Proceedings of the International MultiConference of Engineers and Computer Scientists 2010 Vol II,
IMECS 2010, March 17 - 19, 2010, Hong Kong

Resilient Back-propagation Neural Network for Approximation 2-D GDOP

Chien-Sheng Chen, and Szu-Lin Su

***Abstract*—Geometric dilution of precision (GDOP) represents the geometric effect on the relationship between measurement error and positioning determination error. If the measurement variances are equal in each other, GDOP could be the most appropriate selection criterion of location measurement units. The object of this paper is to obtain the optimal position estimates from the available measurement. The conventional matrix inversion method for GDOP calculation has a large amount of operation, which would be a burden for real time application. This paper employs an artificial neural network approach, namely, the resilient back-propagation (Rprop) method to implement GDOP. This paper also presents two novel architectures to implement the Rprop-based GDOP for the 2D location estimation. Simulation results show that the proposed architectures always yield superior estimation accuracy with much reduced computational complexity, compared to conventional implementation methods for GDOP. The proposed architectures are applicable to cellular communication systems regardless of the number of the measurement units.**

***Index Terms*—Geometric dilution of precision (GDOP), Back-propagation neural network (BPNN), Resilient back-propagation (Rprop), Time of arrival (TOA)**

I. Introduction

The geometric dilution of precision (GDOP) concept was originally used as a criterion for selecting the optimal 3D geometric configuration of satellites in GPS. When enough measurements are available, the optimal measurements selected with the minimum GDOP can help reduce the adverse geometry effects, thereby improving the location accuracy. However, excessive or redundant measurements may increase the computational overhead and may not provided significantly improved location accuracy. It is very critical to select a subset with the most appropriate measurement units rapidly before positioning.

The GDOP computation assumes that the pseudo-orange errors are independent and identically distributed [1]. Several methods based on GDOP have been proposed to improve the GPS positioning accuracy [2-5]. Most, if not all, of those methods need matrix inversion to calculate GDOP. Though they can guarantee to achieve the optimal subset, the computational complexity is usually too intensive to be practical. Back-propagation neural network (BPNN), a supervised learning neural network, is the most popular technique for classification and prediction [6]. BPNN was initially proposed in [7-8] to calculate the GDOP function approximation. The BPNN was employed to “learn” the input-output relationship between the entries of a measurement matrix and the eigenvalues of its inverse. Three other input-output relationships were proposed and compared based on simulation results [9]. However, BPNN usually converges slowly and tends to get trapped in local minima easily.

Resilient back-propagation (Rprop) is considered the best algorithm, measured in terms of convergence speed, accuracy and robustness with respect to training parameters [10]. Most location estimation methods of the cellular communication systems have been dedicated in the estimation of 2D environments. Considering both effectiveness and efficiency, this paper proposes two novel architectures and presents four original architectures based on an alternative artificial neural networks method, namely, the Rprop method, to approximate GDOP in the form of 2D formulations. Our simulation results have shown that Rprop-based architectures provide faster convergence speed and the number of training iterations is greatly reduced.

To select the most appropriate set of BSs, which will give the minimum positioning error, GDOP effect must be considered in cellular communication systems. Simulation results show that the proposed architectures using Rprop for GDOP approximation always give the better accuracy comparing with the other architectures. These architectures for approximating GDOP can be applied regardless of the number of the location measurement units.

The remainder of this paper is organized as follows. Section II presents the calculation of GDOP. BPNN and Rprop are described in Section III. The six types of mapping for GDOP approximation based on Rprop are proposed in Section IV. Simulation results are presented in Section V, followed by conclusion in Section VI.

