

Investigation of Three Channels Oversampled FIR Filter Bank

S.R.Chougule, Smt.R.S.Patil

Abstract— This paper proposes design algorithm for real valued, even order oversampled three channel FIR filter banks with proper selection of frequency bands and filter order, which reduces inband aliasing and amplitude distortion. This filter bank is designed using direct form-II transposed structure. Analysis of two dimensional (2D) images are carried out using three channel oversampled filter bank. This system is suitable for any 2-D image types. DCT Image compression technique is used to compare output of filter banks with respect to filter orders by using mean square error and peak signal to noise ratio. This algorithm of oversampled filter bank shows output near perfect replica of input image. Output image matches with input image at PSNR equal to 37 dB and 33 dB respectively.

Index Terms— oversampled, FIR, filter bank, subsampled, Multirate

I. INTRODUCTION

Filter banks (FB) have found many applications in image/audio/video processing, discrete multitone modulation and channel equalization and so on^[1,2]. Most past research work focused on critically sampled FBs which have the equal number of channels with equal subsampling factor. However, oversampled filter banks (OFB) whose number of channels are greater than their subsampling factor, have gained great interests recently, because they can provide more degree of design freedom, residual redundancy left in subband signals and improved noise and erasure resistance^[3,4]. In many applications, especially in image and video processing, linear phase (LP) property of filters is always crucial. Moreover, simple symmetric extension methods can be employed for LP filters to accurately handle the boundaries of finite length signals. For various practical purposes, only causal FIR oversampled linear phase filter banks (OLPFB) with real-valued coefficients are in consideration^[1]. In various speech, image, and communications applications, digital filter banks [FB] have been used extensively. In this paper, we consider the maximally decimated -channel filter banks as shown in Fig. 1. In the analysis stage, the signal is passed through a bank of analysis filters, each of which

preserves a frequency band, and the output signals are then decimated suitably to preserve the system's overall sampling rate (thus justifying its name maximally decimated filter bank). The resulting subband signals can be treated (coded, processed, and/or transmitted) independently. In the synthesis stage, the subband signals are combined by interpolators and a set of synthesis filters to form the reconstructed signal. Filter banks that yield

the output as a time-delayed version of the input, i.e., are called perfect reconstruction filter banks (PRFB's). In some applications, i.e., image processing, it is very crucial for all analysis as well as synthesis filters to have linear phase (either symmetric or antisymmetric). Additionally, linear-phase filters allow us to use simple symmetric extension methods to accurately handle finite-length signals' boundaries^[2].

Review of previous work: Taking a step towards design of oversampled filter bank, initially uniform and dyadic filter banks are designed using real valued casual FIR filters^[5]. Uniform filter bank was developed for four channel and dyadic filter bank developed for two level symmetric filter bank. Here we found that we are not able to develop perfect reconstruction output because of spectral gap between transition band. Our main goal is to develop oversample filter bank which gives the output near input image^[3,4]. Harteneck, Stephan Weiss, and Robert W. Stewart^[3,4] have developed one oversampled filter bank for one dimensional signal. There is open problem to design oversample filter bank without inband aliasing and without amplitude distortion for two dimensional image. Number of researcher have developed filter bank using lattice structure filters. But we have developed filter banks using direct form -II transposed structure real valued filters for even order. We have tried above filter banks with different filter orders for two dimensional images.

Outline of the Paper: Section-II shows oversampled three channel filter bank structure, which is designed for subsampling factor 2. Section III gives design procedure of filter bank. Section IV highlights implementation and result discussion. Section V and VI show conclusion and references.

Prof. Mrs. Sangeeta R. Chougule: Professor and Principal of Bharati Vidyapeeth's College Of Engg. Kolhapur, Maharashtra, India. 416013.
Email: sr.1001chougule@rediffmail.com

Dr. Mrs. R. S. Patil: Principal Of D.Y.Patil College of Engineering Talsande Kolhapur, Maharashtra India 416013

