares by Pauca [11] and alternating least squares algorithm by Pattero [12]. They differ in the measure cost function for measuring the divergence between V and WH or by regularization of the W and/or H matrices.

Two simple cost functions studied by Lee and Seung are the squared error (or Frobenius norm) and an extension of the Kullback-Leibler divergence to positive matrices. Each cost function leads to a different NMF algorithm, usually minimizing the divergence using iterative update rules. Using the Frobenius norm for matrices, the objective function or minimization problem can be stated as

$$\underset{W,H}{\overset{min}{\Vert}} \| V - WH \|_{F}^{2} \tag{3}$$

where, W and H are non-negative. The method proposed by Lee and Sung [2] based on multiplicative update rules using Forbenus norm, popularly called multiplicative method (MM) which can be described as follows.

1.4.1 MM Algorithm

- i. Initialize W and H with non-negative values.
- ii. Iterate for each c, j, and i until within approximation error converge or after l iterations:

(a)
$$H_{cj} \leftarrow H_{cj} \frac{(W^T V)_{cj}}{(W^T W H)_{cj} + e}$$
 (4)

(b)
$$W_{ic} \leftarrow W_{ic} \frac{\left(VH^T\right)_{ic}}{\left(WHH^T\right)_{ic} + e}$$
 (5)

In steps ii (a) and ii (b), e, a small positive parameter equal to 10^{-9} , is added to avoid division by zero. As observed from the MM Algorithm, W and H remain non-negative during the updates.

Lee and Seung [2] proved that with the above update rules objective function (1) achieve monotonic convergence and is non-increasing, and they becomes constant if and only if W and H are at a stationary point. The solution to the objective function is not unique.

1.5 Document clustering with NMF

Ding C et al. [8] shows that when Frobenius norm is used as a divergence and adding an orthogonality constraint $H^{T} H$ = I, NMF is equivalent to a relaxed form of k-means clustering. Wei Xu et al. were the first ones to use NMF for document clustering [6] where unit euclidean distance constraint was added to column vectors in W. This work was extended by Yang et al. [7] adding the sparsity constraints because sparseness is one of the important characters of huge data in semantic space. In both of the works the clustering has been based on the interpretation of the elements of the matrices.

"There is an analogy with the SVD in interpreting the meaning of the two non-negative matrices U and V. Each element u_{ij} of matrix U represents the degree to which term f_i W belongs to cluster j, while each element v_{ij} of matrix V indicates to which degree document i is associated with cluster j. If document i solely belongs to cluster x, then v_{ix} will take on a large value while rest of the elements in i^{th} row vector of V will take on a small value close to zero." [6]

From the above statement, we can summarize the relationship as $W \approx UV$. From the work of Kanjani K [9] it is seen that the accuracy of algorithm from Lee and Seung [2] is higher than their derivatives [9], [10]. In our model, the original multiplicative update proposed by Lee and Seung [2] is undertaken.

2. METHODOLOGY

The following section describes the proposed model in detail. It includes the clustering method with the proposed model and the parallel implementation strategy of k-means. The latter parts contain explanation to the underlying architecture of Hadoop Distributed File System (HDFS) over which k-means algorithm is implemented.

2.1 The Proposed Model

From hereinafter our model is termed as KNMF (k-means with NMF algorithm). In KNMF the document clustering is

done on basis of the similarity between the extracted features and each document. Let, extracted feature vectors be $F=\{f_1,f_2,f_3...,f_k\}$ computed by NMF. Let the documents in the term-document matrix be $V = \{d_1,d_2,d_3...,d_n\}$, then document d_i is said to be the member of cluster f_x if, the angle between d_i and f_x is minimum.

2.1.1 The methodology adapted

- i. Construct the term-document matrix V from the files of a given folder using term frequency-inverse document frequency
- ii. Normalize length of columns of V to unit Euclidean length.
- iii. Perform NMF based on Lee and Seung [2] on V and get W and H using (1).
- iv. Apply cosine similarity to measure distance between the documents d_i and extracted features/vectors of W. Assign d_i to w_x if the angle between d_i and w_x is smallest. This is equivalent to k-means algorithm with a single turn.

