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# A Reasoning Method on Computational Network and Its Applications

Nhon Do, Hien Nguyen

*Abstract*—**Knowledge base is an important component of expert systems and intelligent programs. Therefore, researching and development of models for knowledge representation play an important role in Artificial Intelligent. Besides known models, the Computational Network model is useful for representing computational knowledge of human. In this paper, an extensive model of computational network has been presented. This model will use Sample Problems such as the experience of human about practical problem for simulating the way of human thinking, and give good solutions for the practical problems faster and more effectively. This extensive model is applied to construct an automatic system for solving algebraic problems of middle school.**

***Keywords*—artificial intelligence, expert systems, knowledge representation.**

## I. Introduction

Knowledge base has an important role in the design of expert systems. Building expert systems require designing knowledge bases and inference engine. Nowadays there are many various knowledge models have been suggested and applied, such as some model in [1, 2, 3, 4, 8]. Furthermore, we already knew some common methods for knowledge representation in designing knowledge base systems (see [5, 7, 9,11]). Nevertheless, those methods also have several drawbacks which make knowledge representation difficult to implement, especially for areas in general education such as plane geometry, analytic geometry, algebra, etc. Thus, the computational network has been introduced to represent human's knowledge, especially for computational knowledge (see [17]). Also, it has been applied to construct supporting programs in education (see [15]).

In order for dealing with a practical problem, in [10], the author pointed out that we try to recognize something familiar with unknown problem:

"*Relate the given situation to previous knowledge. Look at the unknown and try to recall a more familiar problem that has a similar unknown or involves similar principles.*"

(G. Polya)

This corresponds to searching relating problems which have been solved before, and then proposing an appropriate solution for the problem. Besides that, we can use the result of relating problems for solving the problem. Such relating problems are called *sample problems*. In this paper, we extend the computational network by adding sample problems component to the knowledge of the system. This extension allows us to represent human's thought about finding sample problems before.

In addition, this extensive model can be applied in constructing a system for automatically solving algebraic problems in middle school.

## II. The Extensive Model of Computational Network

### A. Computational Network:

**Definition 2.1:** A *computational network* is a pair (M, R), in which $M = \{x_1, x_2, ..., x_n\}$ is a set of variables with simple values (or unstructured values), and $R = \{r_1, r_2, ..., r_m\}$ is a set of computational relations over the variables in the set M. Each computational relation $r \in R$ has the following form:

(i) An equation over some variables in M, or

(ii) Deductive rule $r : u(r) \rightarrow v(r)$, with $u(r) \subset M$, $v(r) \subset M$, and there are corresponding formulas to determine (or to compute) variables in v(r) from variables in u(r).We also define the set $M(r) = u(r) \cup v(r)$.

Given a computational network (M, R), the popular problem arising from reality applications is that to find a solution to determine a set $H \subseteq M$ from a Goal.

This problem is denoted by the symbol **(H, Goal)**, with **H** is the hypothesis and **Goal** is the goal of the problem.

**Definition 2.2:** Given a Computational Network (M, R).

(i) For each $A \subseteq M$ and $r \in R$, denote $r(A) = A \cup M(r)$ be the set obtained from A by applying r. Let $S = [r_1, r_2, ..., r_k]$ be a list consisting relations in R, the notation $S(A) = r_k(r_{k-1}(\ldots r_2(r_1(A)) \ldots ))$ is used to denote the set of variables obtained from A by applying relations in S.

(ii) The list $S = [r_1, r_2, ..., r_k]$ is called a *solution* of the problem (H, Goal) ($H \subseteq M$) if S(H) satisfied Goal. Solution S is called a *good solution* if there is not a proper sublist S' of S such that S' is also a solution of the problem. The problem is *solvable* if there is a solution to solve it.