II. OVERSAMPLED FILTER BANK

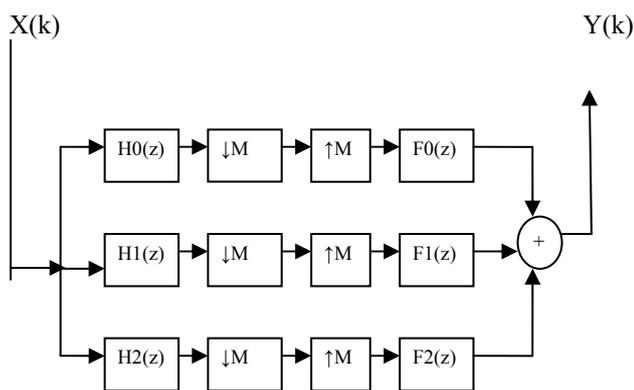


Figure.1: Three channel oversampled FIR Filter bank.

This proposed filter bank uses equal subsampling factor. This filter bank preserves the property of alias free output of two dimensional Lena image. In this design $H_0(z)$ is lowpass filter and $H_2(z)$ is highpass filter which covers all frequency components of input signal except frequency $\pi/2$ i.e. 0.5 normalized frequency which is shown in fig.2. $H_1(z)$ is selected as a band pass filter of passband normalized frequency range of 0.3 to 0.7. $H_0(z)$, $H_1(z)$ and $H_2(z)$ are subsampled by M . All frequencies must be covered by at least one filter. To fill the spectral gap between $H_0(z)$ and $H_2(z)$, one bandpass filter $H_1(z)$ is selected. Frequency spectrum of all analysis and synthesis filters is shown in Fig.3. This filter bank is designed using 1D FIR filter design technique which is then transformed into two dimensional FIR filter using frequency transformation technique. Requirement for filter bank is that it yields the perfect or near-perfect reconstruction property, i.e. $y(k) = x(k-\Delta)$, where Δ is a fixed and delay chosen a priori, and therefore common zeros in all analysis filters $H_i(z)$ are ruled out as information is lost at these frequencies.

Initially filter order 4 of all three filters of FB and subsample factor 2 for all channels have been selected, then keeping subsample factor 2 constant and order of filters are changed as shown in table1 for analysis of image. This filter bank is developed with different filter orders and resultant of each applied for compression. The analysis of resultant of FBs with different filter orders is carried out and with desired filter orders output shows near perfect reconstruction of input. The normalized frequency responses of each filter bank with different filter orders are shown in figure 3. The main aim of this paper is selecting filter orders, design of filters and design of FB for oversampled condition to achieve output near input image.

III FILTER BANK DESIGN

A basic multirate filter bank is shown in Figure 2a. Multirate filter banks are so named because they effectively alter the sampling rate of a digital system, as indicated by the decimators (downsamplers) following the analysis filters, A_0 and A_1 , and the expanders (upsamplers) preceding the synthesis filters, S_0 and S_1 . Properly designed analysis and synthesis filters combined with the properties of decimation and expansion allow filter banks to partition a wideband input signal into multiple frequency bands (often called subbands or channels) and to recombine these subband signals back into the original signal. In the case of Figure 2, the analysis filters, A_0 and A_1 , are typically complementary lowpass and highpass filters that mirror each other about the digital frequency, $\pi/2$, as shown in Figure 2b. Such filters are often called quadrature mirror filters (QMF), since $\pi/2$ corresponds to one fourth the sampling frequency.

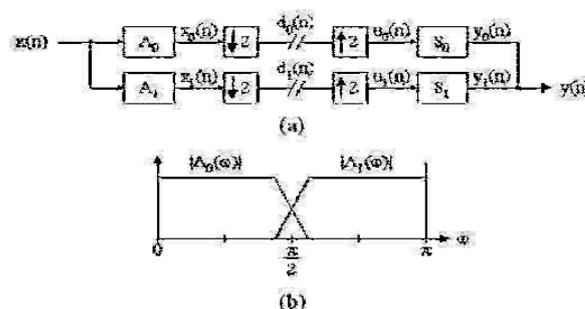


Figure2. [a] Block diagram of a simple two-channel multirate system, and [b] approximate magnitude responses of analysis filters, A_0 and A_1 .

