Fuzzy Threshold Values in the Robust Design of Controllers for Probabilistic Uncertain Systems

N. Nariman-zadeh and A. Jamali

***Abstract*—In this paper, optimal multi-objective Pareto design of robust state feedback controllers for an inverted pendulum with parameters probabilistic uncertainty is presented. The objective functions that have been considered are, namely, the probabilities of failure of summation of rising time and overshoot of cart (SRO-C) and the probabilities of failure of summation of rising time and overshoot of pendulum (SRO-P). It is also shown that multi-objective reliability Pareto optimization of the robust state feedback controllers using a new multi-objective uniform-diversity genetic algorithm (MUGA) with fuzzy threshold values includes those may be obtained by various crisp threshold values of probability of failure and, thus, remove the difficulty of selecting suitable crisp values defining the robustness of the stochastic performance of the controller. Besides the multi-objective Pareto optimization of such robust feedback controllers using MUGA unveils some very important and informative trade-offs among those objective functions.**

***Index Terms*—Pareto, Robust control, Reliability, Uncertainty.**

I. Introduction

The synthesis of control policies can be regarded as optimization problems of certain performance measures of a controlled system. A very effective means of solving such optimum controller design problems is genetic algorithms (GAs) and other evolutionary algorithms (EAs). Some early application of GAs in optimum design of controllers are reported in [1] [2]. In addition to the most applications of EAs in the design of controllers for certain systems, there are also much research efforts in robust design of controllers for uncertain systems in which both structured or unstructured uncertainties may exist [3]. Most of the robust design methods such as $\mu - analysis$, H_2 or H_∞ design are based on different norm-bounded uncertainty [4]. As each norm has its particular features addressing different types of performance objectives, it may not be possible to achieve all the robustness issues and loop performance goals simultaneously. Recently, GAs have also been deployed for multi-objective robust control design considering the robust stability and the mixed H_2 and H_∞ norms, simultaneously [5]. Indeed the designing robust control method for uncertain systems is a computationally complex problem. Recently, there have been many efforts for designing robust control methods. In these robust design methods, probabilistic uncertainty propagates through the uncertain parameter of plants. These methods reduce the conservatism involved with such robust design methods or to account more for the most likely plants with respect to uncertainties, the probabilistic uncertainty, as a weighting factor, must be considered accordingly. In fact, probabilistic uncertainty specifies set of plants as the actual dynamic system to each of which a probability density function (PDF) is assigned [6]. Therefore, such additional information regarding the likelihood of each plant allows a reliability-based design in which probability is incorporated in the robust design. The notion of stochastic robustness and probability of instability have been first mentioned by Stengel [7] and Stengel and Ryan [8]. The analysis of Monte Carlo Simulation (MCS) has also been introduced by Stengel to evaluate stochastic stability and performance of probabilistic uncertain systems.

In this paper, fuzzy threshold values have been used for optimal reliability-based multi-objective Pareto design of state feedback controller for an inverted pendulum with probabilistic uncertain parameters. It is shown that a small change in the crisp threshold values causes to have very different Pareto fronts and, therefore it is very difficult to select a suitable crisp threshold values which is needed to compute the probability of failure in RBDO design. Moreover, it is also shown that the Pareto optimal design points that are obtained using fuzzy threshold values include the best Pareto design points which are obtained when a wide range of crisp threshold values are instead used. A multi-objective uniform-diversity genetic algorithm (MUGA) that has been proposed by authors [9], [10] is used for multi-objective optimization. The objective functions that have been considered are, namely, the probabilities of failure of summation of rising time and overshoot of cart (S_{RO}-C) and the probabilities of failure of summation of rising time and overshoot of pendulum (S_{RO}-P). Therefore, optimum robust state feedback controllers are found in a Pareto sense using the probabilistic measures of those objective functions including their probability of failure by hybrid use of MCS and MUGA

II. Stochastic Robust Analysis

In real control engineering practice, there exist a variety of typical sources of uncertainty which have to be compensated through robust control design approach [9]. Those uncertainties include plant parameter variations due to environmental condition, incomplete knowledge of the parameters, age, unmodeled high frequency dynamics, and etc. Two categorical types of uncertainty, namely, structured uncertainty and unstructured uncertainty are generally used in classification. The structured uncertainty concerns about the model uncertainty due to unknown values of parameters

