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An Integer Programming Formulation for the Lot Streaming Problem in a Job Shop Environment with Setups

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***Abstract*—This paper aims at solving the lot streaming problem in a job shop environment, where setup times are involved. The proposed integer programming formulation sufficiently describes the processing dynamics of individual sublots and enables the simultaneous determination of schedules on machines and sublot sizes. Small instances of job shop problems with consistent sublots can thus be optimally solved. Computational results confirm that, by applying the lot streaming strategy, both idling times of machines and completion times of operations are significantly reduced. In view of setups, various types of setups are incorporated in the model. The influence of setup times on the performance of lot streaming is also intensively examined. In addition, the efficiency of the formulation with special constraints is evaluated.**

***Index Terms*—lot streaming; job shop scheduling; integer programming**

I. Introduction

THE purpose of this paper is to solve the lot streaming problem in a job shop environment, where setup times are involved. The *job shop* scheduling problem can be briefly described as follows: A set of jobs and a set of machines are given. Each machine can process at most one job at a time. Each job consists of a sequence of operations, which need to be processed during an uninterrupted time period of a given length on a given machine. A *schedule* is an allocation of the operations to time intervals on the machines. The objective is to find a schedule of minimum length (*makespan*). This class of problems is proved to be NP-hard.

With respect to *lot streaming*, a job is actually a *lot* composed of identical items. In classical job shop scheduling problems a lot is usually indivisible. The entire lot must be completed before being transferred to its successor operation, which leads to low machine utilization and long completion times. Lot streaming techniques, on the other hand, provide the possibility of splitting a lot into multiple smaller sublots, which can be treated individually and immediately transferred to the next stage once they are completed. Different sublots of the same job can thus be simultaneously processed at different operation stages. As a result of operation overlapping, the production can be considerably accelerated. However, due to the complex interaction between sublots and machines, job shop problems with the application of the lot streaming strategy is difficult to formulate mathematically.

In the last years, a majority of researches focused on solving lot streaming problems in a flow shop production system [1], [9], [10], [4], [5], [2], [7]. The job shop scheduling problem, on the contrary, has received little attention. Dauzère-Pérès and Lasserre [6] introduced an iterative heuristic to solve the lot streaming problem in a job-shop environment. By adopting the modified shifting bottleneck procedure, a good solution can be obtained within a few iterations. For the same problem, Buscher and Shen [3] presented an advanced tabu search algorithm which outperforms the previous heuristic. This algorithm is also able to reach the theoretical lower bounds for some hard benchmark instances in scheduling. In [8] a model for the lot streaming problem with setups was established. Aside from the makespan objective, cost-based measurements are integrated as well. Under the assumption that the sublot sizes are given, several examples were tested with the conclusion that equal-sized sublots provide better solutions in general.

The integer programming formulation presented in this paper is based on the study of [8]. Necessary modifications are conducted in the first place. The model is then further developed to increase efficiency. Moreover, instead of employing fixed sublot sizes, test instances are solved with the determination of sublot sizes. According to our observation, optimal solutions are generally obtained with unequal-sized sublots, which obviously contradicts the assertion of [8].

The remainder of the paper is organized as follows: In the next section, an integer programming formulation for solving the lot streaming problem in a job shop production system with setups is developed. Various types of setups are incorporated in the model. Section 3 provides a detailed analysis of computational results. The computational results focusing on various aspects are then presented in detail. Brief conclusions are summarized in Section 4.

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II. Model formulation

A. Notations

C_{max}	makespan
n	total number of jobs
m	total number of machines
s	total number of sublots
i, i'	job indices, $i, i' = 1, \ldots, n$
k, k'	machine indices, $k, k' = 1, \ldots, m$
j, j'	sublot indices, $j, j' = 1, \ldots, s$
H	sufficiently large number
D_i	demand of job i, i.e. the initial lot size
M_k	machine k
O_{ijk}	the operation of the jth sublot of job i on machine k

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t_{ijk}	start time of operation O_{ijk}
p^u_{ik}	unit processing time of job i on machine k
r_{ik}	setup time of job i on machine k
X_{ij}	production quantity of the jth sublot of job i
A	set of pairs of operations constrained by precedence relations
L	set of the last operations of sublots
δ_{ijk}	binary variable which equals 1 if setup is required before processing operation O_{ijk}; 0 otherwise
$Y_{iji'j'k}$	binary variable which equals 1 if Operation O_{ijk} is processed prior to Operation $O_{i'j'k}$; 0 otherwise.