The concept of multirate filtering relies on the two processes that effectively alter the sampling rate, decimation and expansion. Decimation or downsampling by a factor of M essentially means retaining every M^{th} sample of a given sequence. Decimation by a factor of M can be mathematically defined as

$$X(z)_{\downarrow M} = \frac{1}{M} \sum_{m=0}^{M-1} X(W_M^m z^{1/M}) \quad \text{where} \quad W^M = e^{j2\pi/M} \tag{1}$$

Expansion or upsampling by a factor of M essentially means inserting $M-1$ zeros between each sample of a given sequence. Expansion by a factor of M can be mathematically defined as,

$$x(n)_{\uparrow M} = \begin{cases} x(n/M) & \text{for integer } n/M \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

or equivalently,

$$X(z)_{\uparrow M} = X(z^M) \tag{3}$$

The QMF bank is prone to three types of distortion: aliasing distortion, amplitude distortion, and phase distortion. Practical filters have a non-zero transition band. In order for the QMF bank to remain lossless, then, the analysis filters must overlap as shown in Figure 2b. Hence, the analysis filters are not strictly band-limiting and the subsequent decimation causes aliasing. Fortunately, aliasing can be completely cancelled by properly designing the synthesis filters, S_0 and S_1 [6].

$$Y(z) = \frac{1}{2} [A_0(z)S_0(z) + A_1(z)S_1(z)]X(z) + \frac{1}{2} [A_0(-z)S_0(z) + A_1(-z)S_1(z)]X(-z) \tag{4}$$

The aliasing component in $y(n)$ becomes zero when

$$A_0(-z)S_0(z) + A_1(-z)S_1(z) = 0 \tag{5}$$

or, more specifically, when

$$S_0(z) = A_1(-z) \text{ and } S_1(z) = -A_0(-z) \tag{6}$$

Thus, aliasing in one branch is completely cancelled by the synthesis filter in the opposite branch. The entire alias-free filter bank can now be expressed as the single transfer function,

$$H(z) = \frac{1}{2} [A_0(z)A_1(-z) + A_1(z)A_0(-z)] \tag{7}$$

Then, letting $z = e^{j\omega}$

$$Y(e^{j\omega}) = |H(e^{j\omega})| e^{j\phi(\omega)} X(e^{j\omega}) \tag{8}$$

If $|H(e^{j\omega})|$ isn't constant and non-zero for all ω (allpass), the filter bank suffers from amplitude distortion. Likewise, if $H(e^{j\omega})$ does not have linear phase (if $\phi(\omega)$ is not of the form $a \cdot \omega + b$) the filter suffers from phase distortion. It is important, at this point, to emphasize that these two conditions apply to the filter bank as a whole (Equation 8), not the individual analysis and synthesis filters. When the filter bank is free from aliasing, amplitude distortion, and phase distortion, it is called a perfect reconstruction (PR) filter bank.

Polyphase representation is, in simple terms, a method of re-organizing the coefficients of a given transfer function, $h(n)$. The transfer function can be represented in terms of its even and odd coefficients.

$$H(z) = \sum_{n=-\infty}^{\infty} h(2n)z^{-2n} + z^{-1} \sum_{n=-\infty}^{\infty} h(2n+1)z^{-2n} \tag{9}$$

Therefore,

$$H(z) = E_0(z^2) + z^{-1} E_1(z^2) \tag{10}$$

Where

$$E_0(z) = \sum_{n=-\infty}^{\infty} h(2n)z^{-n} \text{ and } E_1(z) = \sum_{n=-\infty}^{\infty} h(2n+1)z^{-n} \tag{11}$$

This idea can be extended to further decomposition by a factor of M [6]. Then $H(z)$ takes on the "Type 1 polyphase" form

$$H(z) = \sum_{p=0}^{M-1} z^{-p} E_p(z^{2M}) \tag{12} \text{ Where}$$

$$E_p(z) = \sum_{n=-\infty}^{\infty} e_p(n)z^{-n} \tag{13} \text{ With}$$

$$e_p(n) = h(Mn+p), \quad 0 \leq p \leq M-1 \tag{14}$$

The "Type 2 polyphase" form is a permuted version of the Type 1 form and is given by