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in a known structure. In conventional optimum control system design, uncertainties are not addressed and the optimization process is accomplished deterministically. In fact, it has been shown that optimization without considering uncertainty generally leads to non-optimal and potentially high risk solution [11]. Therefore, it is very desirable to find robust design whose performance variation in the presence of uncertainties is not high. Generally, there exist two approaches addressing the stochastic robustness issue, namely, robust design optimization (RDO) and reliability-based design optimization (RBDO), [12]. With the aid of ever increasing computational power, there have been a great amount of research activities in the field of robust analysis and design devoted to the use of Monte Carlo simulation [13]. In fact, Monte Carlo simulation (MCS) has also been used to verify the results of other methods in RDO or RBDO problems when sufficient number of sampling is adopted [14]. Monte Carlo simulation (MCS) is a direct and simple numerical method but can be computationally expensive. In this method, random samples are generated assuming pre-defined probabilistic distributions for uncertain parameters. The system is then simulated with each of these randomly generated samples and the percentage of cases produced in failure region defined by a limit state function approximately reflects the probability of failure.

Let X be a random variable, then the prevailing model for uncertainties in stochastic randomness is the probability density function (PDF), $f_X(x)$ or equivalently by the cumulative distribution function (CDF), $F_X(x)$, where the subscript X refers to the random variable. This can be given by

$$F_X(x) = \Pr(X \le x) = \int_{-\infty}^{x} f_X(x)dx \tag{1}$$

where $\Pr(.)$ is the probability that an event ($X \le x$) will occur. Some statistical moments such as the first and the second moment, generally known as mean value (also referred to as expected value) denoted by $E(X)$ and variance denoted by $\sigma^2(X)$, respectively, are of the most important ones. They can also be computed by

$$E(X) = \int_{-\infty}^{\infty} x dF_X x = \int_{-\infty}^{\infty} x f_X(x)dx \tag{2}$$

and

$$\sigma^2(X) = Var(X) = \int_{-\infty}^{\infty} (x - E(X))^2 dF_X(x) = \int_{-\infty}^{\infty} (x - E(X))^2 f_X(x)dx. \tag{3}$$

In the robust design approach, it is desired to minimize the variability of a stochastic response as a function of w (w represents time or frequency) due to the uncertain probabilistic parameters about a deterministic behavior. In this approach, let $h(\mathbf{p}, w)$ be the random response of an uncertain plant due to uncertain parameters **p**, and let $\hat{h}(w)$ be the response of the certain deterministic plant [15] The variability is then obtained by defining the error between each stochastic response of uncertain probabilistic plant and the response of the certain deterministic plant. Therefore, the sum of squared error (SSE) can be used for obtaining variability about deterministic behavior as follows

$$\nu = \sum_{i=1}^{N} \sum_{j=1}^{N_w} \left(h_i(\mathbf{p}, w_j) - \hat{h}(w_j) \right)^2 \tag{4}$$

where h_i with $i = 1, 2, \dots N$ is a random response, and w_j with $j = 1, 2, \dots N_w$ is time or frequency sample.

In the reliability-based design approach, it is required to define reliability-based metrics via some inequality constraints. Therefore, in the presence of uncertain parameters of plant (**p**) whose PDF or CDF can be given by $f_\mathbf{p}(\mathbf{p})$ or $F_\mathbf{p}(\mathbf{p})$, respectively, the reliability requirements can be given as

$$P_f^i = \Pr(g_i(\mathbf{p}) \le 0) = \varepsilon \quad (i = 1, 2, \dots, k) \tag{5}$$

In (5), P_f^i denotes the probability of failure (i.e., $g_i(\mathbf{p}) \le 0$) of the i^{th} reliability measure and k is the number of inequality constraints (i.e. limit state functions) and ε is the highest value of desired admissible probability of failure. It is clear that the desired value of each P_f^i is zero. Therefore, taking into consideration the stochastic distribution of uncertain parameters ($\mathbf{p}$) as $f_p(\mathbf{p})$, equation (5) can now be evaluated for each probability function as

$$P_f^i = \Pr(g_i(\mathbf{p}) \le 0) = \int_{g_i(\mathbf{p}) \le 0} f_\mathbf{p}(\mathbf{p}) d\mathbf{p} \tag{6}$$