Model Computational network which was presented in [15], [17] was used in the design of many applications in intelligent systems. However, the alternating current problems in practical domain of knowledge have specific

Nhon Do is currently a senior lecturer in the faculty of Computer Science at the University of Information Technology, Ho Chi Minh City, Vietnam. He got his MSc and Ph.D. in 1996 and 2002 respectively, from University of Sciences – National University of Ho Chi Minh City. His research interests include Artificial Intelligence, computer science, and their practical applications, especially intelligent systems and knowledge base systems. (email: nhondv@uit.edu.vn)

Hien Nguyen is a master student at University of Sciences – National University of Ho Chi Minh City, Vietnam. His research interests include Artificial Intelligence, computer science, especially knowledge representation. He is currently a teaching assistant in the faculty of Computer Science at the University of Information Technology, Ho Chi Minh City, Vietnam. (email: hiennd@uit.edu.vn)

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characteristics so that a suitable model will be proposed to design a system to solve them.

*B. The Extensive model of Computational Network:*

When dealing with a practical problem, a convenient way to proceed is considering whether we have met a similar or relating problem before or not. If so, then the solution for the problem can be obtained effectively. Or we determine whether the result of relating problems can be used to solve the practical problem or not. This leads to a requirement that model of knowledge base needs to be furnished a new component which can capture this behavior of human.

**Definition 2.3:** A *sample problem* has three components

**( $M_P$, Goal , Sol)**

in which:

(i) ($M_p$, Goal) is a problem of computational network

(ii) Sol: is a solution of problem ($M_p$, Goal)

Example 2.1:

SAMPLE PROBLEM FOR SOLVING THE QUADRIC INEQUATION

Object = $a.x^2 + b.x + c > 0$ assuming $a > 0$

We have rules for solving the quadric equation

Rule $r_1$: $\{\text{Object}\} \to \{a, b, c\} \quad (a \neq 0)$

Rule $r_2$: $\{a, b, c\} \to \{\text{delta} = b^2 - 4ac\}$

Rule $r_3$: Compare delta and number 0

Rule $r_{3.1}$: $\{\text{delta} < 0\} \to \{\text{Root} = \mathbb{R}\}$

Rule $r_{3.2}$: $\{\text{delta} = 0\} \to \left\{\text{Root} = \mathbb{R} \setminus \left\{-\frac{b}{2a}\right\}\right\}$

Rule $r_{3.3}$:

$$\{\text{delta} > 0\} \to \{\text{Root} = (-\infty, x_1) \cup (x_2, +\infty)\}$$

(where $x_1 = \frac{-b - \sqrt{\Delta}}{2a}$ , $x_2 = \frac{-b + \sqrt{\Delta}}{2a}$ )

Solution of problem is Sol = [$r_1$, $r_2$, $r_3$]

Now, we have the sample problem S for solving the quadric inequation as follow:

S = ($M_p$, Goal, Sol)

which: $M_p$ = {Object, a > 0}

Goal = (“SOLVE”, [Object])

Sol = [$r_1$, $r_2$, $r_3$]

**Definition 2.4:** The *Extensive model of Computational network (ECN)* is a model which consists of three following components:

**(M, Sample, R)**

(i) **M** = $M_{num} \cup M_{func}$ is a set of attributes or objects, with simple valued (real or Boolean valued) or functional valued.

$M_{num}$ = {$O_1$, $O_2$, $O_3$, …} is the set of simple valued objects.

$M_{func}$ = {$f_1$, $f_2$, $f_3$, …} is the set of functional valued objects.

(ii) **Sample** = {$S_1$, $S_2$, $S_3$,…} is a set of sample problems.

(iii) **R** = $R_{sample} \cup R_{knowledge} \cup R_{Heuristic}$ is a set of deduce rule, and R is the union of three subsets of rules $R_{sample}$ , $R_{knowledge}$ , $R_{Heuristic.}$ Each rule r has the form r: u(r)→v(r), with u(r) is the hypotheses of r and v(r) is the conclusion of r. A rule is also one of the three cases below:

- Case 1: r $\in R_{sample.}$ For this case, r is a rule for searching sample problem in **Sample**
- Case 2: r $\in R_{knowledge.}$ For this case, r is a rule of knowledge domain.
- Case 3: r $\in R_{Heuristic.}$ For this case, r is a heuristic rule of knowledge domain.

