Neural Network-based Decoupled Sliding Mode Controller Design for Discrete-time Nonlinear MIMO Systems by SPSA Algorithm

Ching-Hung Lee, *Member, IAENG*, Hao-Yuan Hsueh, and Jen-Chieh Chien

Abstract—In this paper, a neural network-based sliding-mode controller design approach with decoupled method is proposed for a class of nonlinear discrete-time uncertain multi-input-multi-output (MIMO) systems. The neural network is used to generate the proper control inputs by simultaneous perturbation stochastic approximation (SPSA) algorithm. The decoupled method simplifies the design complexity to achieve asymptotic stability for the uncertain nonlinear system with external disturbance. The proposed control scheme does not need the exactly system model to avoid the mathematical derivation. In addition, the frictional force analysis for the nonlinear inverted double pendulum system is considered to investigate the relationship between controller and frictional force. Simulation results are presented to illustrate the effectiveness of our approach.

Index Terms—nonlinear control, neural network system, decouple, sliding mode control, simultaneous perturbation stochastic approximation

I. INTRODUCTION

Sliding-Mode control (SMC) has been suggested as an approach for the control of systems with nonlinearities, uncertain dynamics and bounded input disturbances [1, 2]. SMC technique provides fast error convergence and strong robustness for control system [4, 5]. However, SMC requires system dynamic model of the plant and bounds on modeling uncertainty to formulate control laws with guaranteed stability. For solving this problem, the intelligent controllers, developed by principles of fuzzy logic, neural network (NN), and genetic algorithms, should be designed to achieve the control objective [1-5,7].

Recently, NN-based stable and on-line adaptive control has been paid much attention in nonlinear adaptive control [1, 3, 4, 7]. To develop the neural-network-based controller, the corresponding adaptive laws should be derived by the measurement of objective function. The usual used methods are gradient method and Lyapunov approach. However, it is difficult or impossible to directly obtain the gradient of objective function or the derivation of adaptive laws based on Lyapunov approach is difficult and complex to obtain. Therefore, stochastic approximation algorithms such as Kiefer-Wolfowitz finite difference gradient approximation and simultaneous perturbation stochastic approximation (SPSA) were proposed to solve these problems [13-15]. By using SPSA, we can only measure the objective function to solve the optimization problem for providing the adaptive laws of NN.

Ching-Hung Lee is with Department of Electrical Engineering, Yuan-Ze University, Chung-li, Taoyuan 320, Taiwan. (phone: +886-3-4638800, ext: 7119; fax: +886-3-4639355, e-mail: chlee@saturn.yzu.edu.tw).

In this paper, we propose a new neural-network-based decoupled sliding-mode controller scheme to solve the control of a class of multi-input-multi-output (MIMO) nonlinear dynamic systems via the SPSA algorithm. The considered nonlinear system has uncertainty, variant parameters, and external disturbance. We use the decoupled architecture and sliding-mode control approach for the input variables of NN system for simplifying the computational complexity. The weights of the NN are updated according to the results of SPSA algorithm for the purpose of controlling the system states to stay in sliding surface. The NN controller generates the proper control signal for the unknown nonlinear dynamic system. Through SPSA laws, we can adjust the parameters to achieve the stability objective. Finally, the frictional force analysis for the nonlinear inverted double pendulum system is considered to investigate the relationship between controller and frictional force. Simulation results are presented to illustrate the effectiveness of our approach.

This paper is organized as follows. Section II introduces the problem formulation and used neural network. The proposed new neural-network-based decoupled sliding-mode controller scheme (DSMCNN) is introduced in Section III. Section IV shows the simulation results. Conclusion is given in Section V.

II. PRELIMINARIES

A. System description

It is known that the coupling phenomenon is existed in many real-world systems. The so-called couple phenomenon is that a control force will influence the states simultaneous. Consider the ith sub-system of discrete-time nonlinear system represented by the following model

$$\Omega_i : \begin{cases} x_{i1}(k+1) = x_{i1}(k) + x_{i2}(k) \cdot T_s \\ x_{i2}(k+1) = x_{i2}(k) + \left[f_{i1}(x_i(k)) \right. \\ \qquad \left. + g_{i1}(x_i(k)) \cdot u_i(k) + d_{i1}(k) \right] \cdot T_s \\ x_{i3}(k+1) = x_{i3}(k) + x_{i4}(k) \cdot T_s \\ x_{i4}(k+1) = x_{i4}(k) + \left[f_{i2}(x_i(k)) \right. \\ \qquad \left. + g_{i2}(x_i(k)) \cdot u_i(k) + d_{i2}(k) \right] \cdot T_s \end{cases} \quad (1)$$

