Influence of Parameter Variations on System Identification of Full Car Model

Fengchun Sun, Yan Cui

Abstract—The full car model is used extensively in the system identification of a vehicle suspension system to capture the realistic dynamics of a suspension system. Influence of parameter variations on the full car model was analyzed on frequency domain. The magnitude and phase of the transfer function between the output (sprung mass vertical acceleration, pitch rate, raw rate and front tire vertical acceleration) and the front road input changed as parameters changed. System identification of the full car model using Matlab was done.

Index Terms—System Identification, Full Car Model, Time Domain, Frequency Domain

I. INTRODUCTION

A key aspect of vehicle research field is improving car ride quality and handling performance. Performance improvement can be achieved through analysis and modification of vehicle dynamics via the design suspension system controller [1,2,3]. High fidelity mathematical models that accurately capture the dynamics of the car suspension system are critical in this design task. An excellent means to determine such models is the use of system identification techniques.[3]

System identification methods, that address the problem of building mathematical models of dynamic systems based on observed data from the system, are often employed in automotive applications to determine unknown vehicle parameters that cannot be measured directly. A number of system identification methods are available in literature for automatic determination of system parameters. Several of these methods have been applied to vehicle parameter estimation, including: least squares, extended kalman filter, maximum likelihood and recursive prediction error.

The system identification procedure has a natural logical flow: first, collect data, then choose a model set and pick the optimal model in the set, as shown in Figure 1. All of the system methods rely on a model of the vehicle in order to estimate a specific set of vehicle parameters. Often the vehicle parameters can be estimated using vehicle models from different directions of the vehicle dynamics, including: vertical dynamics, longitudinal dynamics and lateral dynamics.



Figure 1: System Identification Flow Chart

In this paper, system identification of parameters from the vertical direction is investigated. The road excitations are used as inputs and the vehicle responses are used as outputs, as show in Figure 2.



Figure 2: System Identification of Vehicle Parameters from Vertical Dynamics

Car models can be either mathematical models, neural network models or multi-body physical models. Several models have been developed with research related to the system identification of vehicle parameters from vertical dynamics. These include: quarter-car, half car and full car models.

To understand which parameters are easily and cost effectively identified, the influence of parameter variations on system identification process should be considered. In [4, 5], the quarter car model and half car model were analyzed with the help of the parameter variations. It shows that the variation of the spring damper characteristic has a more significant influence on the quarter-car model and half car model at resonant frequencies than the sprung mass and spring stiffness. The seven degrees-of-freedom full car model can capture more dynamic characteristics of vehicle. So it is

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Fengchun. Sun. is with National Engineering Lab of Electric Vehicle, Beijing Institute of Technology, Beijing 100081 China. (e-mail: <u>sunfch@bit.edu.cn</u>)

Yan. Cui. is with the National Engineering Lab of Electric Vehicle, Beijing Institute of Technology, Beijing 100081 China. (e-mail: <u>cuiyan0127@hotmail.com</u>, phone: 86-138-1188-1907)

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used more extensively in the system identification of vehicle suspension system [5].

An eigenvalue analysis can be used in many engineering systems where part of the characteristics of the system can be determined [6]. The analysis of full car model eigenvalues is present in this paper.

The vehicle is a dynamic system that exhibits vibration in response to excitation inputs. The frequency response transfer function between vibration (e.g., car body acceleration, and excitation inputs) and excitation inputs is analyzed.

The system identification of the full car model using random road input is shown at the end the paper.

II. FULL CAR MODEL

The full car model has 7 degrees of freedom, which are the vertical displacement of the vehicle body, pitch angle, roll angle, vertical displacement of all four wheels. A schematic of this model is shown in Figure 1. The suspension between each axle and the body is modeled using a linear viscous damper and spring element. The tire is modeled using a simple linear spring without damping.