II. Calculation of GDOP

Originally, the concept of GDOP has been widely used to indicate the geometric effect of 3D satellite configurations in GPS. Using a 3D Cartesian coordinate system, the distances between satellite i and the user can be expressed as

$$r_i = \sqrt{(x - X_i)^2 + (y - Y_i)^2 + (z - Z_i)^2} + C \cdot t_b + v_{ri} \qquad (1)$$

Chien-Sheng Chen is with the Institute of Computer and Communication Engineering, Department of Electrical Engineering, National Cheng Kung University, Taiwan (corresponding author to provide phone: 886-6-2532106 #5038; fax: 886-6-2434897; e-mail: jasonasetest@ yahoo.com.tw).
Szu-Lin Su is with the Institute of Computer and Communication Engineering, Department of Electrical Engineering, National Cheng Kung University, Taiwan(e-mail: ssl@ee.ncku.edu.tw).

ISBN: 978-988-18210-4-1
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

where (x, y, z) and (X_i, Y_i, Z_i) are the locations of the user and satellite i, respectively. C is the speed of light, t_b denotes the time offset and v_{ri} is pseudorange measurements noise. Equation (1) is linearized by using Taylor's series expansion around the approximate user position $(\hat{x}, \hat{y}, \hat{z})$ and neglecting the higher order terms. Defining $\hat{r}_i$ as r_i at $(\hat{x}, \hat{y}, \hat{z})$, we can obtain

$$\Delta r = r_i - \hat{r}_i \cong e_{i1}\delta_x + e_{i2}\delta_y + e_{i3}\delta_z + C \cdot t_b + v_{ri} \quad (2)$$

where $\delta_x, \delta_y, \delta_z$ are, respectively, coordinate offset of x, y, z,

$e_{i1} = \frac{\hat{x} - X_i}{\hat{r}_i}$, $e_{i2} = \frac{\hat{y} - Y_i}{\hat{r}_i}$, $e_{i3} = \frac{\hat{z} - Z_i}{\hat{r}_i}$, and

$\hat{r}_i = \sqrt{(\hat{x} - X_i)^2 + (\hat{y} - Y_i)^2 + (\hat{z} - Z_i)^2}$.

(e_{i1}, e_{i2}, e_{i3}), $i = 1, 2, ..., n$, denote the line-of-sight (LOS) vector from the user to the satellites.

The linearized equations can be expressed in a vector form as

$$z = H\delta + v \quad (3)$$

where $z = \begin{bmatrix} r_1 - \hat{r}_1 \\ r_2 - \hat{r}_2 \\ \vdots \\ r_n - \hat{r}_n \end{bmatrix}$, $\delta = \begin{bmatrix} \delta_x \\ \delta_y \\ \delta_z \\ c \cdot \Delta t_b \end{bmatrix}$, $v = \begin{bmatrix} v_{r1} \\ v_{r2} \\ \vdots \\ v_{rn} \end{bmatrix}$, and

$H = \begin{bmatrix} e_{11} & e_{12} & e_{13} & 1 \\ e_{21} & e_{22} & e_{23} & 1 \\ \vdots & \vdots & \vdots & \vdots \\ e_{n1} & e_{n2} & e_{n3} & 1 \end{bmatrix}$ is a geometry matrix.

The vector variable δ in Eq. (3) can be solved with the least-square (LS) algorithm, namely,

$$\hat{\delta} = (H^T H)^{-1} H^T z \quad (4)$$

Now making the assumption that the pseudorange errors are uncorrelated and have equal variances σ^2, the error covariance matrix can be expressed as

$$E[(\hat{\delta} - \delta)(\hat{\delta} - \delta)^T] = \sigma^2 \cdot (H^T H)^{-1} \quad (5)$$