where $i=1,..., n$, $x_i(k) = [x_{i1}(k)\ x_{i2}(k)\ x_{i3}(k)\ x_{i4}(k)]^T$ is the measurable state vector, $f_{i1}(x)$ and $g_{i1}(x)$, where $i=1,..n$, are nonlinear discrete-time functions, u_i is the control inputs, and $d_{i1}(t)$, $d_{i2}(t)$, $i=1,..n$, are external bounded disturbances, i.e., $|d_{i1}(t)| \leq D_{i1}, |d_{i2}(t)| \leq D_{i2}$, $i=1,... n$. For this ith sub-system, our control objective is to design the input u_i such that state vector x_i converges to zero. Obviously, the decoupling method should be adopted to decouple the state x_{i2} and x_{i4}.

Thus we utilize the method of literature [4, 5, 12] to design decoupled control in the discrete-time form.

For sub-system (1), we first define the following sliding surfaces

$$s_{i1}(k+1) = c_{i1}(x_{i1}(k+1) - z_i(k+1)) + x_{i2}(k+1)$$
$$s_{i2}(k+1) = c_{i2}(x_{i3}(k+1)) + x_{i4}(k+1) \tag{2}$$

where $z_i(k+1) = -z_{Ui}\text{sat}(s_{i2}(k+1)/\phi_{i2})$, with $0<z_{Ui,}<1$, (sat($\cdot$) denotes the saturation function).

In the design of decoupled sliding-mode controller, we can give an equivalent control such that the states can stay on sliding surface. The equivalent control can be obtained by setting s_{i1} are equal to zero. From equation (2), we can observe that if s_{i1} achieves to zero, then x_{i1} is equal to z_i and x_{i2} also achieves to zero. We also find that z_i and x_{i1} achieve to zero simultaneously when s_{i2} achieves to zero. Therefore, if we derive s_{i1} toward to zero, the control objective can be achieved. In addition, the selection of coefficients c_{i1} and c_{i2} will affect the behavior in the transient state of the system. For achieving favorable transient response, proper choice of sliding factor is necessary.

According the above discussion, s_{i1} will be fed into the neural-network (NN) to generate the proper control signals. In addition, our control objective can also be transferred as to derive s_{i1} to approach to zero.
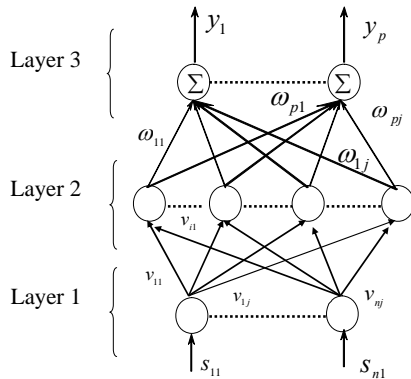

Figure1: Network diagram of the used NN system.

B. NEURAL-NETWORK SYSTEM

Fig. 1 shows the schematic diagram of a three layer NN system. Layer 1 accepts input variables. Its nodes represent input linguistic variables. The nodes only transmit input variables to the next layer directly,

$$O_j^{(1)}(k) = s_{i1}(k) \tag{3}$$

where $s_{i1}(k)$ is input at discrete-time index k. Layer2 is a hidden layer.

$$net_j^{(2)}(k) = \sum_i v_{ij}(k) \cdot s_{i1}(k) \tag{4}$$

$$O_j^{(2)}(k) = \frac{1}{(1+e^{-net_j^{(1)}(k)})^2} \tag{5}$$

where $v_{ij}(k)$ is the weight between the layer 1 and layer 2.

Finally, Layer 3 is the output layer. The links between layer 2 and layer 3 are connected by weighting value

$$O_p^{(3)}(k) = \sum_{j=1}^R \omega_{pj}(k)O_j^{(2)}(k). \tag{6}$$

III. DESIGN OF DECOUPLED NN SLIDING-MODE CONTROLLER

Most of control systems contain the un-model uncertainty, parameters variation, or external disturbance. Therefore, the