Figure 3. Full Car Model

A. State Space for the Full Car Model

The full car model can be presented using the state space model. The state space for full car model is shown in equation (1) and (2).

$$\boldsymbol{x}_{full} = \boldsymbol{A}_{full} \boldsymbol{x}_{full} + \boldsymbol{B}_{full} \boldsymbol{u}_{full}$$
(1)

$$\mathbf{y}_{full} = \mathbf{C}_{full} \mathbf{x}_{full} + \mathbf{D}_{full} \mathbf{u}_{full}$$
(2)

The state vector is shown in equation (3)

$$\boldsymbol{x}_{full} = \begin{pmatrix} z & \dot{z} & \theta & \dot{\theta} & \varphi & \dot{\varphi} & z_{ufl} & \dot{z}_{ufl} & z_{ufr} & \dot{z}_{url} & \dot{z}_{url} & \dot{z}_{urr} & \dot{z}_{urr} \end{pmatrix}^{T} (3)$$

The input and output are shown in equation (4) and equation (5).

$$\boldsymbol{u}_{full} = \begin{pmatrix} z_{rfl} & z_{rfr} & z_{rrl} & z_{rrr} \end{pmatrix}^T$$
(4)

$$\mathbf{y}_{full} = \begin{pmatrix} z & \ddot{z} & \theta & \dot{\theta} & \varphi & \dot{\varphi} & \ddot{z}_{tfl} & \ddot{z}_{tfr} & \ddot{z}_{trl} & \ddot{z}_{trr} \end{pmatrix}^{T}$$
(5)

ISBN: 978-988-19251-2-1 ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online) The representative values for the parameters of the system used in the simulation are shown in TABLE I. The eigenvalues of the full car model are analyzed and the influence of parameter variations, such as changes in sprung mass, spring damping and tire stiffness, are investigated through the full car model transfer function on frequency domain.

TABLE I
PRESENTATIVE VALUES FOR PARAMETERS OF FULL CAR MODEL

Parameters	Description	Value	Unit
m _s	sprung mass	1200	kg
$\mathbf{I}_{\mathbf{ heta}}$	pitch inertial of sprung mass	2100	m4
$\mathbf{I}_{\mathbf{\phi}}$	roll inertial of sprung mass	1800	m4
$c_{fl} \ c_{fr} \ c_{rl} \ c_{rr}$	spring damping	6000	Ns/m
$k_{fl} \ k_{fr} \ k_{rl} \ k_{rr}$	spring stiffness	50000	N/m
$m_{tfl} \ m_{tfr} \ m_{trl} \ m_{trr}$	tire mass	125	kg
$k_{tfl} \ k_{tfr} \ k_{trl} \ k_{trr}$	tire stiffness	400000	N/m

III. ANALYSIS OF SYSTEM EIGENVALUES

The stability of a large system is determined from the eigenvalues of the system. The system eigenvalues are functions of all control and design parameters. [6] A change in any of these parameters affects the system performance and is reflected in a shift in the system eigenstructure (eigenvalues and eigenvectors). In this paper only the eigenvalues are considered.

TABLE II shows the eigenvalues and corresponding natural frequencies, damping ratios of the full car model with representative values, which is shown in TABLE I. Modes 1 through 3 are the car body modes. Modes 4 through 7 are the wheel hop modes.

TABLE II EIGENVALUES, NATURAL FREQUENCY AND DAMPING RATIO FOR FULL CAR MODEL

CARMODEE					
Mode	Eigenvalue	Nature Frequency	Damping Ratio		
1	$-3.6823 \pm 7.5300i$	1.3341	0.4393		
2	$-10.0875 \pm 9.8077 i$	2.2392	0.7170		
3	$-19.0356 \pm 3.8728 i$	3.0917	0.9799		
4	$-19.1302 \pm 40.4495 i$	7.1214	0.4275		
5	$-23.6039 \pm 45.3213i$	8.1327	0.4619		
6	$-24.5844 \pm 51.3259i$	9.0575	0.4320		
7	$-24.0000 \pm 54.9909i$	9.5493	0.4000		

IV. INFLUENCE OF PARAMETER VARIATIONS ON TRANSFER FUNCTIONS

The transfer function is commonly used in the analysis of a single-input single-output system in the frequency domain. In this case, front left road profile (z_{rfl}) was used as the single system transfer function input. For the dynamic analysis of this model, the transfer function between state variables x(s) and the front left road profile $z_{rf}(s)$ can be defined as equation (6).