B. Integer programming formulation

It is assumed that each job consists of m operations and must pass through each machine exactly once. All machines are available at time zero. Furthermore, the total number of sublots is given and consistent sublot sizes are considered. In addition, transport times are negligible. With the notations and assumptions, the model can then be summarized as follows:

$$\min C_{max} \tag{1}$$

Subject to:

$$\sum_{j=1}^{s} X_{ij} = D_i \quad \forall i \tag{2}$$

$$X_{ij} \geq 0 \quad \forall i, j \tag{3}$$

$$\delta_{ijk} \leq X_{ij} \quad \forall i, j, k \tag{4}$$

$$t_{ijk'} \geq t_{ijk}+r_{ik}\cdot\delta_{ijk} + p^u_{ik}\cdot X_{ij} \quad \forall (O_{ijk}, O_{ijk'}) \in A \tag{5}$$

$$t_{i(j+1)k} \geq t_{ijk} + r_{ik} \cdot \delta_{ijk} + p^u_{ik} \cdot X_{ij} \quad \forall i, k, j < s \tag{6}$$

$$C_{max} \geq t_{isk} + r_{ik} \cdot \delta_{isk} + p^u_{ik} \cdot X_{is} \quad \forall i, O_{isk} \in L \tag{7}$$

$$\begin{aligned} & t_{ijk} \geq t_{i'j'k} + r_{i'k} \cdot \delta_{i'j'k} + p^u_{i'k} \cdot X_{i'j'} - H \cdot Y_{iji'j'k} \\ & t_{i'j'k} \geq t_{ijk} + r_{ik} \cdot \delta_{ijk} + p^u_{ik} \cdot X_{ij} - H \cdot Y_{i'j'ijk} \\ & Y_{iji'j'k} + Y_{i'j'ijk} = 1 \quad \forall i \neq i', j, j' \end{aligned} \tag{8}$$

$$\delta_{i1k} = 1 \quad \forall i, k \tag{9}$$

$$\begin{aligned} & \delta_{i(j+1)k} \geq Y_{iji'j'k} - Y_{i(j+1)i'j'k} \\ & \forall i \neq i', j < s, j', k. \end{aligned} \tag{10}$$

In our model we employ the conventional makespan objective function (1). Constraints (2) ensure that all required units are produced. Constraints (3) are the non-negativity conditions. Since sublot sizes may equal 0, the actual number of sublots is possibly smaller than the given number s. This adds flexibility to the formulation with the fixed total number of sublots (s). Obviously, no setup is necessary, if the corresponding sublot doesn't exist. Constraints (4) are therefore used to avoid redundant setups.

Constraints (5) represent the precedence relations of the operations that belong to the same sublot. In the model of [8] similar constraints are considered, which apply to the operations of different sublots as well. However, it should be pointed out that the operations of different sublots are not constrained by the precedence relation, since sublots are treated as separate jobs.

When attached setup times are taken into consideration, the setup of a certain machine cannot begin until the corresponding sublot has been transferred to this machine. Constraints (5) fulfil this requirement. On the other hand, detached setups can be performed in advance, with no regard to the availability of sublots. The constraints can then be slightly modified as:

$$t_{ijk'}+r_{ik'}\cdot\delta_{ijk'} \geq t_{ijk}+r_{ik}\cdot\delta_{ijk}+p^u_{ik}\cdot X_{ij} \quad \forall (O_{ijk}, O_{ijk'}) \in A \tag{11}$$

Constraints (6) state that a sublot can only be scheduled on a certain machine after the sublots with smaller indices of the same job finish their processing. For instance, the second sublot cannot be processed prior to the first sublot of the same job. Due to the simultaneous determination of sublot sequences and sublot sizes, constraints (6) can be employed without loss of generality. In the meantime, these constraints provide the basis for the concise formulation of setup times.

Constraints (7) indicate that the makespan is defined by the latest completion time of the last operation of the sublot with the maximal index (s). In the model developed by [8], the makespan is similarly calculated, while the sublot precedence constraints (6) are neglected. This, however, is not always correct, since the sth sublots are not necessarily scheduled at the end.

Theoretically, constraints (6) can be removed. The makespan is thus determined by:

$$C_{max} \geq t_{ijk} + r_{ik} \cdot \delta_{ijk} + p^u_{ik} \cdot X_{ij} \quad \forall i, j, k. \tag{12}$$

Owing to the complex interaction between sublots and machines, this formulation generally requires more iterations to solve an identical problem.