*C. Example:*

Knowledge domain about algebra of middle school can be modeled by the ECN model with its components as follows:

1. **M** = $M_{num} \cup M_{func}$ , in which:

- $M_{num}$ = {Real number; numeric expressions; Boolean expressions}

E.g.: “2”, “4.5” , “9+7+1”, “16*18+84*18”, “1/7 < 7/8”

- $M_{func}$ = {functional valued objects}

E.g.:

$$a^5, a^4 + 2a^2 + 9,\ x^2 - y^2,\ x^3 - y^3,\ (x + y)^2 \geq 0$$

2. **Sample** = {

- Sample problems for solving simple and quadratic equations;
- Sample problems for solving simple and quadratic inequations;
- Sample problems for computing the numeric expressions;
- Etc.

}

3. **R** = $R_{sample} \cup R_{knowledge} \cup R_{Heuristic}$ , in which:

- $R_{sample:}$ set of rules for searching sample problems in **Sample** set**.**

E.g.:

o Rule for searching the kind of Sample Problems

o Rule for comparising the goal of Sample Problems

o Rule for determining the Sample Problem

- $R_{knowledge}$: **s**et of rules in knowledge domain about algebra of middle school.

E.g.:

o $(X - Y)^2 = X^2 - 2 * X * Y + Y^2$

o $\{X^2 \geq 0\} \Rightarrow \{X \text{ in } \mathbb{R}\}$

o $\{X \geq Y,\ Y \geq Z\} \Rightarrow \{X \geq Z\}$



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*D. Model of problem on Extensive model of Computational Network*

**Definition 2.5:** Given a extension model of Computational Network (M,Sample,R). *Model of problem* in ECN is denoted by the symbol:

**(P, O, F) ➝ Goal**

In which:

(1) **P** = $\{m_1, m_2, m_3, \ldots\}$ is a set of parameter in problem
(2) **O** = $\{O_1, O_2, O_3, \ldots\}$ is a set of Objects in problem
(3) **F** = $\{fact_1, fact_2, \ldots\}$ is a set of Facts in problem
(4) **Goal**: the purpose of problem.

The solution and good solution of problems on ECN have been defined as definition 2.2

**Definition 2.6:** Given a model ECN (M, Sample, R), and a problem H → G on this model. It is easy to verify that there exists an unique maximum set $\overline{H}$ such that the problem H → $\overline{H}$ is *solvable*.

The set $\overline{H}$ is called the closure of H.

**Lemma:** Given a model ECN (M, Sample, R), by definition 2.6, problem H → $\overline{H}$ is solvable. Then, there exists an relation $S = [r_1, r_2, ..., r_k]$ such that $S(H) = \overline{H}$.

**Theorem 2.1:** Given a model ECN (M, Sample, R). The following statements are equivalent.

(i) Problem H → G is solvable.
(ii) $\overline{H} \supseteq G$
(iii) There exists a list of relations S such that $S(H) \supseteq G$

Proof:

* The equivalent of (i) and (iii) can been checked easily.
* (i) => (ii): Problem H → G is solvable, but by definition 2.6, $\overline{H}$ is a maximum set such that problem H → $\overline{H}$ is solvable, so $G \subseteq \overline{H}$.
* (ii) => (iii): Problem H → $\overline{H}$ is solvable, by lemma, there exists an relation $S = [r_1, r_2, ..., r_k]$ such that $S(H) = \overline{H}$. But $G \subseteq \overline{H}$, so $S(H) \supseteq G$

*E. Algorithms on Extensive model of Computational network*

Give the problem (P,O,F)➝ Goal on model ECN as definition 2.4, base on theorem 2.1, these are algorithms for finding the solution of this problem.