This integral is, in fact, very complicated particularly for systems with complex $g(\mathbf{p})$ [16] and Monte Carlo simulation is alternatively used to approximate (6). In this case, a binary indicator function $I_{g(\mathbf{p})}$ is defined such that it has the value of 1 in the case of failure ($g(\mathbf{p}) \le 0$) and the value of zero otherwise,

$$I_{g_i(p)} = \begin{cases} 0 & g_i(\mathbf{p}) > 0 \\ 1 & g_i(\mathbf{p}) \le 0 \end{cases} \tag{7}$$

Consequently, the integral of (6) can be rewritten as

$$P_f{}^i = \int_{-\infty}^{\infty} I_{g_i(\mathbf{p})}(G(\mathbf{p}), C(\mathbf{k})) f_\mathbf{p}(\mathbf{p}) d\mathbf{p} \tag{8}$$

where $G(\mathbf{p})$ is the uncertain plant model and $C(\mathbf{k})$ is the controller to be designed in the case of control system design problems. Based on Monte Carlo simulation, the probability using sampling technique can be estimated using

$$P_f^{\ i} = \frac{1}{N}\sum_{i=1}^{N} I_{g_i(p)}\left(G(\mathbf{p}), C(\mathbf{d})\right) \tag{9}$$

In other words, the probability of failure is equal to the number of samples in the failure region divided by the total number of samples. Evidently, such estimation of P_f approaches to the actual value in the limit as $N \to \infty$ [13]. In this paper, Hammersley Sequence Sampling (HSS) has been used to generate samples for probability estimation of failures. The binary crisp indicator function $I_{g_i(p)}$ of (1) that has been adopted in many research works to compute the occurrence of failure is, however, very prone sensitive to the definition of the limit state functions $g_i(p)$. Consequently, the optimum design of controllers could significantly lead to different optimum results based on the boundary definition of those limits state functions for the objective functions. In fact, some robust non-dominated optimum designs may be eliminated from the Pareto front for such specified crisp indicator function $I_{g_i(p)}$. Therefore, a Gaussian fuzzy membership indicator function [9] is rather used in this work to circumvent to difficulty of selecting the suitable crisp indicator function. In this case, equation (7) is replaced by

$$I_{g_i(p)} = \frac{1}{2}\left[1 + erf\left(\frac{|g_i(p)| - \mu_i}{\sqrt{2}\ \sigma_i}\right)\right] \tag{10}$$

where the Gaussian error function is defined by

$$erf(y) = \frac{2}{\sqrt{\pi}}\int_0^y e^{-t^2}\,dt \tag{11}$$

In (10) μ_i and σ_i stand for mean and variance of the limit state function $g_i(p)$ of each objective function respectively. Thus, the use of the Gaussian degree of membership $I_{g_i(p)}$ given by (10) ensures gradual variation of the probability of failure based on selected admissible mean and variance of each limit state function in reliability-based design

III. Multi-objective Pareto Optimization

Multi-objective optimization which is also called multi-criteria optimization or vector optimization has been defined as finding a vector of decision variables satisfying constraints to give optimal values to all objective functions [17] . In general, it can be mathematically defined as:

find the vector $X^* = \left[x_1^*, x_2^*, \ldots, x_n^*\right]^T$ to optimize

$$F(X) = \left[f_1(X), f_2(X), \ldots, f_k(X)\right]^T \tag{12}$$

subject to *m* inequality constraints

$$l_i(X) \le 0 \quad , \quad i = 1\ to\ m \tag{13}$$

and *p* equality constraints

$$h_j(X) = 0 \quad , \quad \mathrm{j=1\ to\ p} \tag{14}$$

Where, $X^* \in \Re^n$ is the vector of decision or design variables, and $F(X) \in \Re^k$ is the vector of objective functions. Without loss of generality, it is assumed that all objective functions are to be minimized. Such multi-objective minimization based on the Pareto approach can be conducted using some definitions:

Definition of Pareto Dominance: A vector $U = [u_1, u_2, \ldots, u_k] \in \Re^k$ dominates to vector $V = [v_1, v_2, \ldots, v_k] \in \Re^k$ (denoted by $U \prec V$) if and only if $\forall i \in \{1,2,\ldots,k\}$, $u_i \le v_i \ \wedge\ \exists j \in \{1,2,\ldots,k\} : u_j < v_j$. It means that there is at least one u_j which is smaller than v_j whilst the rest u's are either smaller or equal to corresponding v's.