Constraints (8) are adopted to determine the sequences on machines and to prevent overlapping of operations. If $Y_{iji'j'k}$ takes the value 1, only the first set of constraints is relevant, which indicate that operation $O_{i'j'k}$ must be processed after the completion of operation O_{ijk}. If $Y_{iji'j'k}$ equals 0, the second set of constraints operate in a similar manner.

In the model of [8], the last set of constraints in (8) are neglected. This, in the first place, contradicts the definition of the binary variable $Y_{iji'j'k}$. Moreover, if $Y_{iji'j'k}$ and $Y_{i'j'ijk}$ both equal 1, constraints (8) and (8) are fulfilled at the same time, which, however, leads to infeasible solutions.

In view of setups, constraints (9) ensure that the machines are properly adjusted before processing the first sublot of each job.

Note that only one setup is essential, if sublots of the same job are consecutively scheduled on a certain machine. In terms of [6], this is a scheduling problem with *sequence-dependent setup times*, which is difficult to solve. Instead of approximate modelling, constraints (10) formulate this situation precisely. According to (6), operation O_{ijk} should always be scheduled before $O_{i(j+1)k}$. If these two operations are processed directly one after the other, $\delta_{i(j+1)k}$ takes the value 0 automatically (see figure 1). As long as there is an operation of any other job in between, the right side of the corresponding inequation equals 1, which forces $\delta_{i(j+1)k}$ to be 1 (see figure 2). Therefore, constraints (10) ensure that all the consecutively scheduled sublots of the same job are processed under a single setup.

C. Extensions

1) No-wait: An important class of machine scheduling problems is characterized by a no-wait production system, where a job must be processed from start to completion

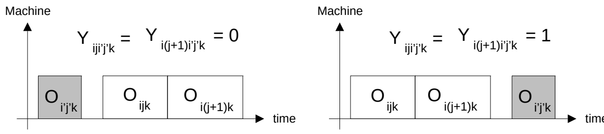

Fig. 1. Illustration of constraints (10) (1)

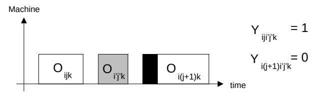

Fig. 2. Illustration of constraints (10) (2)

without intermediate buffer between machines. In comparison to constraints (5), the no-wait requirement can be simply expressed as:

$$t_{ijk'} = t_{ijk} + r_{ik} \cdot \delta_{ijk} + p^u_{ik} \cdot X_{ij} \quad \forall (O_{ijk}, O_{ijk'}) \in A. \quad (13)$$

2) Non-idling: On the other hand, if a non-idling environment occurs, where all sublots of the same job must be continuously processed on a particular machine, constraints (6) can be modified as follows:

$$t_{i(j+1)k} = t_{ijk} + r_{ik} \cdot \delta_{ijk} + p^u_{ik} \cdot X_{ij} \quad \forall i, k, j < s. \quad (14)$$

Obviously, only the setup before the first sublot of each job is required. The other binary variables related to setups are then equal to 0:

$$\delta_{ijk} = 0 \quad \forall i, k, j \neq 1. \quad (15)$$

Moreover, only the sequence of jobs is relevant. Binary variables $Y_{iji'j'k}$ can thus be simplified as $Y_{ii'k}$, which significantly reduces the complexity of the formulation. The modified constraints concerning $Y_{ii'k}$ are summarized as follows:

$$\begin{cases} t_{i1k} \geq t_{i'sk} + r_{i'k} \cdot \delta_{i'sk} + p^u_{i'k} \cdot X_{i's} - H \cdot Y_{ii'k} \\ t_{i'1k} \geq t_{isk} + r_{ik} \cdot \delta_{isk} + p^u_{ik} \cdot X_{is} - H \cdot Y_{i'ik} \\ Y_{ii'k} + Y_{i'ik} = 1 \quad \forall i, k, i' \neq i. \end{cases} \quad (16)$$

In order to prevent overlapping on machines, we can take advantage of the attribute of the non-idling case and need to compare only the start time of the first sublot of a certain job with the completion time of the last sublot of another job.

3) Non-intermingle: Another situation especially associated with the lot streaming problem is the non-intermingling setting. This case requires that no interruption from any other job is allowed while processing a particular job. Therefore, constraints (14) are the sufficient but not necessary conditions, whereas (15) and (16) must be satisfied.