**Algorithm 2.1:** Find a solution of problem (P,O,F)➝Goal

```
Step 1: Solution ← empty;
        Solution_found ← false;
        H ← O ∪ P;
Step 2: Repeat
        Find a sample problem S
        if (S found) then
          begin
            H ← H ∪ S.Goal;
            Add S.Sol to Solution;
          end;

        if (Goal is satisfied) then
            Solution_found ← true;
        Until Solution_found or not(S found);
Step 3: if Solution_found then
            goto step 5;
        else goto step 4;
Step 4: Repeat
        Hold ← H;
        Select r ∈ R_knowledge ∪ R_Heuristic;
        while not Solution_found and (r found) do
          if (applying r from H produces new facts)
          then
            begin
              H ← H ∪ M(r);
              Add r to Solution;
            end;
          if (Goal is satisfied) then
              Solution_found ← true;
          Select new r ∈ R_knowledge ∪ R_Heuristic;
        end while;
        Until Solution_found or (H = Hold);
Step 5: if Solution_found then
            Solution is a solution of the problem;
        else
            There is no solution found;
```

**Algorithm 2.2:** Find a sample problem from problem (P,O,F)➝Goal

```
Step 1: SP ← Sample
        Sample_found← false
Step 2: Repeat
        Select S in SP
        if kind of F = kind of S.M_p then
          begin
            if kind of Goal = kind of S.Goal then
                Sample_found ← true
            Else if S.M_p ⊆ H then
                Sample_found ← true
          end
        SP ← (SP – S)
      Until SP = {} or Sample_found
Step 3: if Sample_found then
            S is a sample problem of the problem;
        else
            There is no sample problem found;
```

For the Extensive model of computational net (M, Sample, R), in many cases the problem (P,O,F)➝Goal has a solution S in which there are some redundant variables arise unexpectedly. For those situations, we need to omit these redundant variables of each step in the solving process. The following theorem shows the way to analyze the solution to determine necessary variables to be computed at each step.

**Theorem 2.2:** Given a ECN model. Let $[r_1, r_2, ..., r_m]$ be a good solution of the problem (P,O,F) ➝ Goal denote $A_0 = H$, $A_i = [r_1, r_2, ..., r_i](H)$, with i = 1, ..., m. Then there exists a list $[B_0, B_1, ..., B_{m-1}, B_m]$ satisfying the following conditions:

(1) $B_m$ = Goal,
(2) $B_i \supseteq A_i$, with i=0, 1, ..., m.
(3) For i = 1,..., m, $[r_i]$ is a solution of the problem $B_{i-1} \to B_i$ but not to be a solution of the problem $B \to B_i$, with B is any proper subset B of $B_{i-1}$.

**Algorithm 2.3:** Find a good solution from a solution $S = [r_1, r_2, ..., r_k]$ of the problem (P,O,F)→Goal on the ECN model (M,Sample,R).

Step 1: NewS ← [ ];
H ← O ∪ P;
V ← Goal;
Step 2: for i := k downto 1 do
if $v(r_i) \cap V \neq \emptyset$ then
begin
Insert $r_i$ at the beginning of NewS;
$V \leftarrow (V - v(r_i)) \cup (u(r_i) - H)$;
end
Step 3: NewS is a good solution.

## III. THE SYSTEM FOR SOLVING ALGEBRAIC PROBLEMS IN MIDDLE SCHOOL AUTOMATICALLY

By the Extensive model of Computational Network and the above algorithms, we have represented the knowledge base about algebra of middle school. In addition, the system for automatic solving algebraic problems has been built successfully.

### A. *Designing knowledge base:*

The knowledge about algebra of middle school can be represented by the ECN model in section II.C

### B. *Implementation and Testing:*

The knowledge of algebra in middle schools was built in section A. Based on the architecture of the system in [14], the system called *"The system for solving automatically algebraic problems of middle school"* has been built by using MAPLE and C#. The solution proposed by this system is intuitively natural and close with the way of thinking and solving of human. Moreover, this system was tested on all various kinds of algebraic exercises in middle-school algebra program of Vietnam ([12]).