Definition of Pareto front: For a given MOP, the Pareto front $\mathcal{PF}^*$ is a set of vectors of objective functions which are obtained using the vectors of decision variables in the Pareto set $\mathcal{P}^*$, that is, $\mathcal{PF}^* = \{F(X) = (f_1(X), f_2(X), \ldots, f_k(X)) : X \in \mathcal{P}^*\}$. Therefore, the Pareto front $\mathcal{PF}^*$ is a set of the vectors of objective functions mapped from $\mathcal{P}^*$.

In this work, a new multi-objective uniform-diversity genetic algorithm method called MUGA that has been proposed by authors [9], [10] is used for multi-objective reliability-based optimization

IV. The Single Inverted Pendulum and State Feedback Controller Design Method

One of the important benchmark of control problem is inverted pendulum that has been shown in Fig. (1). The linear steady state equations of inverted pendulum are given as

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\\ \dot{x}_4\end{bmatrix} = \frac{1}{I_0}\begin{bmatrix}0 & IM-(M_1 l_s)^2 & 0 & 0\\ 0 & -If_r & -(M_1 l_s)^2 g & M_1 l_s C\\ 0 & 0 & 0 & IM-(M_1 l_s)^2\\ 0 & M_1 l_s f_r & MM_1 l_s g & -CM\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\\ x_4\end{bmatrix} + \frac{1}{I_0}\begin{bmatrix}0\\ I\\ 0\\ -M_1 l_s\end{bmatrix}u \tag{15}$$

where $M = M_1 + M_2$, $I_0 = IM - (M_1 l_s)^2$, and x_1, x_2, x_3 and x_4 stand for the position of cart, the velocity of cart, the angle of pendulum with the vertical axis and the angular velocity of pendulum, respectively. And M_1, M_0, l_s, f_r, I and C are the mass of cart, the mass of pendulum, the length of arm, friction coefficient of cart, moment of inertia of pendulum and rotating friction coefficient of pendulum, respectively. In the case of stochastic robust design, parameters of the plant given by (15) vary according to *a priori* known probabilistic