The variances are functions of the diagonal elements of $(H^T H)^{-1}$. GDOP is a measure of accuracy for positioning systems and can be defined as

$$GDOP = \sqrt{tr(H^T H)^{-1}}. \quad (6)$$

III. The traditional BPNN algorithm and the Rprop algorithm

It has been known that BPNN is capable of learning and realizing both linear and nonlinear functions [6]. The learning process of BPNN can be considered as one of gradient descent methods that minimizes some measure, e.g., mean-square value of the difference between the actual output vector of network and the desired output vector. Define an error function F

$$F = \frac{1}{2}\sum_k (T_k - O_k)^2 \quad (7)$$

where T_k is the output vector of the network while O_k is the desired output vector. Then, the gradient decent algorithm is employed to adapt the weights (namely, synapses) as follows

$$\Delta w_{ij}(t) = -\varepsilon \frac{\partial F}{\partial w_{ij}}(t) \quad (8)$$

where ε is a pre-determined learning rate, and w_{ij} denotes the weight connecting neuron i to neuron j. The major drawbacks of traditional BPNN include slow learning process and the tendency to be trapped easily in local minima.

Comparing to the traditional BPNN algorithm, the Rprop algorithm offers faster convergence and is usually more capable of escaping from local minima [10]. In a sense, Rprop is a first-order algorithm and its time and memory requirement scales linearly with the number of parameters. In practice, Rprop is easier to implement than BPNN. Besides, a hardware implementation for Rprop has been presented in [11].

IV. The Proposed Network Architectures for GDOP Approximation

Without having to invert a matrix, the traditional BPNN learns the relationships between the entries of a measurement matrix and the eigenvalues of its inverse to estimate GDOP [7-8]. Three other relationships for training based on traditional BPNN are employed in [9]. In this paper, the original four different input-output mapping types based on BPNN for GDOP calculation will be extended to the employment of Rprop. In addition, we propose two new mapping architectures for 2D scenarios.

If more measurements are available, the optimal measurements selected with the minimum GDOP can prevent the poor geometry effects and have the potential of obtaining greater location accuracy. Instead of using all visible satellites, four satellites are usually sufficient for GPS positioning. As such, we take only four BSs from among seven with better geometry to estimate the MS location in cellular communication networks.

The geometry matrix composed of four location measurement units in 2D environments is

$$H = \begin{bmatrix} e_{11} & e_{12} & 1 \\ e_{21} & e_{22} & 1 \\ e_{31} & e_{32} & 1 \\ e_{41} & e_{42} & 1 \end{bmatrix}.$$

From equation (6), $H^T H$ is a 3x3 matrix and it has three eigenvalues, λ_i, $i = 1, 2, 3$. Therefore, the three eigenvalues

ISBN: 978-988-18210-4-1
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

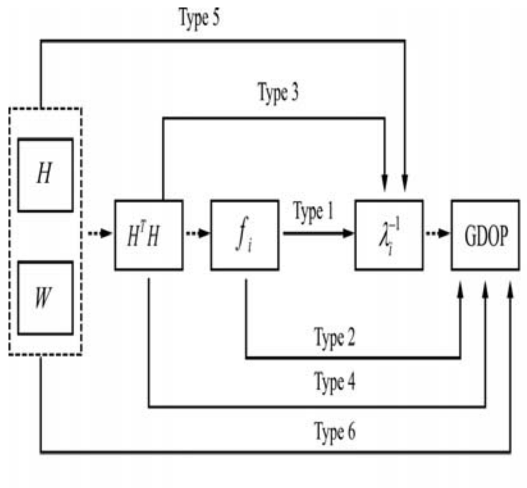

Fig. 1. The input-output relationships for six types of mapping using Rprop.

of $(H^T H)^{-1}$ is $\lambda_i^{-1}, i = 1, 2, 3$. From the algebra theory, the trace of a matrix is equal to the sum of its eigenvalues. Consequently, GDOP also can be expressed as

$$GDOP = \sqrt{tr(H^T H)^{-1}} = \sqrt{\lambda_1^{-1} + \lambda_2^{-1} + \lambda_3^{-1}} \quad (9)$$

We present the six types of Rprop mapping architectures for GDOP prediction in 2D scenarios and the mapping relationship with the three layer 'input p - hidden neuron number - output q ' structures. These six types of architectures are described by a block diagram shown in Fig. 1.