To run the parallel version of k-means algorithm, Hadoop is started in local reference mode and pseudo-distributed mode and the k-means job is submitted to the JobClient. The time taken from i though iii and the total time taken were noted separately.

2.1.2 Steps in indexing the documents in a folder

- i. Determine if the document is new; update the index of non-updated document.
- ii. If it's up to date then do nothing . While the document is new, create a Lucene Document, if it's not updated then delete the old document and create new Lucene Document.
- iii. Extract the words from the document.
- iv. Remove the stop-words.
- v. Apply Stemming.
- vi. Store the created Lucene Document in index.
- vii. Remove stray files.

The Lucene Document contains three fields namely *path*, *contents* and *modified*. These fields respectively store the full-path of the document, the terms and modified date (to seconds). The field *path* is used to uniquely identify documents in the index; the field *modified* is used to avoid re-indexing the documents if it's not modified. In the step vii the documents which have been removed from the folder but with entries in the index are removed from index. This step has been followed to keep the optimal word dictionary size.

In our document stop-words are filtered using the wordlist from the project of Key Phrase Extraction Algorithm [4] which defines some 499 stop-words. This worldlist can be modified by the users if further stop-words are required to be maintained. After the removal of stop-words, the document is stemmed by the Porter algorithm [3].

2.2 Parallel implementation of k-means

The parallel implementation strategy of k-means algorithms in multi-core is described [20] as:

"In k-means [9], it is clear that the operation of computing the Euclidean distance between the sample vectors and the centroids can be parallelized by splitting the data into individual subgroups and clustering samples in each subgroup separately (by the mapper). In recalculating new centroid vectors, we divide the sample vectors into subgroups,

compute the sum of vectors in each subgroup in parallel, and finally the reducer will add up the partial sums and compute the new centroids. "

In the same paper, it was noted that the performance of k-means algorithm with map-reduce increased in an average 1.937 times than its serial implementation without map-reduce. From as low as 1.888 times in Synthetic Time Series (sample = 100001 and features = 10) to as high as 1.973 times in KDD Cup 999 (sample = 494021 and features = 41). It also adds that it was possible to achieve 54 times speedup on 64 cores.

Indeed, the performance upgrading with the increase of number of cores was almost linear. This paper was the source for the foundation of Mahout project² which include the implementation strategy of k-means algorithm in map-reduce over the Hadoop. Implementation of k-means in mapreduce is also presented in lectures from [17]. Drost, I [16] describes k-means of Mahout project. In [18] Gillick et al. studied the performance of Hadoop's implementation of mapreduce and has suggested performance enhancing guidance as well.

2.2.1 MapReduce in KNMF

In the method proposed in this work, since the final clusters are the features extracted from the NMF algorithms, the parallelization strategy of map-reduce can be applied to compute the distance between the data vectors and the feature vectors. Since it requires only one iteration, it can be considered as having only one map-reduce operation. Furthermore, since the cluster centred computation isn't needed, only one map operation is sufficient. The map operation intakes the list of feature vectors and individual data vectors and outputs the closest feature vector for the data vector.

For instance, we have list of data vectors $V = \{v1, v2, ..., vn\}$ and list of feature vectors $W = \{w1, w2, w3\}$ computed by NMF. Then,

$\langle v_i, W \rangle \rightarrow map \rightarrow \langle v_i, w_x \rangle$

where w_x is the closest (cosine similarity) feature vector to data vector v_i

2.3 Hadoop

Hadoop is a distributed file system written in Java with an additional implementation of Google's MapReduce framework [19] that enables application based on map-reduce paradigm to run over the file system. It provides high throughput access to data which is makes it suitable for working with large scale data (typical block size is 64 Mb).

2.3.1 Hadoop Distributed File System (HDFS)³

It is a distributed file system specifically designed for fault-tolerant which can be deployed on low-cost hardware. It is based on master/slave architecture. The master nodes are called namenodes and every cluster has only one namenode. It manages the filesystem namespace and access to files by client (opening, closing, renaming files).