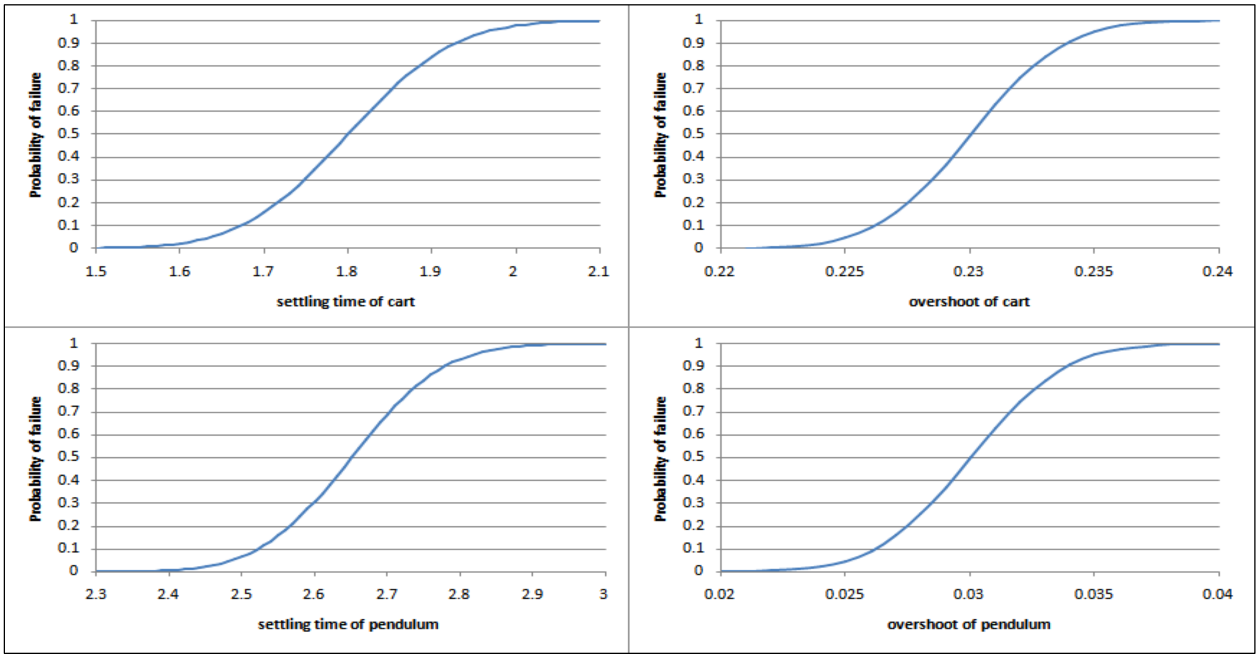

Fig. (2): fuzzy threshold values for calculating the probability of failure

distribution functions around a nominal set of parameters. The nominal values of plant are given in Table (I). In this study, linear state-feedback controllers are used for control of the inverted pendulum. The equation of the state feedback controller is

$$u = K_1(x_1 - 0.2) + K_2 x_2 + K_3 x_3 + K_4 x_4 \quad (16)$$

where the design vector $\mathbf{k} = (K_1, K_2, K_3, K_4)$ has to be optimally determined based on probabilistic uncertain values of those parameters in the Pareto multi-objective optimum approach.

Table (I): the nominal values of single inverted pendulum

M_1	0.36kg
M_0	4kg
l_s	0.451m
f_r	10
C	0.00145

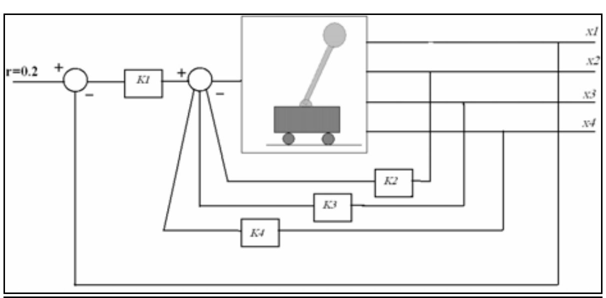

Fig. (1): Block diagram of SIP and state feedback controller.

The performance of a controlled closed-loop system is usually evaluated by variety of goals. In this paper, a bi-objective optimum robust linear state feedback controller is considered whilst each of those objective function is the probability of failure of S_{RO}-C and S_{RO}-P based on the fuzzy membership definition of failure as described before. Such pre-defined fuzzy membership functions for the probability failure definition of overshoot and settling time of both cart and pendulum based on (10) and (11) are depicted in Fig. (2): The probability of failure of each function can now be computed using equation (11) using MCS approach with the Hammersly random distribution sampling.

Table (II): Optimum values of objective functions and the corresponding gains of state feedback of Pareto front points

	Objective functions		**Gains of the controller**			
	Prob. of failure of S_{SO}-C	**Prob. of failure of S_{SO}-P**	K_1	K_2	K_3	K_4
A	1.23	0.01761	47.24	86.61	496.06	125.98
B	1.13	0.0177	47.24	86.61	500	125.98
C	1.021724	0.07173	62.99	86.61	362.20	129.92
D	0.974335	0.072337	62.99	86.61	366.14	129.92
E	0.954336	0.074546	62.99	86.61	370.07	129.92
F	0.882893	0.081522	62.99	86.61	366.14	133.85
G	0.863531	0.084496	62.99	86.61	370.07	133.85
H	0.810603	0.088883	62.99	86.61	374.01	133.85
I	0.591567	0.093727	59.05	78.74	334.64	114.17
J	0.354947	0.096733	66.93	86.61	374.01	125.98
K	0.26116	0.136599	94.49	114.17	500	153.54
L	0.253011	0.141357	94.49	114.17	496.06	153.54
M	0.174052	0.142995	94.49	110.23	500	153.54
N	0.166823	0.247558	94.49	110.23	448.81	149.60
O	0.162381	0.385868	66.93	78.74	334.64	98.42
P	0.128591	0.451299	98.42	110.23	440.94	141.73
Q	0.01	1	94.49	90.55	366.14	82.67