adaptive property and robustness analysis should be considered. In this section, we introduce the decoupled neural network sliding-mode controller for obtaining the equivalent control. This approach does not need the system dynamics model exactly. It uses the model free concept to develop the adaptive DSMCNN control scheme. Fig. 2 shows the decoupled adaptive control scheme. First, the system state variables are transferred into SMC variables s_{11} and s_{21} by the decoupled method (shown in equations (2)). Then, the NN can generate the proper control signals such that the system outputs approach to zero. The adaptive laws of NN's parameters can be derived by the following description.
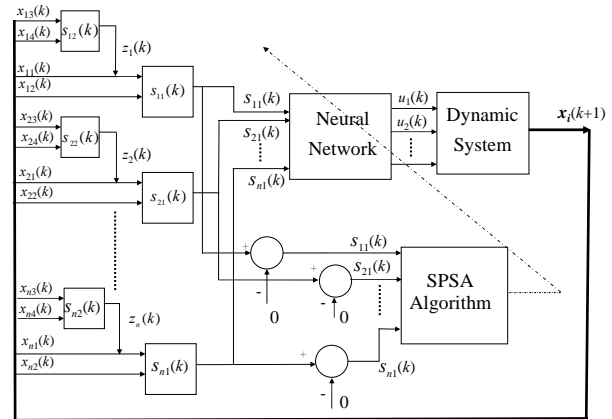

Figure 2:The proposed DSMCNN control scheme via SPSA.

Our control goal is to minimize the following cost function:

$$E(k) = \frac{1}{2}\sum_{i=1}^n (s_{i1}(k))^2 \tag{7}$$

By the gradient-descent method, the update laws of NN's parameters are

$$w_{pj}(k+1) = w_{pj}(k) + \Delta w_{pj}(k) = w_{pj}(k) + \eta\left(-\frac{\partial E(k)}{\partial w_{pj}}\right) \tag{8}$$

$$v_{ij}(k+1) = v_{ij}(k) + \Delta v_{ij}(k) = v_{ij}(k) + \eta\left(-\frac{\partial E(k)}{\partial v_{ij}}\right). \tag{9}$$

To avoid deriving the gradient of objective function, we here utilize the SPSA algorithm to approximate

$$\frac{\partial E(k)}{\partial w_{pj}} = \frac{E(w_{pj}(k) + R_{pj}(k) \cdot \Delta_{pj}(k)) - E(w_{pj}(k))}{R_{pj}(k) \cdot \Delta_{pj}(k)} \tag{10}$$

$$\frac{\partial E(k)}{\partial v_{ij}} = \frac{E(v_{ij}(k) + R_{ij}(k) \cdot \Delta_{ij}(k)) - E(v_{ij}(k))}{R_{ij}(k) \cdot \Delta_{ij}(k)} \tag{11}$$

where $R_{ij}(k)$ is perturbation due to the equation and $\Delta_{ij}(k)$ is a vector whose elements are either 1 or -1 in random.

Therefore, we should calculate, $E(w_{pj}(k)+R_{pj}(k)\cdot\Delta_{pj}(k))$, $E(w_{pj}(k))$, $E(v_{ij}(k)+R_{ij}(k)\cdot\Delta_{ij}(k))$, and $E(v_{ij}(k))$ at first. However, in experimental applications, we cannot feed two control forces to evaluate the corresponding objective values at the same time. In this paper, we use mathematics derivation to improve the problem.

From equation (7), we have

$$\frac{\partial E(k)}{\partial w_{pj}} = s_{i1}(k) \cdot \frac{\partial s_{i1}(k)}{\partial w_{pj}} \tag{12}$$

$$\frac{\partial E(k)}{\partial v_{ij}} = s_{i1}(k) \cdot \frac{\partial s_{i1}(k)}{\partial v_{ij}} . \qquad (13)$$

Next, we use modified SPSA to approximate $\frac{\partial s_{i1}(k)}{\partial w_{pj}}$.

Substitute system (1) into the sliding surface (2), we have

$$s_{i1}(k+1) = c_{i1}(x_{i1}(k) + x_{i2}(k) \cdot T_s - z_i(k+1)) + x_{i2}(k)$$
$$+ \left[f_{i1}(x_i(k)) + g_{i1}(x_i(k)) \cdot u_i(k) + d_{i1}(k) \right] \cdot T_s). \qquad (14)$$

The perturbed results is shown below:

$$s_{\Delta i1}(k+1) = c_{i1}(x_{i1}(k) + x_{i2}(k) \cdot T_s - z_i(k+1)) + x_{i2}(k)$$
$$+ \left[f_{i1}(x_i(k)) + g_{i1}(x_i(k)) \cdot u_{\Delta i}(k) + d_{i1}(k) \right] \cdot T_s). \qquad (15)$$