Type 1: Three inputs are mapped to three outputs.

$$f_1(\lambda) = \lambda_1 + \lambda_2 + \lambda_3 = trace(H^T H) \quad (10a)$$

$$f_2(\lambda) = \lambda_1^2 + \lambda_2^2 + \lambda_3^2 = trace[(H^T H)^2] \quad (10b)$$

$$f_3(\lambda) = \lambda_1 \lambda_2 \lambda_3 = \det(H^T H) \quad (10c)$$

The network has the input-output pairs:

Input: $(f_1, f_2, f_3)^T$

Output: $(\lambda_1^{-1}, \lambda_2^{-1}, \lambda_3^{-1})^T$

The mapping from f to λ^{-1} is nonlinear and cannot be determined analytically. After the training period, this mapping relationship can be approximated perfectly by neural network. GDOP is calculated by taking the square root of the sum of the outputs.

Type 2: Three inputs are mapped to one output.

Input: $(f_1, f_2, f_3)^T$

Output: $GDOP$

Type 3: Six inputs are mapped to three outputs.

The type of mapping is used to approximate the eigenvalues inverse from the elements of matrix $(H^T H)$. The matrix $(H^T H)$ is a symmetric matrix and can be expressed as

$$H^T H = \begin{bmatrix} B_{11} & B_{12} & B_{13} \\ & B_{22} & B_{23} \\ sym & & B_{33} \end{bmatrix} \quad (11)$$

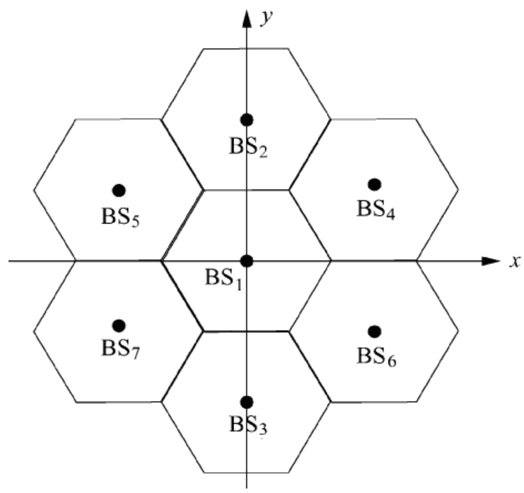

Fig. 2. Seven-cell system layout.

The network has the structure with the following input-output pair:

Input: $(B_{11}, B_{12}, B_{13}, B_{22}, B_{23}, B_{33})^T$

Output: $(\lambda_1^{-1}, \lambda_2^{-1}, \lambda_3^{-1})^T$

Type 4: Six inputs are mapped to one output.

This is a type of mapping from the elements of the matrix $(H^T H)$ to approximate GDOP. The network has the following input-output relationship:

Input: $(B_{11}, B_{12}, B_{13}, B_{22}, B_{23}, B_{33})^T$

Output: $GDOP$

Type 5: Twelve inputs are mapped to three outputs.

The elements of matrix H and W are utilized to approximate the inverse of the eigenvalues without having to calculate $H^T H$. The network has the following mapping architecture:

Input: $(e_{11}, e_{12}, e_{21}, e_{22}, e_{31}, e_{32}, e_{41}, e_{42})^T$

Output: $(\lambda_1^{-1}, \lambda_2^{-1}, \lambda_3^{-1})^T$

Type 6: Twelve inputs are mapped to one output.

This architecture is proposed to train the mapping for approximating GDOP from the elements of matrix H and W.

The network has the following input-output relationship:

Input: $(e_{11}, e_{12}, e_{21}, e_{22}, e_{31}, e_{32}, e_{41}, e_{42})^T$

Output: $GDOP$

Our simulation results have shown that our proposed Type 5 and Type 6 need fewer hidden neurons and the number of training iterations. Thus they have much reduced computational load, and are more practical. It is note that all above architectures for obtaining GDOP are applicable regardless of the number of the location measurement units.