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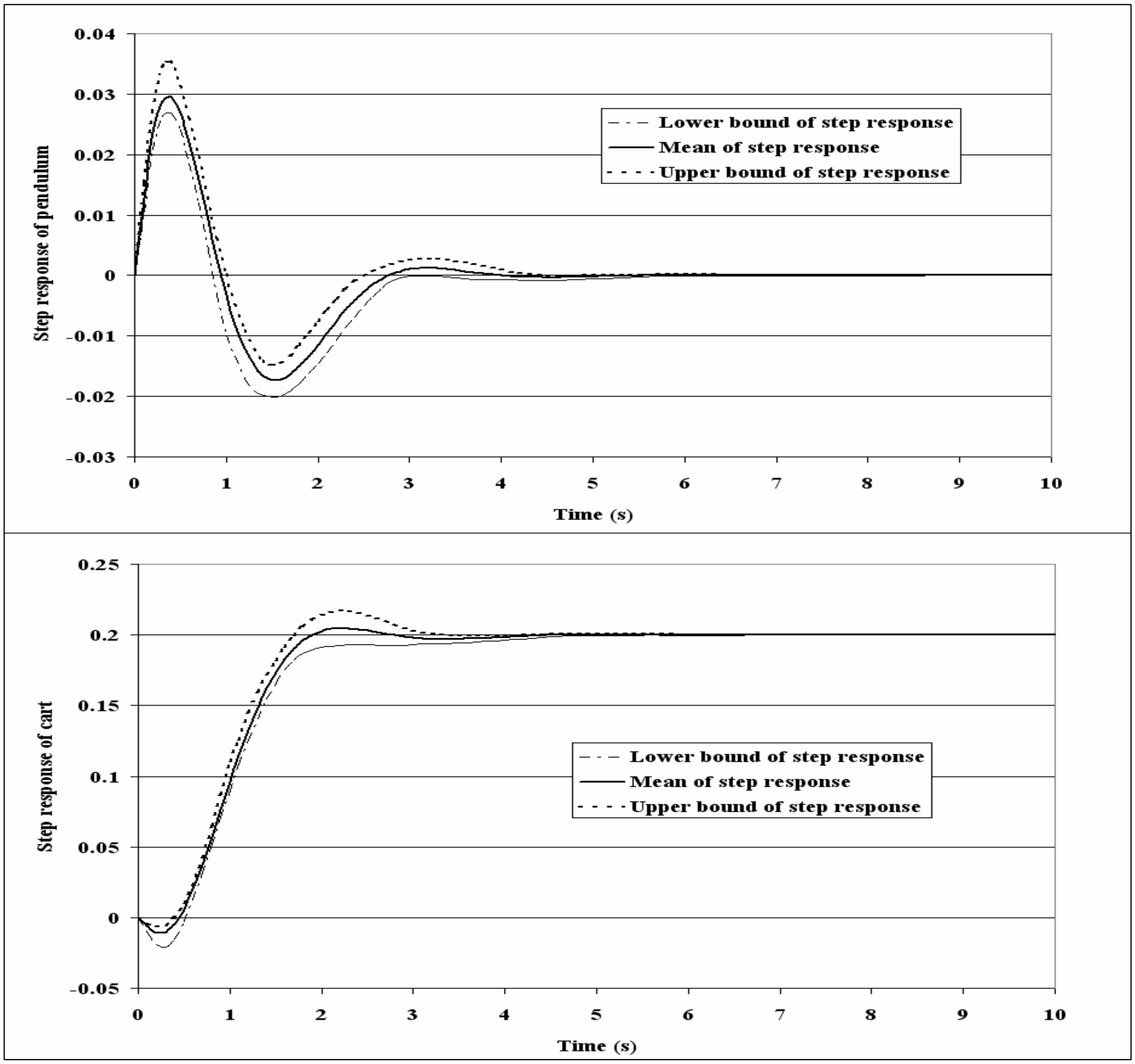

Fig. (3): Probabilistic time response behaviors of optimum design point M (7000 samples).