² http://lucene.apache.org/mahout/

³ http://hadoop.apache.org/core/

Proceedings of the International MultiConference of Engineers and Computer Scientists 2010 Vol I, IMECS 2010, March 17 - 19, 2010, Hong Kong



Figure 1 : Clustering with our model

The task of datanode is to manage the data stored in the node (each file is stored in one of more blocks). It's responsible for read/write requests from clients (creation, deletion, replication of blocks).

All HDFS communication protocols are layered on top of the TCP/IP protocol. Files in HDFS are write-once (read many) and have strictly one writer at any time. The blocks of a file are replicated for fault tolerance. The block size and replication factor are configurable per file. Hadoop can be run in Local(stand-alone), Pseudo-distributed mode or Fully-distributed mode.

2.3.2 MapReduce framework in Hadoop

same set of nodes.

The input and output to the map-reduce application can be shown as follows:

(input) $<k1,v1>\rightarrow$ map $\rightarrow<k2,v2>\rightarrow$ reduce <k3,v3> (output) The input data is divided and processed in parallel across different machines, processes in map phase and the reduce combines the data according the key to give final output. For this sort of task the framework should be based on master-slave architecture. Since HDFS is itself based on master-slave architecture, MapReduce framework fits well in Hadoop. Moreover usually the compute nodes and the storage nodes are the same, that is, the map-reduce framework and the distributed filesystem are running on the

This configuration allows the framework to effectively schedule tasks on the nodes where data is already present, resulting in very high aggregate bandwidth across the cluster.

To implement map-reduce framework in Hadoop, there is a single master called JobTracker per job. Job is the list of task submitted to the MapReduce framework in Hadoop. The master is responsible for scheduling the jobs' component tasks on the slaves, monitoring them and re-executing the failed tasks. There can be one slave or tasktracker per cluster-node. The slaves execute the tasks as directed by the master.

3. IMPLEMENTATION

Since Hadoop, Lucene and Mahout are built with Java natively, it would be easy for the interoperability between the

ISBN: 978-988-17012-8-2 ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online) components developed with Java. Considering this fact, Java was chosen as the programming language for the implementation ⁴ of our proposed model. Before the documents can be clustered, they need to be indexed. For the purpose of indexing, Lucene APIs have been utilized. The documents are determined whether they are up-to-date in index. If it's up to date then do nothing what follows. If the document is new, a Lucene Document is created, if it's not updated then old documents are deleted and new Lucene Document is created. The stop-words are removed using key-words from [4] and Potter Stemming [3] is applied. Indexing is described in detail in Section 2.1.2.

Clustering has more complex steps than indexing of the documents. It involves creation of document-term matrix followed by the application of KNMF. Section 2.1.1 describes the steps in detail. Figure 1 shows the files to be clustered in the left side frame and the resulting clusters and folders along with their corresponding files is shown in the lower middle frame. The upper middle frame shows the extracted themes. The settings regarding number of clusters, convergence delta for of k-means and NMF, stop words list is done from the right side frame.

4. EXPERIMENTS AND RESULTS

Table i: List of	Topics of 20	New Groups

comp.graphics comp.os.ms-windows.misc comp.sys.ibm.pc.hardware comp.sys.mac.hardware comp.windows.x	rec.autos rec.motorcycles rec.sport.baseball rec.sport.hockey	sci.crypt sci.electronics sci.med sci.space
Misc.forsale	Talk.politics.misc talk.politics.guns talk.politics.mideast	talk.religion.mis c alt.atheism soc.religion.christ ian

⁴ This application (named Swami) is distributed under GNU GPL v3.0. It is currently tested only on GNU/Linux and can be downloaded from http://www.wakhok.ac.jp/~dipesh/swami

Proceedings of the International MultiConference of Engineers and Computer Scientists 2010 Vol I, IMECS 2010, March 17 - 19, 2010, Hong Kong

4.1 Data set Description

20 News Groups⁵ is a popular data set for text clustering and classification. It has a collection about 20,000 documents across 20 different newsgroups from Usenet. Each newsgroup is stored in a subdirectory, with each article stored as a separate file. Some of the newsgroups are closely related with each other while others are highly unrelated. Table i shows the topics of the newsgroups arranged by Jason Renn⁶