Using the SPSA algorithm to approximate $\frac{\partial s_{i1}(k)}{\partial w_{pj}}$, we

obtain

$$\frac{\partial s_{i1}(k)}{\partial \omega_{pj}} = \frac{s_{i1}(w_{pj}(k) + R_{pj}(k) \cdot \Delta_{pj}(k)) - s_{i1}(w_{pj}(k))}{R_{pj}(k) \cdot \Delta_{pj}(k)}$$
$$= \frac{s_{\Delta i1}(k+1) - s_{i1}(k+1)}{R_{pj}(k) \cdot \Delta_{pj}(k)} \qquad (16)$$
$$= \frac{u_{\Delta i}(k) - u_i(k)}{R_{pj}(k) \cdot \Delta_{pj}(k)} \cdot (g_{i1}(x_i(k)) \cdot T_s)$$

$$\frac{\partial s_{i1}(k)}{\partial v_{ij}} = \frac{s_{i1}(v_{ij}(k) + R_{ij}(k) \cdot \Delta_{ij}(k)) - s_{i1}(v_{ij}(k))}{R_{ij}(k) \cdot \Delta_{ij}(k)}$$
$$= \sum_{i=1}^n \frac{u_{\Delta i}(k) - u_i(k)}{R_{ij}(k) \cdot \Delta_{ij}(k)} \cdot (g_{i1}(x_i(k)) \cdot T_s). \qquad (17)$$

Substituting equations (16) and (17) into equations (12) and (13), we can obtain

$$\frac{\partial E(k)}{\partial \omega_{pj}} = s_{i1}(k) \cdot \frac{u_{\Delta i}(k) - u_i(k)}{R_{pj}(k) \cdot \Delta_{pj}(k)} \cdot (g_{i1}(x_i(k)) \cdot T_s). (18)$$

$$\frac{\partial E(k)}{\partial v_{ij}} = s_{i1}(k) \cdot \frac{u_{\Delta i}(k) - u_i(k)}{R_{ij}(k) \cdot \Delta_{ij}(k)} \cdot (g_{i1}(x_i(k)) \cdot T_s). \quad (19)$$

Without derivation from the system dynamics model, we should evaluate the objective function twice for obtaining

the gradient terms $\frac{\partial s_{i1}(k)}{\partial w_{pj}}$ and $\frac{\partial s_{i1}(k)}{\partial v_{ij}}$. As above, we can

obtain the same result without repetition evaluations. In addition, the control signal is proportional to the difference between the control input and perturbed control one. That is, the control input does not feed into the nonlinear system for evaluating the objective function. This provides a simple approach for real-time control system. In this paper, this approach is applied on a double pendulum on cars.
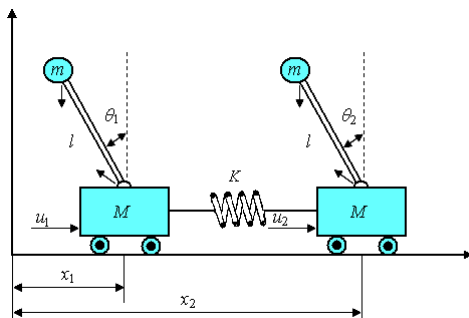

Fig. 3: System diagram of two inverted pendulums connected by a moving spring mounted on two carts.

IV. SIMULATION RESULTS

Herein, we apply the proposed approach to solve the stabilization of the two inverted pendulums connected by a moving spring mounted on two carts which is modified from [19]. Fig. 3 is the diagram of inverted double pendulums on cars systems which includes the cars which can move horizontally and each car has a pendulum which swings left and right. In addition, these two cars are connected by a spring. In the diagram, u_1 and u_2 are control input forces of the cars, $X=[x_1 \ x_2]^T$ is the output position of the car, $\theta=[\theta_1 \ \theta_2]^T$ is the angle between vertical axis and pendulum, and k is the spring constant, m is mass of load, M is mass of car, l is length of pendulum, r is the radius of load, and m_l is mass of pendulum, and L is the initial distance of two cars. In this paper, we consider the stabilization of X and θ, this is different from the previous literature.

For the further analysis of the dynamic system characteristic, we have to establish the mathematical model. Then we have the following motion equations.

$$M_1 \ddot{x}_1 = u_1 - H_1 + K(x_2 - x_1 - L)$$
$$M_2 \ddot{x}_2 = u_2 - H_2 - K(x_2 - x_1 - L) \qquad (20)$$

where H denotes the pendulums' horizontal forces

$$H_1 = m_1 \ddot{x}_1 + m_1 l_1 (\dot{\theta}_1^2) \sin\theta_1 - m_1 l_1 \ddot{\theta}_1 \cos\theta_1$$
$$H_2 = m_2 \ddot{x}_2 + m_2 l_2 (\dot{\theta}_2^2) \sin\theta_2 - m_2 l_2 \ddot{\theta}_2 \cos\theta_2 \qquad (21)$$