V. Illustrative Example

Multi-objective uniform-diversity genetic algorithm (MUGA) [9] [10] is used for Pareto multi-objective optimization of linear robust state feedback controllers of the probabilistic uncertainties single inverted pendulum whose parameters are varied with Gaussian distributions within the limits of $\pm 25\%$ of the nominal values of plant parameters. A population size of 70 has been chosen with crossover probability P_c and mutation probability P_m as 0.85 and 0.09, respectively. Two objectives, namely, probability of failure of S_{RO}-C and the probability of failure of S_{RO}-P are considered simultaneously in a Pareto optimization process to obtain some important trade-offs among conflicting objectives. The optimization process of the robust linear statefeedback controller given by (16) is accomplished by 200 Monte Carlo evaluations using Gaussian distribution for each candidate control law during the evolutionary process. Consequently, total number of 17 non-dominated optimum design points have been obtained and shown in Table (II). The optimum design point Q and A simply demonstrate the best values for S_{RO}-C and S_{RO}-P, respectively, whilst the optimum design point M is the trade-off design and may be compromisingly chosen from that table of Pareto front points. Evidently, the design point M exhibits a significant improvement in the probability of failure of either S_{RO}-C or S_{RO}-P. Comparing to the design points Q and A, respectively.

However, in order to show the robustness behavior of the robust trade-off optimum point M, a MCS with 7000 evaluations has been accomplished to measure the probability of failures of both S_{RO}-C and S_{RO}-P based of the fuzzy membership variations given in Fig. (2) It is now very evident from the values of the probabilities given in Table (II) that the robust trade-off optimum point M exhibits very good performance. Fig. (3) depicts the time step response of trade-off design point M in a MCS with 7000 evaluations. Such robust behavior of design point M is very clear from Fig. (3) for both cart and pendulum.

In order to compare the results of probabilistic design using fuzzy membership definition with those using crisp

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definition of thresholds, three different Pareto fronts are obtained corresponding to the crisp threshold values are given in Table (III). It should be noted that the order of sets (1), (2) and (3) is from large values of the crisp threshold to small ones. In fact, large values of crisp threshold increase the possibility of finding a design point satisfying all those thresholds for the particular Monte Carlo Simulation (MCS). This, in turn, leads to lower values of probability of failure for MCS as shown in set (3). It is now possible to translate back the optimum design point M obtained using fuzzy membership values of probabilities of failure into each Pareto front of crisp sets (1), (2) and (3), which are correspondingly shown in Table (III). It is now evident from this table that all non-dominated optimum design points obtained from those three different sets of crisp threshold values are almost inclusive in the optimum design points obtained by the approach of fuzzy threshold values.

Table (III): The values of objective function of the best optimal points obtained by three different sets of crisp threshold values

Set No.	Crisp threshold values				Best Pareto point		Cross-validation of design point M	
	Settling time of cart	Overshoot of cart	Settling time of pendulum	Overshoot of pendulum	Prob. of failure of S_{SO}-C	Prob. of failure of S_{SO}-P	Prob. of failure of S_{SO}-C	Prob. of failure of S_{SO}-P
1	2.1	0.24	3	0.04	0.01	0.01	0.01	0.03
2	1.95	0.235	2.85	0.035	0.03	0.01	0.02	0.03
3	1.8	0.23	2.7	0.03	0.09	0.42	0.1	0.42

VI. CONCLUSION

A multi-objective uniform-diversity genetic algorithm (MUGA) has been proposed and successfully used to optimally design linear state feedback controllers from a reliability-based point of view in a probabilistic approach with fuzzy threshold values. The objective functions which often conflict with each other were appropriately defined using some probabilistic metrics in time domain. The multi-objective optimization of robust linear state feedback controllers led to the discovering some important trade-offs among those objective functions. The framework of such hybrid application of multi-objective GAs and Monte Carlo Simulation of this work for the Pareto optimization of both robust and reliability-based approach using some non-commensurable stochastic objective functions is very promising and can be generally used in the optimum design of real-world complex control systems with probabilistic uncertainties.

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