4.2 Experiment

For the purpose of experimentation, clustering was done using up to 10 groups. Each group containing five documents were taken randomly and added to a folder. The folder was indexed after removing the stop-words using KEA stop-words [4] and applying Porter stemming [3]. Then the clustering was done and results were noted. Other five documents were taken out randomly from another group, added to the folder, indexed and clustered accordingly. In this way a total of 10 groups with 50 documents were clustered. Clustering results were noted for three cases, without using Hadoop, using Hadoop in local mode and finally using Hadoop in pseudo-distributed mode.

In addition to this, for KNMF the following parameters were used

- i. NMF: convergence parameter = 0.001 and maximum iteration = 10.
- ii. K-Means: k = number of news groups in folder, convergence parameter = 0.001, maximum iteration = 1, distance measure = cosine

Since the length of W was not normalized as suggested by Xu et al. [6] there was no unique solution. For this purpose the experiments with the highest values of AC among the three cases as mentioned above was taken.

The performance of the clustering algorithm evaluated by calculating the accuracy defined in [6] is illustrated as follows:

Given a document d_i , let l_i and α_i be the cluster label and the label provided by the document corpus, respectively. The AC is defined as follows:

$$AC = \frac{\sum \delta(\alpha_i, map(l_i))}{n}$$
(6)

where n denotes the total number of documents, $\delta(x, y)$ is the delta function that equals one if x = y and equals zero otherwise, and map(l_i) is the mapping function that maps each cluster label l_i to the equivalent label from the document corpus.

4.3 Results

Table ii shows the time taken by KNMF algorithm on the 20 Newsgroup collection on a Linux Ubuntu 8.04 machine (1.66Ghz Intel Pentium Dual-core, 1G RAM) with and without MapReduce (Map = 2). The numbers of clusters were denoted by k and the accuracy is denoted by AC. Further, the column *Without Hadoop* and *Local Reference*

Table ii: Results

k	AC	Without Hadoop	Local Reference mode	Pseudo-Distri buted mode
2	0.80	0.558/0.597	0.390/1.580	1/2
3	0.75	0.898/0.958	0.796/2.0	1/2
4	0.66	1.090/1.159	1.074/2.307	2/2
5	0.60	1.961/2.111	2.155/3.457	2/2
6	0.56	4.086/5.295	4.122/5.617	1/1
7	0.68	6.340/7.158	6.262/7.653	2/1
8	0.625	8.710/9.874	8.615/10.025	2/2
9	0.533	12.435/14.088	12.503/14.027	3/3
10	0.60	26.963/30.615	26.700/30.648	3/3

mode shows time taken by NMF and KNMF (The value at numerator is time taken by NMF and the value at denominator is total time taken by KNMF). The column *pseudo-distributed mode* shows time taken by Map phases (The numerator value is the time taken by map phase 1 and the denominator value is the time taken by map phase 2).

The figure 2 shows the time taken during the clustering phase which is calculated from table ii as (the total time taken - time by NMF). For the pseudo-distributed mode of Hadoop, the time taken by map phase is considered as the time taken for clustering. It can be seen that the time taken by the map phase of pseudo-distributed mode of Hadoop is guite steady and rises only when the number of clusters increase to 8. The time taken by local reference and serial implementation or without using Hadoop exceeds time taken by pseudo-distributed mode of Hadoop for cluster size equivalent to 10.



Figure 2: Time taken by the clustering phase (k-means with 1 turn)

⁵ http://kdd.ics.uci.edu/databases/20newsgroups/20newsgroups.html

⁶ http://people.csail.mit.edu/jrennie/

Proceedings of the International MultiConference of Engineers and Computer Scientists 2010 Vol I, IMECS 2010, March 17 - 19, 2010, Hong Kong

5. CONCLUSIONS

In this work, a new working model for document clustering was given along with development of application based on this model. This application can be used to organise documents into sub-folders without having the knowledge about the contents of the document. This really improves the performance of information retrieval in any scenario. The accuracy of model was tested and found to be 80% for 2 clusters of documents and 75% for 3 clusters and the results averages to 65% when for 2 through 10 clusters. NMF has shown to be a good measure for clustering document and this work has also shown similar results when the extracted features are used as the final cluster labels for k-means algorithm. To scale the document clustering the proposed model uses the map-reduce implementation of k-means from Apache Hadoop Project and it has shown to scale even in a single cluster computer when clusters size exceeded 9 i.e. 40 documents averaging 1.5 kilobytes.