V. SIMULATION RESULTS

We consider a center hexagonal cell (where the home BS resides) with six adjacent hexagonal cells of the same size, as shown in Fig.2. Each cell has a radius of 5 km and the MS location is uniformly distributed in the center cell [12]. In our simulations, only a subset of the four BSs is selected for

ISBN: 978-988-18210-4-1
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

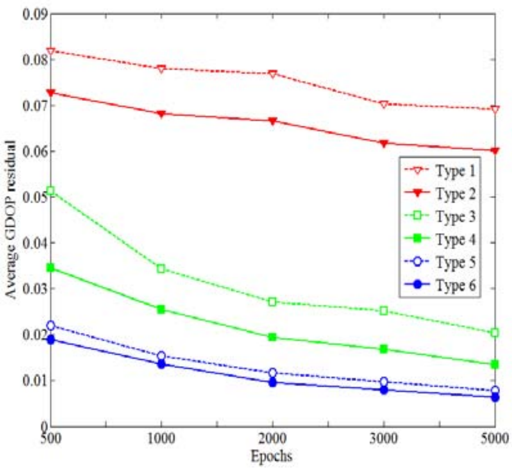

Fig. 3. The GDOP residual of convergence versus the epochs.

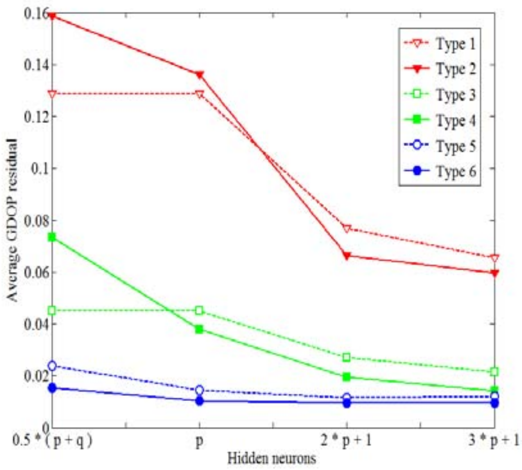

Fig. 4. The GDOP residual with various neurons numbers of the hidden layer.

location process. Thus, the measurements could be divided into 35 (C_4^7) possible subsets. GDOP is computed for all possible subsets and the one with the smallest GDOP is selected. The minimum GDOP subset of the four BSs is used to estimate the MS location.

The dominant error for wireless location systems is usually due to the NLOS propagation effect. The NLOS propagation model is based on the uniformly distributed noise model [13] and assumed to be uniformly distributed over $(0, U_i = 300\ \text{m})$, for $i = 1, 2,..7$,where U_i is an upper bound. Single hidden layer are the most widely used method among various learning methods for neural networks. Therefore, the number of hidden layers is set at one. The GDOP residual is defined as difference between the actual GDOP and the estimated GDOP. The prediction accuracy of the GDOP is measured in terms of the GDOP residual.

Figure 3 provides the average GDOP residual of convergence varies as the number of training iterations (epochs) increases. The average GDOP residual decreases as the number of epochs increase. It also can be found that the effect of the number of training iterations of 2000 epochs provides a better performance. Therefore, the number of epochs of Rprop is less than the traditional BPNN with 20000 epochs [9] and Rprop requires much less convergence time.

When there are too few hidden neurons, a bigger error may occur. Increasing the number of hidden neurons can alleviate this situation but will affect the speeds of convergence simultaneously. General rules for determining the number of hidden neurons are: (i) $0.5*(p+q)$; (ii) p; (iii) $2*p+1$; (iv) $3*p+1$. The average GDOP residual decrease as the number of hidden layer neurons increasing, as shown in Fig. 4. The hidden layer neurons with $(2p+1)$ give reasonably accurate results, where p is the number of input neurons.