4) Special constraints: According to the sublot precedence constraints (6), operation $O_{i'j'k}$ should be scheduled prior to operation $O_{i'(j'+1)k}$. Figure 3 illustrates all possible positions of a third operation O_{ijk}. The corresponding values of the binary variables $Y_{iji'j'k}$ are listed in table I.

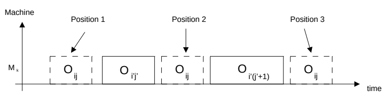

Fig. 3. Illustration of condition (17)

TABLE I
ILLUSTRATION OF CONDITION (17)

	Position 1	Position 2	Position 3
$Y_{iji'j'k}$	1	0	0
$Y_{iji'(j'+1)k}$	1	1	0

The following constraints describe these attributes of $Y_{iji'j'k}$:

$$Y_{iji'j'k} \leq Y_{iji'(j'+1)k} \quad \forall i, j, j' < s, i' \neq i. \quad (17)$$

According to figure 4 and table II, the other constraints

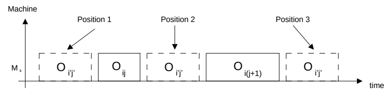

Fig. 4. Illustration of condition (18)

TABLE II
ILLUSTRATION OF CONDITION (18)

	Position 1	Position 2	Position 3
$Y_{iji'j'k}$	0	1	1
$Y_{i(j+1)i'j'k}$	0	0	1

concerning $Y_{iji'j'k}$ can be expressed as:

$$Y_{iji'j'k} \geq Y_{i(j+1)i'j'k} \quad \forall i, j', j < s, i' \neq i \quad (18)$$

[7] contains similar constraints to solve lot streaming flow shop problems. The function of these constraints in a complex job shop environment will be discussed in the next section.

III. COMPUTATIONAL RESULTS

A. Benefit of lot streaming

The integer programming formulation addressed in the previous section was implemented in the optimization software Lingo 9.0 on a personal computer (Athlon 64X2 4800+, 2450MHZ). However, only small instances of job shop problems can be solved optimally. While solving a 2·2 problem with 4 sublots requires less than 2 seconds, the optimal solution for a 3·3 problem with 4 sublots cannot be obtained within 6 hours.

In our study 96 instances of job shop problems, which consist of 2 to 3 jobs and 2 to 6 machines, are tested. Each of the instances with 2 jobs is solved by adopting 1 to 4 sublots. The instances containing 3 jobs are solved employing 1 to 3 sublots. The mean improvement of makespan is summarised in table III.

The average improvement of makespan amounts to 23.44% by applying 2 sublots. With regard to 3 sublots, the benefit of lot streaming is abruptly reduced to 7.32%. In general, 74.83% of potential makespan reduction is already achieved by the application of 2 sublots, whereas the proportion of 4 sublots is marginal. This observation suggests employing merely 2 sublots, so that the most advantage of lot streaming can be obtained while saving computing time.

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TABLE III
MEAN IMPROVEMENT OF MAKESPAN

job· machine	2 sublots		3 sublots		4 sublots	
	Improvement*	Percentage*	Improvement	Percentage	Improvement	Percentage
2·2	6.04%	100.00%	0.00%	0.00%	0.00%	0.00%
2·3	20.61%	71.03%	5.98%	20.84%	2.39%	8.13%
2·4	25.02%	63.54%	9.82%	25.00%	4.53%	11.46%
2·5	32.89%	69.45%	10.08%	21.28%	4.40%	9.26%
2·6	32.04%	67.06%	11.12%	23.26%	4.65%	9.69%
3·3	23.08%	77.88%	6.92%	22.12%	–	–
Mean	23.44%	74.83%	7.32%	18.75%	3,19%	7,71%

* Improvement and percentage are calculated by: $(C_{max,s-1} - C_{max,s}) / C_{max,1}$ and $(C_{max,s-1} - C_{max,s}) / (C_{max,1} - C_{max,4})$, respectively. $C_{max,s}$ represents the makespan obtained by applying s sublots.