$$H(z) = \sum_{p=0}^{M-1} z^{-(M-1-p)} R_p(z^{2M}) \tag{15} \text{ Where}$$

$$R_p(z) = E_{M-1-p}(z) \tag{16}$$

By combining the polyphase representation and the noble identities, it is possible to implement more efficient analysis filter/decimator and expander/synthesis filter blocks Harteneck, Stephan Weiss, and Robert W. Stewart^[3,4] says that to design oversampled filter bank, add one more channel in fig.2 as bandpass filter to pass frequency components near to $\pi^*1/2$. For practical implementation selection of third channel frequency band and pass band ranges of first two channels are most important. Normalized frequency response of three channel filter bank with different filter orders are shown in fig.3. Implementation of this filters of filter banks using type-I direct form II transposed structure. These filters are transformed into two dimensional forms for analysis of 2D image. In fig.1 three channel FIR filter bank is shown, which shows that decimation and interpolation factor is less than no. of channels as per the oversampled condition. In the oversampled case more design freedom and improved numerical properties as compared with critically sampled FB's, and they have noise reducing properties^[1]. The design freedom increased since for a given oversampled analysis FB, there exists a whole class of synthesis FB's providing near perfect reconstruction [NPR]

After design of filters of filter bank in one dimension it is transformed into two dimension using frequency transformation technique as below.

$$H(w_1, w_2) = B(W) \mid \cos w = T(w_1, w_2)$$

$B(W)$ is the Fourier transform of the one dimensional filter.

$$B(W) = \sum_{n=-N}^N b(n) e^{-jwn} \quad (17)$$

$$T(w_1, w_2) = \sum_{n_1} \sum_{n_2} t(n_1, n_2) e^{-jw_1 n_1} e^{-jw_2 n_2} \quad (18)$$

The returned filter h is the inverse Fourier transform of $H(w_1, w_2)$. Filters of analysis and synthesis filter banks design by using above design method.

IV EXPERIMENTAL RESULTS AND DISCUSSION

Initially design of proposed oversampled FIR filter bank using type-I direct form II transposed structure FIR filters are developed in one dimensional (1D) form. Initially input signal is divided into 2 channels but at transition gap some frequency components are lost means not passed for further processing. Therefore to overcome this problem extra bandpass filter is selected at the spectral gaps and all the frequency components are passed at output stage. Design of downsampler and upsampler of analysis and synthesis filter bank is carried out by using bilinear interpolation technique. In downsampler block of each channel size of image with rows and columns are reduced by downsample factor. Similarly at the upsampler block, image size is restored by selecting same upsampler factor. Initially subsample factor 2 which is at oversample rate and filter order 4 for all channels are selected and output of filter bank applied for compression. Similarly with different filter orders investigation of oversampled filter bank near perfect reconstruction of input image is carried out. One can apply this filter bank for different types of images. Here for filter bank investigation of oversample factor with proper filter orders are carried out, which shows output resultant near perfect input. Resultant of Lena (gif image format) image for different filter orders and subsample factor 2 with DCT compression is as shown in figure.4.

Resultant of Barba (tif image format) image for different filter orders and subsample factor 2 with DCT compression is as shown in figure.5. Out of the image compression techniques available, transform coding is the preferred method. Since energy distribution varies with each image, compression in the spatial domain is not an easy task. Images do however tend to compact their energy in the frequency domain making compression in the frequency domain much more effective. Transform coding is simply the compression of the images in the frequency domain. Transform coefficients are used to maximize compression. For lossless compression, the coefficients must not allow for the loss of any information. The DCT is fast. It can be quickly calculated and is best for images with smooth edges like photos with human subjects. The DCT coefficients are all real numbers unlike the Fourier Transform. The Inverse Discrete Cosine Transform (IDCT) can be used to retrieve the image from its transform representation.