Some kinds of algebra problem can be solved by system are following:

- Compute the numerical expressions
- Solve equations and basic inequalities
- Solve the system of equation in two variables
- Factorize a polynomial
- Simplify algebraic expressions

Furthermore, our system can be applied to solve parameters rproblems such as:

- Find the parameters which satisfy some conditions of the problem,
- Solve equations and inequations based on parameters,
- etc.

Example 3.1: Factorize the following expression:

$$xy(x+y)+yz(y+z)+xz(x+z)+2xyz$$

Solution is given by our system:

$$\begin{aligned}
&xy(x+y)+yz(y+z)+xz(x+z)+2xyz\\
&= x^2y+xy^2+y^2z+yz^2+x^2z+xz^2+2xyz\\
&= (xy^2+xyz)+(y^2z+yz^2)+(xyz+xz^2)+(x^2y+x^2z)\\
&= yx(y+z)+yz(y+z)+xz(y+z)+x^2(y+z)\\
&= (y+z)(yx+yz+xz+x^2)
\end{aligned}$$

$$\begin{aligned}
&= (y+z)((xz+x^2)+(yx+yz))\\
&= (y+z)(x(z+x)+(x+z)y)\\
&= (y+z)(x+z)(x+y)
\end{aligned}$$

Example 3.2: Consider the following equation:

$$x^2 - 2.m.x + 2m - 1 = 0$$

with m is a parameter, and x is a variable.

Find m such that this equation has at least one root.

Solution is given by our system:

"The quadratic equation:

$$x^2 - 2.m.x + 2m - 1 = 0$$

have coefficients:

$$a = 1 \qquad b = 2b' = 2m \qquad c = 2m - 1$$

We have: $\text{delta}' = (b')^2 - a*c$

$$= (-m)^2 - (2m-1)$$
$$= m^2 - 2m + 1$$

This equation has at least one root if and only if:

$$\begin{aligned}
&\text{delta}' \geq 0\\
&\Leftrightarrow m^2 - 2m + 1 \geq 0\\
&\Leftrightarrow (m-1)^2 \geq 0
\end{aligned}$$

(true for all m)

So this equation has at least one root $\Leftrightarrow m \in \mathbb{R}$

"


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Example 3.3: Consider the following equation:

$$\left(m^2-4\right)x+2=m$$

with m is a parameter, and x is a variable.

Solve this equation based on m.

Solution is given by our system:

"Solve the equation:

$$\left(m^2-4\right)x+2=m$$

$$\left(m^2-4\right)x=-2+m$$

The coefficient of x has a set of roots:

$$\{-2 \quad , \quad 2\}$$

+ if parameter $m=-2$, then:

"This equation has no root"

+ if parameter $m=2$, then:

"This equation has set of roots is real number set "

+ if parameter $m \notin \{-2\ ,\ 2\}$, then:

$$x=\frac{-2+m}{m^2-4}=\frac{1}{m+2}$$

"

## IV. Conclusion and future work

When dealing with practical problems, a human usually makes use of relating problems to solve them. In this paper, the reasoning method using Sample Problems as relating problems is presented and applied to the Computational Network model. This method is especially helpful for searching solution of problems on this model. Hence, the solving process of the system becomes more natural and similar to human's way of thinking.

Using the extensive model of Computational Network, the system for solving automatically algebra problems is constructed successfully. The solution by this system is clear step-by-step, natural, precise and has reasoning way like human's. In addition, this system has received good comments from teachers and pupils used. Moreover, our system can solve parameters problems which are difficult problems and cannot be solved by usual systems.

Moreover, the method using Sample Problems can be applied to other knowledge base models, such as ontology in [6] and [16]. In particular, the ontology COKB-ONT which was presented in [13] and [19] is a useful tool for designing practical knowledge bases, modeling complex problems. In the future, we will also continue our research in extending ontology COKB-ONT for designing a system that can solve the problems of plane geometry and physics (one-way electric, circuit electric, etc.) based on reasoning method using Sample Problems.

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