Based on the good ability of estimating of the neural network structure stated above, the Rprop algorithm can be applied to predict GDOP value after the training period. Figure 5 shows cumulative distribution functions (CDFs) of the GDOP residual for the six types of mapping architectures

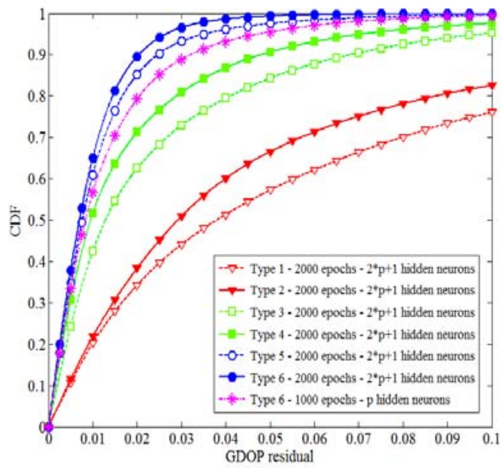

Fig. 5. The CDFs curves of GDOP residual using the six types of mapping architectures.

with $(2p+1)$ hidden neurons after 2000 training iterations. GDOP is equal to the square root of the sum of $\lambda_i^{-1}, i=1,2,3$, which are the outputs of three-output architectures. The one-output architectures approximate GDOP with much better accuracy than those of the three-output architectures. The type 1 mapping architecture predicts the eigenvalues inverse and then obtains GDOP value with poor accuracy. The results show that the proposed Type 5 gives the best performance among the three-output architectures and the proposed Type 6 provides much better accuracy than all the other one-output architectures.

Figure 6 shows how the average GDOP residual is affected by the upper bound on uniform NLOS error. By comparing the average GDOP residual, the proposed Type 5 and Type 6 are shown to have better performance. It can also be observed that the sensitivity of these types with respect to the NLOS effect is not obvious.

The second NLOS propagation model is based on the distance-dependent NLOS error model. The NLOS range error for the i th range was taken to be $\xi_i = \chi_i \cdot r_i$, for

ISBN: 978-988-18210-4-1
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

$i = 1, 2,..7$ where $\chi_i = 0.2$ is a proportionality constant [13]. It makes intuitive sense to view NLOS errors as being proportional to the distance traveled by the signal. It can also be seen in the CDF curves of the GDOP residual, as shown in Fig. 7. The Type 1 and Type 2 provided relatively poor GDOP estimation performance. It was observed that the proposed Type 5 and Type 6 always provide much better GDOP mapping accuracy as compared with the other types.

No matter which NLOS propagation model is considered, our simulation results have shown that the proposed Rprop-based architecture provides faster convergence and the required number of training iterations is greatly reduced. The proposed Type 6 with p hidden neurons and 1000 epochs renders superior performance to other architectures, such as those with $2p+1$ neurons and 2000 epochs. In order to minimize the computational load, the proposed Type 6 with the aforementioned parameters was used in location estimation because it offers satisfactory prediction performance.

VI. Conclusion

The matrix inversion method for GDOP calculation is rather time consuming and presents a calculation burden. This paper presents novel Rprop-based architectures to approximate GDOP. In order to eliminate the poor geometry influence and improve the positioning accuracy, the minimum GDOP approximation can be used and optimal geometric configuration with four measurements is obtained. Our results show that the proposed architectures for predicting GDOP have high degree of accuracy and the number of training iterations is greatly reduced.

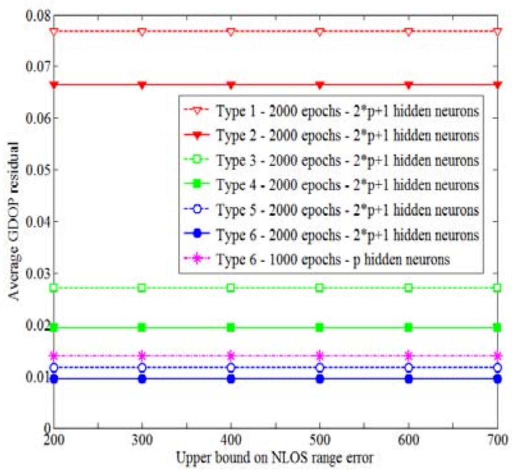

Fig. 6. Average GDOP residual versus the upper bound on uniform NLOS error.