The rotating motion equations for pendulums are

$$(V_1 \sin\theta_1 + H_1 \cos\theta_1) \cdot l_1$$
$$= (m_1 l_1^2 + \frac{1}{3} m_1 m_l^2 + \frac{2}{5} m_1 r_1^2 + m_1 \cdot (l_L + r_1)^2) \cdot \ddot{\theta}_1$$
$$(V_2 \sin\theta_2 + H_2 \cos\theta_2) \cdot l_2$$
$$= (m_2 l_2^2 + \frac{1}{3} m_2 l_L^2 + \frac{2}{5} m_2 r_2^2 + m_2 \cdot (l_L + r_2)^2) \cdot \ddot{\theta}_2 \qquad (22)$$

where

$$V_1 = m_1 g - m_1 l_1 \ddot{\theta}_1 \sin\theta_1 - m_1 l_1 \dot{\theta}_1^2 \cos\theta_1$$
$$V_2 = m_2 g - m_2 l_2 \ddot{\theta}_2 \sin\theta_2 - m_2 l_2 \dot{\theta}_2^2 \cos\theta_2 \qquad (23)$$

Herein, we select $m=m_1=m_2$, $M=M_1=M_2$, and $r=r_1=r_2$ for simplifying the design. Define the state variables as $q_{11}=\theta_1$, $q_{12}=\dot{\theta}_1$, $q_{13}=x_1$, $q_{14}=\dot{x}_1$, $q_{21}=\theta_2$, $q_{22}=\dot{\theta}_2$, $q_{23}=x_2-x_1-L$, and $q_{24} = \dot{x}_2 - \dot{x}_1$. Therefore, system (20) can be represented as

$$q_{11}(k+1) = q_{11}(k) + q_{12}(k) \cdot T_s$$
$$q_{12}(k+1) = q_{12}(k)$$
$$+ \left(\frac{mlg \sin q_{11}(k)(M+m) - m^2 l^2 q_{12}^2(k) \cos q_{11}(k) \sin q_{11}(k)}{(ml^2 + \frac{1}{3} ml_L^2 + \frac{2}{5} mr^2 + m(l_L + r)^2 - \frac{(ml \cos q_{11}(k))^2}{M+m})} \right)$$
$$+ \frac{ml \cos q_{11}(k) K(q_{23}(k)) + mlu_1 \cos q_{11}(k)}{(ml^2 + \frac{1}{3} ml_L^2 + \frac{2}{5} mr^2 + m(l_L + r)^2 - \frac{(ml \cos q_{11}(k))^2}{M+m})}$$
$$\cdot \frac{T_s}{(M+m)}$$

$$q_{13}(k+1) = q_{13}(k) + q_{14}(k) \cdot T_s$$

$$q_{14}(k+1) = q_{14}(k)$$
$$+ (\frac{u_1 - ml(q_{12}^2(k))\sin q_{11}(k) - K(q_{23}(k))}{M + m}) \cdot T_s$$

$$- (\frac{ml(\frac{q_{12}(k+1) - q_{12}(k)}{T_s})\cos q_{11}(k)}{M + m}) \cdot T_s$$

$$q_{21}(k+1) = q_{21}(k) + q_{21}(k) \cdot T_s$$
$$q_{22}(k+1) = q_{22}(k)$$
$$+ (\frac{m \lg \sin q_{21}(k)(M+m) - m^2 l^2 q_{22}^2(k)\cos q_{21}(k)\sin q_{21}(k)}{(ml^2 + \frac{1}{3}ml_L^2 + \frac{2}{5}mr^2 + m(l_L + r)^2 - \frac{(ml\cos q_{21}(k))^2}{M+m})}$$
$$+ \frac{-m\,l\,\cos q_{21}(k)K(q_{23}(k)) + m\,l\,u_2\cos q_{21}(k)}{(m\,l^2 + \frac{1}{3}m\,l_L^2 + \frac{2}{5}m\,r^2 + m(l_L+r)^2 - \frac{(m\,l\,\cos q_{21}(k))^2}{M+m})})$$
$$\cdot \frac{T_s}{(M+m)}$$

$$q_{23}(k+1) = q_{23}(k) + q_{24}(k)$$
$$q_{24}(k+1) = q_{24}(k)$$
$$+ (\frac{u_2 - u_1 - ml[(q_{22}^2(k))\sin q_{21}(k) - (q_{12}^2(k))\sin q_{11}(k)]}{M+m}) \cdot T_s$$
$$- (\frac{ml[(q_{22}(k+1) - q_{22}(k))\cos q_{21}(k)]}{M+m})$$
$$- (\frac{ml[(q_{12}(k+1) - q_{12}(k))\cos q_{11}(k)]}{M+m})$$
$$- (\frac{2K(q_{23}) \cdot T_s}{M+m})$$