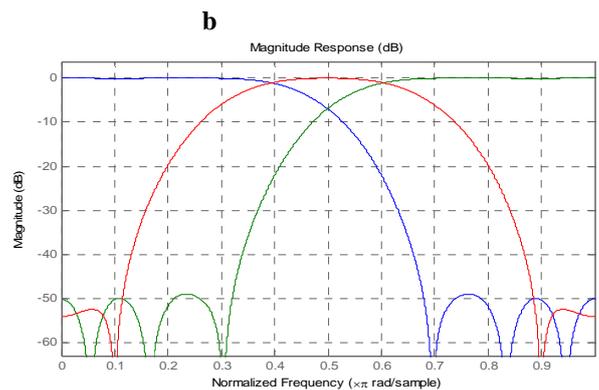
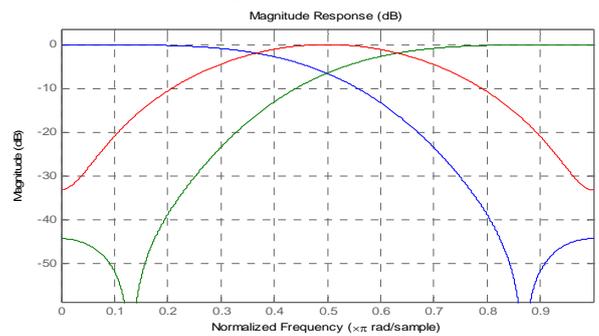
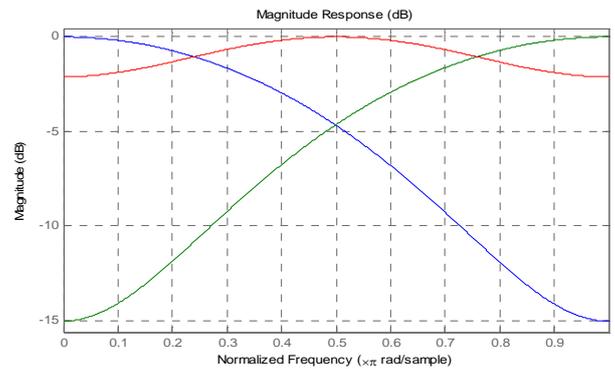


Figure 3: a) Normalised frequency response of 3ch FB for Filter order 4 b) Normalised frequency response of 3ch FB for Filter order 10 c) Normalised frequency response of 3ch FB for Filter order 18.

Here results of compressed image of each oversampled filter bank compared with input image. It is most easily defined via the mean squared error (MSE) which for two $m \times n$ monochrome images I and K where one of the images is considered a noisy approximation of the other is defined as:

$$MSE = \frac{1}{mn} \sum_{i=0}^{m-1} \sum_{j=0}^{n-1} ||I(i, j) - K(i, j)||^2$$

Comparison of all the results of compressed output of filter banks with input image in terms of MSE is as shown in table 1. After image compression all the results are applied for peak signal to noise ratio [PSNR] estimation. The PSNR is most commonly used as a measure of quality of reconstruction in image compression. It is most easily defined via the mean squared error (MSE)

The PSNR is defined as:

$$PSNR = 10 \cdot \log_{10} \left(\frac{MAX_I^2}{MSE} \right) = 20 \cdot \log_{10} \left(\frac{MAX_I}{\sqrt{MSE}} \right)$$

Here, MAX_I is the maximum pixel value of the image. Comparison of all the results of filter banks in terms of PSNR, MSE for different filter orders with oversample factor 2 is as shown in table1. PSNR and MSE calculated with respect to input compressed image. Histogram of input image and Oversampled filter bank output with filter order 10 is as shown in fig. 4i, histogram which gives information in terms of intensity values of input and output images.

Table 1: MSE for different filter orders with oversample factor 2

FILTER ORDER	MSE Lena (Gif image)	PSNR[dB] Lena	MSE Barba (tif image)	PSNR [dB] Barba
6	1.16	47.85	9.03	38.57
10	11.07	37.68	31.87	33.09
20	11.01	37.71	31.64	33.13
28	11.82	37.40	32.27	33.04
42	12.69	37.10	32.84	32.96
80	13.59	36.79	33.44	32.88
120	14.06	36.65	33.88	32.83