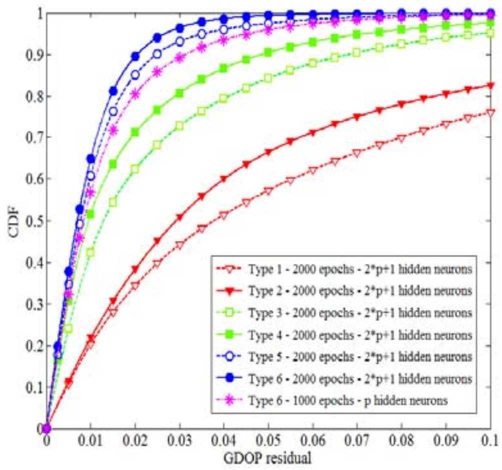

Fig. 7. CDFs for GDOP residual of the six types of mapping architectures.

Reference

[1] E. D. Kaplan, "Understanding GPS: principles and applications (2nd Ed)". Artech House Press, London, UK, 2006.

[2] M. Zhang and J. Zhang, "A Fast Satellite Selection Algorithm: Beyond Four Satellites," *IEEE Journal of Selected Topics in Signal Processing*, vol. 3 pp. 740–747, Oct. 2009.

[3] W. Wang, Z-X Deng, and G.-Q Zhao, "Research of Coastal Station Distribution Algorithm Based on GDOP in Wireless Positioning System," *Int. Conf. Mechatronics and Automation*, pp. 3150-3154, Aug. 2007.

[4] M. Kihara, "Study of a GPS satellite selection policy to improve positioning accuracy," *IEEE PLANS* pp.267-273, Apr. 1994.

[5] Y. Yong and M. Lingjuan, "GDOP Results in All-in-view Positioning and in Four Optimum Satellites Positioning with GPS PRN Codes Ranging,", *IEEE PLANS*, pp. 723-727, Apr. 2004.

[6] D. E. Rumelhart, G. E. Hinton, and R. J. Williams, "Learning representations by back-propagating errors," *Nature*, pp. 533–536, 1986.

[7] D. Simon and H. El-Sherief, "Navigation satellite selection using neural networks," *Neurocomputing*, vol. **7**, pp. 247-258, May 1995.

[8] D. Simon and H. El-Sherief, "Fault-tolerant training for optimal interpolative nets," *IEEE Trans. Neural Networks*, vol. 6 pp. 1531-1535, Nov. 1995.

[9] D.-J. Jwo and K.-P. Chin, "Applying Back-propagation Neural Networks to GDOP Approximation," *The Journal of Navigation*, vol. 55, pp. 97-108, Jan. 2002.

[10] M. Riedmiller and H. Braun, "A direct adaptive method for faster backpropagation learning: The RPROP algorithm," *Proc. IEEE Int. Conf. On Neural Network*, pp. 586-591, 1993.

[11] L. M. Patnaik and K. Rajan, "Target detection through image processing and resilient propagation algorithms," *Neurocomputing*, pp. 123–135, 2000.

[12] L. Cong and W. Zhuang, "Nonline-of-Sight Error Mitigation in Mobile Location," *IEEE Trans. Wireless Commun.*, vol. 4, no. 2, pp. 560–573, March 2005.

[13] S. Venkatraman, J. Caffery, and H.-R. You, "A novel TOA location algorithm using LOS range estimation for NLOS environments," *IEEE Trans. on Veh. Technol.*, vol. 53, pp. 1515–1524, Sept. 2004.

ISBN: 978-988-18210-4-1
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)