REFERENCES

- Cutting, D, Karger, D, Pederson, J & Tukey, J (1992). Scatter/gather: A cluster-based approach to browsing large document collections, In Proceedings of ACM SIGIR.
- [2] Lee, D & Seung, H (2001). Algorithms for non-negative matrix factorization. In T. G. Dietterich and V. Tresp, editors, Advances in Neural Information Processing Systems, volume 13, Proceedings of the 2000 Conference: 556-562, *The MIT Press*.
- [3] Porter, MF (1980). "An algorithm for suffix stripping", *Program*, Vol. 14, No. 3, pages 130-137
- <u>http://tartarus.org/~martin/PorterStemmer/def.txt</u> [4] Key Phrase Extraction Algorithm (KEA)
- http://www.nzdl.org/Kea/
- [5] Guduru, N (2006). Text mining with support vector machines and non-negative matrix factorization algorithm. *Masters Thesis*. *University of Rhode Island, CS Dept.*
- [6] Xu, W, Liu, X & Gong, Y (2003). Document clustering based on non-negative matrix factorization, Proceedings of ACM SIGIR, pages 267–273.
- [7] Yang, CF, Ye, M & Zhao, J (2005). Document clustering based on non-negative sparse matrix factorization. International Conference on advances in Natural Computation, pages 557–563.
- [8] Ding, C, He X, & Simon, HD (2005). <u>On the Equivalence of Nonnegative Matrix Factorization and Spectral Clustering</u>. Proceedings in SIAM International Conference on Data Mining, pages 606-610.
- [9] Kanjani, K (2007). Parallel Non Negative Matrix Factorization for Document Clustering.
- [10] Hoyer, P (2002). Non-Negative Sparse Coding. In Proceedings of the IEEE Workshop on Neural Networks for Signal Processing, Martigny, Switzerland.
- [11] Pauca, V, Shahnaz, F, Berry, MW & Plemmons R (April 22-24, 2004). Text Mining Using Non-Negative Matrix Factorizations. In Proceedings of the Fourth SIAM International Conference on Data Mining, Lake Buena Vista, FL.
- [12] Amy, L & Carl, M (2006). ALS Algorithms Nonnegative Matrix Factorization Text Mining.
- [13] Guillamet, D & Vitria, J (2002). Determining a Suitable Metric when Using Non-Negative Matrix Factorization. In Sixteenth International Conference on Pattern Recognition (ICPR'02), Vol. 2, Quebec City, QC, Canada.
- [14] Berry, M, Dumais, ST & O'Brien, GW(1995). Using Linear Algebra for Intelligent Information Retrieval. Illustration of the application of LSA to document retrieval.
- [15] Landauer, T, Foltz, PW & Laham, D(1998). Introduction to Latent Semantic Analysis. Discourse Processes 25: pages 259–284
- [16] Drost, I (November 2008). Apache Mahout : Bringing Machine Learning to Industrial Strength, In Proceedings of ApacheCon 2008, pages 14-29, New Orleans
- [17] Michels, S (July 5, 2007). Problem Solving on Large-Scale Clusters, Lecture 4.

- [18] Gillick, D, Faria, A & DeNero, J (December 18, 2006). MapReduce: Distributed Computing for Machine Learning.
- [19] Dean, J & Ghemawat, J (December 2004). MapReduce: Simplified Data Processing on Large Clusters, In the Proceedings of the 6th Symp. on Operating Systems Design and Implementation.
- [20] Chu, CT, Kim, SK, Lin, YA, Yu, YY, Bradski, G, Yng, Andrew, & Olukotun, K (2006). Map-Reduce for Machine Learning on Multicore, *NIPS*