$$(24)$$

According the decoupled control scheme introduced above, we define

$$s_{11}(k+1) = c_{11}(q_{11}(k+1) - z_1(k+1)) + q_{12}(k+1)$$
$$s_{12}(k+1) = c_{12}q_{13}(k+1) + q_{14}(k+1)$$
$$s_{21}(k+1) = c_{21}(q_{21}(k+1) - z_2(k+1)) + q_{22}(k+1)$$
$$s_{22}(k+1) = c_{22}q_{23}(k+1) + q_{24}(k+1)$$

$$(25)$$

where

$$z_1(k) = -z_{U1}sat(s_{12}(k)/\phi_2), 0 < z_{U1} < 1$$
$$z_2(k) = -z_{U2}sat(s_{22}(k)/\phi_4), 0 < z_{U2} < 1$$

In the simulation, the following specifications are used: $g=9.8m/s^2$, $l=0.5m$, $m_a=0.8kg$, $m=0.1kg$, $r=0.01m$, $L=0.1m$, $k=1$, $c_1=5$, $c_2=0.5$, $\Phi_1=5$, $\Phi_2=15$, $z_{U1}=\pi/4$, $c_3=5$, $c_4=0.5$, $\Phi_3=5$, $\Phi_4=15$, $z_{U2}=\pi/4$. Initial condition is [$-\pi/12$, 0, $-\pi/12$, 0, 0, 0, 0.1, 0]. The sample time T_s is 0.002 second. The corresponding learning rates for NN are $\eta^w=0.3/T_s$, $\eta^v=0.1/T_s$. Fig. 4 and 5 show the simulation results of system without disturbance and perturbed by impulse, respectively. According to the simulation result of Fig. 4, we can find that the decoupled NN controller and SMC are better than the results of using fuzzy logic control approach. From the comparison results of performance in stabilizing time, we can find that DSMCNN control has the better performance.

Subsequently, we discuss the effect if there are external disturbance. Firstly, when the simulation time $T=15sec$, we add external disturbance such as $\theta_1=-\pi/20$. Fig. 5 shows the simulation results. After adding disturbance, we could obviously find that the disturbance rejection ability of NN performed better than fuzzy logic control and SMC. For the consideration of robustness, we can find that NN only need

smaller effort such that system outputs achieve to stable even system having external disturbance.

In addition, we consider the effect of frictional force. In real system, the frictional force may result in a delay appearance. In the system model, we use the frictional force model as the following description [6]. After considering the frictional force, the horizontal displacement should be

$$M_1\ddot{x}_1 = u_1 - H_1 + k(x_2 - x_1 - L) - F_3$$
$$M_2\ddot{x}_2 = u_2 - H_2 - k(x_2 - x_1 - L) - F_4 \qquad (26)$$

The inverted pendulums on cars with frictional force are

$$(m_1 l_1^2 + \frac{1}{3}m_1 l_L^2 + \frac{2}{5}m_1 r_1^2 + m_1 \cdot (l_L + r_1)^2) \cdot \ddot{\theta}_1$$
$$= (V_1 \sin\theta_1 + H_1 \cos\theta_1) \cdot l_1 - F_1$$
$$(m_2 l_2^2 + \frac{1}{3}m_2 l_L^2 + \frac{2}{5}m_2 r_2^2 + m_2 \cdot (l_L + r_2)^2) \cdot \ddot{\theta}_2$$
$$= (V_2 \sin\theta_2 + H_2 \cos\theta_2) \cdot l_2 - F_2 \qquad (27)$$

Herein, the coulomb frictional and viscous friction models are [6]

$$F_1 = K_{FS1}(\dot{q}_1) + K_{FQ1}\,\text{sgn}(\dot{q}_1)$$
$$F_2 = K_{FS2}(\dot{q}_3) + K_{FQ2}\,\text{sgn}(\dot{q}_3)$$
$$F_3 = K_{FS3}(\dot{q}_5) + K_{FQ3}\,\text{sgn}(\dot{q}_5) \qquad (28)$$
$$F_4 = K_{FS4}(\dot{q}_7) + K_{FQ4}\,\text{sgn}(\dot{q}_7)$$

In the simulation, the following specifications are used: $K_{FS1}=K_{FS2}=K_{FS3}=K_{FS4}=0.5$, $K_{FQ1}=K_{FQ2}=K_{FQ3}=K_{FQ4}=0.1$. Fig. 6 shows the simulation results of our approach. From Fig. 6, we can find that NN has better performance for treating the frictional force. Compare with fuzzy logic control and SMC approaches, we observe that the position of the right car has bigger steady state error, and the proposed approach results smaller error which is close to 0. In addition, the control effort of DSMCNN is smooth. From Fig.7, after adding frictional force, external disturbance and external force, we could obviously find the system still achieve to stable. Therefore, we can conclude that using DSMCNN has the best performance.