TABLE IV
PERFORMANCE OF LOT STREAMING WITH VARIOUS SETUP TIMES

Setup time	2 sublots		3 sublots		4 sublots	
	Improvement*	Percentage*	Improvement	Percentage	Improvement	Percentage
1%	24.49%	74.38%	7.61%	18.69%	3.56%	8.33%
10%	24.28%	74.17%	7.61%	18.74%	3.61%	8.51%
50%	23.61%	74.61%	7.38%	18.65%	3.38%	8.09%
100%	23.28%	74.79%	7.37%	18.99%	3.06%	7.47%
200%	22.75%	74.90%	7.03%	18.49%	3.20%	7.93%
400%	22.21%	76.10%	6.92%	18.95%	2.36%	5.93%

* Improvement and percentage are calculated by: $(C_{max,s-1} - C_{max,s}) / C_{max,1}$ and $(C_{max,s-1} - C_{max,s}) / (C_{max,1} - C_{max,4})$, respectively. $C_{max,s}$ represents the makespan obtained by applying s sublots.

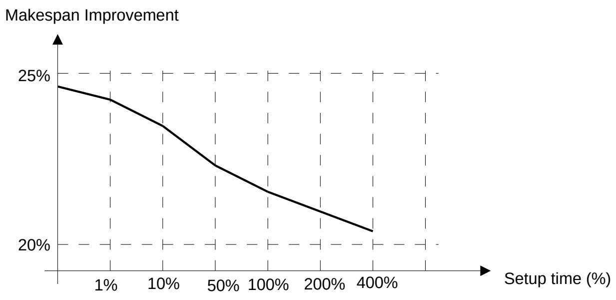

Fig. 5. Makespan improvement with different setup times

B. Impact of setup times

In order to analyze the relationship between setup times and makespan reduction, we adopt the settings from [6]. The setup times are set to be 1%, 10%, 50%, 100%, 200% and 400% of the unit processing times. The computational results of 96 test instances are listed in table IV.

As plotted in figure 5, while the proportion between setup times and processing times rises, the advantage of lot streaming declines in accordance. Nevertheless, the improvement of makespan with 2 sublots exceeds 20%.

Evidently, there is a trade off between the time saved by splitting into sublots and the extra time required due to additional setups. Although the size of setup times imposes a negative influence on the reduction of makespan, lot streaming is still unnegligibly efficient.

C. Solutions with equal-sized sublots

As mentioned in the first section, many researches involved examining the performance of equal-sized sublots. In this respect, constraints (2) are modified as:

$$X_{ij} = D_i / s \quad \forall i, j \qquad (19)$$

In comparison to solving the problem optimally, the sublot sizes are predetermined, which significantly reduces the complexity of the problem. As a result, Lingo program can be remarkably accelerated. In our study, the test instances are also solved by adopting equal-sized sublots. As presented in table V, the deviation of makespan is surprisingly only 3.05% on average, while the necessary iterations to solve identical problems fall sharply. This valuable information suggests that we can take advantage of the trade off between the reduction of computing time and the increment of makespan to solve larger instances of job shop problems. In consequence, satisfying solutions can be obtained within reasonable amount of time.

D. Evaluation of the formulation

One main difference of our formulation compared to the model proposed by [8] is the successful removal of the operation index. In order to compare the efficiency of these two formulations, we implemented their model in Lingo 9.0 as well (after necessary corrections, so that feasible solutions are generated). 45 instances were tested under identical circumstances.

As shown in table VI, our formulation requires significantly fewer iterations for most of the instances. This advantage becomes especially obvious when the problem size increases. The experiment confirms that our formulation is not only straightforward but also more efficient in general.

In our model constraints (17) and (18) are incorporated to describe attributes of the binary variable $Y_{iji'j'k}$. Constraints similar to (17) are also considered in [7]. In terms of [7], the number of iterations required to solve a flow shop problem could be reduced to 60% compared to the model without these restrictions. However, the scheduling reality in a job shop environment is much more complicated. In order to investigate the function of these constraints, 60 instances

TABLE V
PERFORMANCE OF EQUAL-SIZED SUBLOTS

job· machine	2 sublots	3 sublots	4 sublots	Percentage *
2·3	4.11%	4.75%	4.40%	16,68%
2·4	3.40%	4.37%	4.44%	18,11%
2·5	5.78%	7.41%	7.48%	3,65%
2·6	3.09%	4.11%	4.76%	2,50%
3·3	1.91%	2.41%	–	4,94%
Mean	3.05%	3.84%	4.21%	8,19%

$$* \quad \text{Percentage} = \frac{\text{iteration required applying equal sublots}}{\text{iteration required applying consistent sublots}}$$