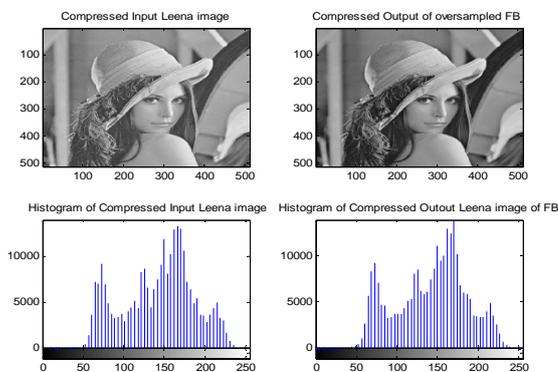
c



d



g



i

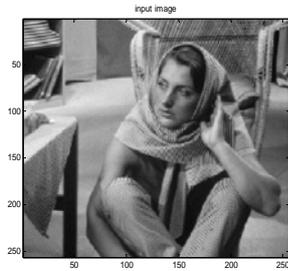


a] Input Lena Image



b

Figure 4: a) Compressed input image b) Compressed image of FB with filter order=2 c) Compressed image of FB with filter order=4 d) Compressed image of FB with filter order=6 g) Compressed image of FB with filter order=28 i) Histograms of Compressed input image and output of FB with filter order=10



a) Input Barba image



b



c



d

Figure5 : a) Input Barba Image b) Compressed oversampled FB output with filter order 4 c) Compressed oversampled FB output with filter order 10 d) Compressed oversampled FB output with filter order 20

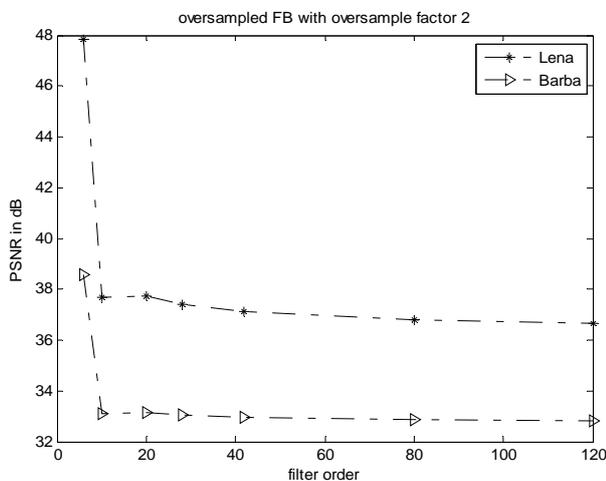


Figure 6: From table 1 graph for oversampled filter bank with oversample factor 2 for images Barba and Leena.

V. CONCLUSION

This filter bank is designed with real valued direct form II structure, FIR filters with even filter orders. The investigation of three channel oversampled filter bank is carried out with oversample factor 2 with different filter orders for 2D images. This filter bank gives result near perfect reconstruction of input image with filter orders from 10 onwards to filter order 120. Below filter order 10 we found distortion in FB output image because of more side lobes in transition gap of filters and some frequency components are lost at transition gap. For more filter order i.e. above 120, narrow frequency band and loss of frequency occurs and histogram becomes narrow means output histogram not matches with input image histogram. The analysis is carried out with the help of image compression. PSNR, MSE and histogram of images is calculated for checking perfect reconstruction.

REFERENCES

- [1] Zhiming Xu and Anamitra Makur "Theory and lattice structure for oversampled linear phase paraunitary filter banks with arbitrary filter length": zhmXU@pmail.ntu.edu.sg, eamakur@ntu.edu.sg
- [2] Trac Duy Tran and Truong Q. Nguyen, "M-channel linear phase FIR filter banks and application in image compression" IEEE Tran. On signal processing vol.45,no.9,Sept1997.
- [3] Moritz Harteneck, Robert W. Stewart and J.M. Paez-Borrillo, "A filterbank design for oversampled filter banks without aliasing in the subbands", Proc. TFFS'97, Warwick, UK, 1997.
- [4] Moritz Harteneck, Stephan Weiss, and Robert W. Stewart, "An oversampled filter bank with different analysis and synthesis filters for the use with adaptive filters" Proc. Asilomar Conf. Signals, Systems and Computers, vol. 2, pp. 1274-1278, Nov.1997.
- [5] S.R.Chougule and R.S. Patil "Uniform and Dyadic FIR filter bank applied for 2D image"NCSPA National conference Sept07 at DYPCOE Pimpri Pune.
- [6] P.P.Vaidyanathan "Multirate systems and filter banks".Pearson Education 1993.
- [7] Moritz Harteneck, Stephan Weiss, and Robert W. Stewart "Design of near perfect reconstruction oversampled filter banks for subband adaptive filters"