V. CONCLUSION

In this study, we have proposed a DSMCNN with SPSA algorithm control scheme for solving the control of a class of nonlinear discrete-time MIMO uncertain systems. The control method provides a simple way to achieve asymptotic stability for uncertain nonlinear system with frictional force and external disturbance. In addition, we use SPSA algorithm to derive the adaptive laws of the NN's parameters, which not only decrease computation complexity but also get over friction. The illustrated example of two inverted pendulums connected by a moving spring mounted on two carts has been introduced to show the effectiveness of our approach.

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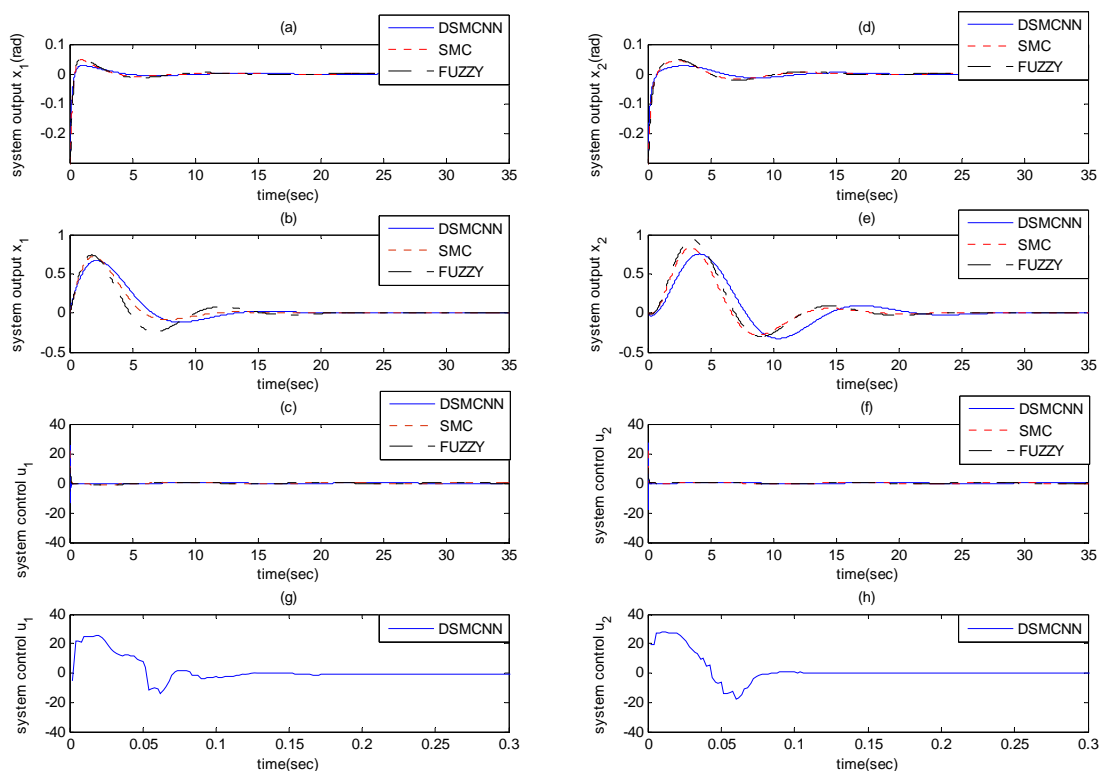
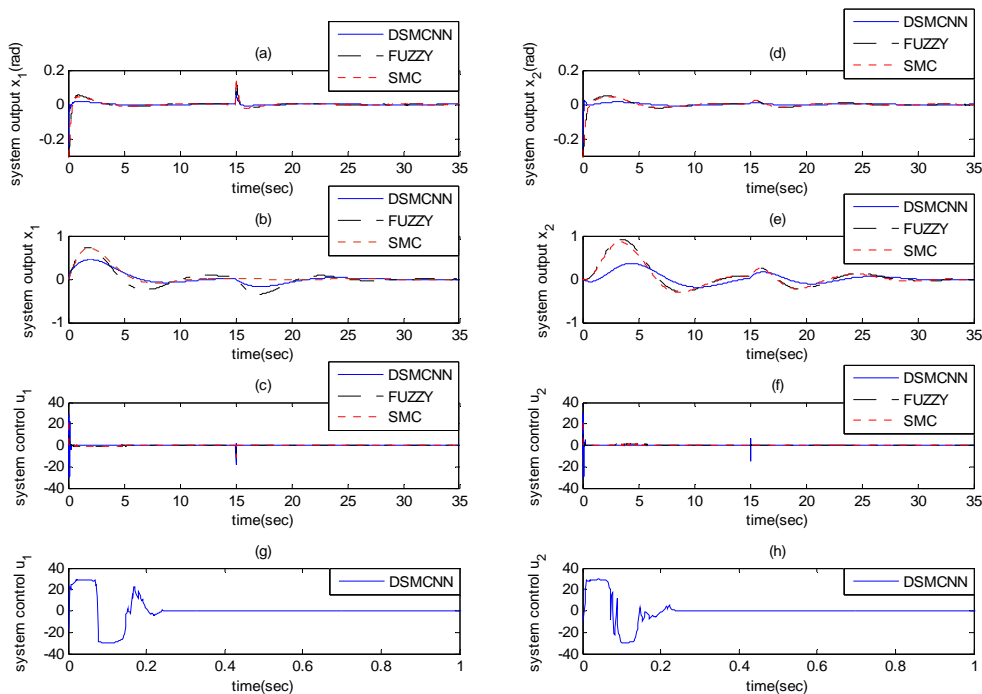Figure 4: The simulation results without disturbance.