TABLE VI
COMPARISON OF THE PERFORMANCE OF TWO FORMULATIONS

Problem size		(a) Iter. (our model)	(b) Iter. (model of Low et al)	Percentage (a/b)
3·3·2	1	339311	1661149	20,43%
	2	10587	10821	97,84%
	3	14750	28919	51,00%
	4	27910	56208	49,65%
	5	18304	31785	57,59%
	6	15267	49228	31,01%
	7	18750	42579	44,04%
	8	11069	11349	97,53%
	9	11169	11494	97,17%
	10	5494	4707	116,72%
	11	17313	24313	71,21%
	12	12117	15198	79,73%
	13	14312	20909	68,45%
	14	18616	20183	92,24%
	15	37712	24313	155,11%
	16	21315	27663	77,05%
	17	21678	14775	146,72%
	18	21778	43153	50,47%
	19	19988	33179	60,24%
				77,06%
2·2·4	20	16485	45645	36,12%
	21	9829	13330	73,74%
	22	36019	52291	68,88%
	23	18457	34088	54,15%
	24	31726	72561	43,72%
	25	33657	37683	89,32%
	26	17932	18015	99,54%
	27	19652	24281	80,94%
	28	16574	30150	54,97%
	29	5412	9431	57,39%
	30	4680	10576	44,25%
	31	14248	22067	64,57%
	32	16422	17778	92,37%
	33	2512	5477	45,86%
	34	3361	17367	19,35%
				61,68%
3·3·3	35	763827	950003	80,40%
	36	2301475	5993093	38,40%
	37	5255761	12591491	41,74%
	38	3162333	8806108	35,91%
	39	2229550	9719360	22,94%
	40	769311	4785587	16,08%
	41	1033909	1615861	63,99%
	42	739141	2404653	30,74%
	43	656268	1410262	46,54%
	44	173255	2248802	7,70%
	45	80564	131995	61,04%
				40,50%

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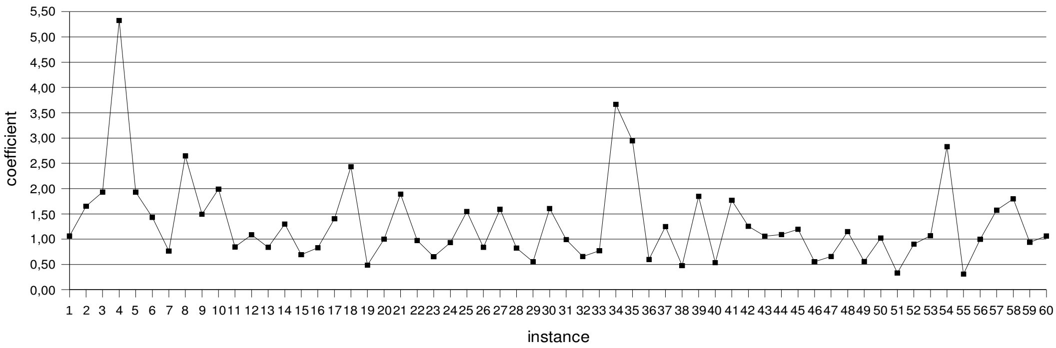

$$\text{coefficient} = \frac{\text{Iterations required without (17) and (18)}}{\text{Iterations required employing (17) and (18)}}$$

Fig. 6. Performance of (17) and (18)

were tested. The results are depicted in figure 6 where the coefficient calculation is given as well.

By employing these constraints, the necessary iterations for some instances, on the one hand, can be reduced to less than 50%. On the other hand, solving some instances with these constraints demands exceedingly more iterations. Unlike in a flow shop environment, no conclusive behaviour pattern of these constraints was recognizable.

IV. Conclusion

This paper addresses solving the lot streaming problem in a job shop environment, where setup times are included. The proposed integer programming formulation sufficiently describes the processing dynamics of individual sublots and enables the simultaneous determination of schedules on machines and sublot sizes. In view of setup times, various types of setups are incorporated. The model is then further developed to fulfil the requirements of special production systems.

Computational results confirm that the makespan can be considerably improved, when lot streaming techniques are applied to the standard job shop problem. Furthermore, detailed analysis is conducted to reveal the relation between setup times and makespan reduction. Although the improvement of makespan declines as setup times increase, lot streaming is still advantageous.

In comparison to the model established by [8], our formulation is not only straightforward but also more efficient in general. However, by the implementation of the optimization-based software Lingo 9.0, only small instances of the job shop problem can be optimally solved within a realistic time span. Thus, the development of effective heuristics to solve large instances of the problem is desirable for future study. For instance, the implementation of metaheuristic is advisable.

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