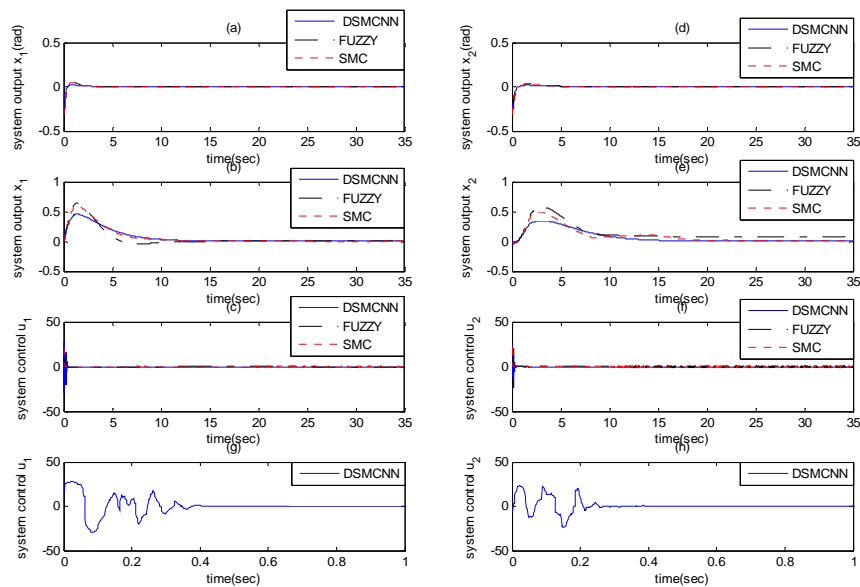Figure 5: The simulation results adding external disturbance and external force.

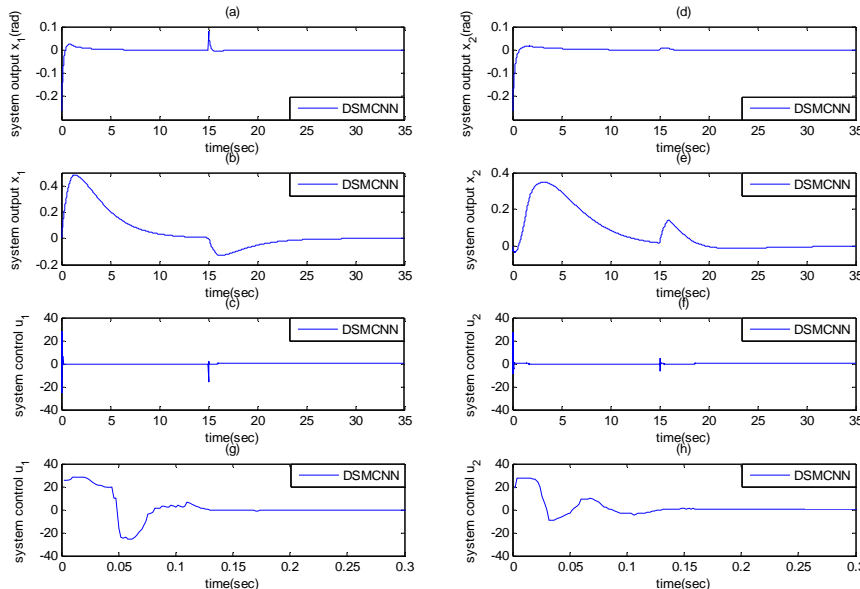Figure 6: The simulation results adding friction models.

Figure 7: The simulation results adding friction, external disturbance and external force models.

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