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$$\frac{\boldsymbol{Y}(s)}{\boldsymbol{z}_{rfl}(s)} = \boldsymbol{C}_{full} \left(\boldsymbol{s} \boldsymbol{I}_{full} - \boldsymbol{A}_{full} \right)^{-1} \boldsymbol{B}_{full_1} + \boldsymbol{D}_{full_1}$$
(6)

In which \boldsymbol{B}_{full_1} is the input matrix related to the front left road input.

The resulting transfer function between the sprung mass vertical acceleration and the front left road input is:

$$\frac{\ddot{z}(s)}{z_{rfl}(s)} = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \frac{\boldsymbol{Y}(s)}{z_{rfl}(s)}$$
(7)

The transfer function between the sprung mass pitch rate and the front left road input is:

$$\frac{\dot{\theta}(s)}{z_{rf}(s)} = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \frac{\boldsymbol{Y}(s)}{z_{rf}(s)}$$
(8)

The transfer function between the sprung mass roll rate and the front left road input is:

$$\frac{\dot{\varphi}(s)}{z_{rfl}(s)} = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix} \frac{Y(s)}{z_{rfl}(s)}$$
(9)

The transfer function between the front left tire vertical acceleration and the front left road input is:

$$\frac{\ddot{z}_{tfl}(s)}{z_{rfl}(s)} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix} \frac{\boldsymbol{Y}(s)}{z_{rfl}(s)}$$
(10)

Simulation shows how parameter variations influence the magnitude and phase of the transfer function between four outputs and front left road input. In this case the parameters are varied from 50% to 150% of values showed in TALBE I. As shown in Figure 4, Figure 5 and Figure 6, the change in the front left spring damping has a significant influence on magnitude and phase of the transfer functions between 3 outputs (sprung mass vertical acceleration, sprung mass pitch rate, sprung mass roll rate) and the front left road input. Figure 7 shows that the front left spring damping variation does not have a significant influence on magnitude of the transfer function between the front left tire vertical acceleration and the front left road input. This indicates that the change in the front left spring damping has a dramatic influence on each mode of full model. As shown in Figure 8 through Figure 11, the front left tire stiffness has a large influence on magnitude above 10 Hz on each mode of the full car model. The sprung mass pitch inertia has a significant influence on pitch rate, but almost no influence on the sprung mass vertical acceleration and the front tire vertical acceleration. Sprung mass roll inertia has a significant influence on roll rate, but almost no influence on other vehicle responses. Changes in the front left spring stiffness slightly influence the sprung mass vertical acceleration and the pitch rate near 0.7-10 Hz.



Figure 4. Influence of front left spring damping variation on transfer function between sprung mass vertical acceleration and front left road input



Figure 5. Influence of front left spring damping variation on transfer function between sprung mass pitch rate and front left road input



Figure 6. Influence of front left spring damping variation on transfer function between sprung mass roll rate and front left road input

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Figure 7. Influence of front left spring damping variation on transfer function between front tire vertical acceleration and front left road input



Figure 8. Influence of front tire stiffness on transfer function between sprung mass Vertical Acceleration and Front left road input



Figure 9. Influence of front tire stiffness on transfer function between sprung mass Pitch Rate and Front Left Road Input



Figure 10. Influence of front tire stiffness on transfer function between Front Tire Vertical Acceleration and Front left Road Input



Figure 11. Influence of front tire stiffness on transfer function between sprung mass Roll Rate and Front Left Road Input

V. SYSTEM IDENTIFICATION OF FULL CAR MODEL

For this paper, estimation is executed on simulated data to demonstrate the capabilities of the system identification approach. Thus, input data are generated and passed through the model yielding output data, or a model response. These input and output data are used in the system identification routines to reconstruct the simulated model. To estimate model parameters, an input / output correlation is executed. The road profile is considered as the input of the model and should be realistically described as a random function. For this work, sinusoidal wave and square wave inputs are also used. Such inputs can be generated on vehicle shaker systems. As with any system identification, initial values, or guesses, of the parameters to be identified are used to initiate the estimation algorithm. In these cases, the initial guesses of parameters are 110% of the representative values. To simulate a more realistic scenario, noise is added to the output at the 5% level.

Six trials were made for system identification for the full car model with random road input. In Fig. 7, the percent errors of parameters in system identification were shown. The front spring damping has relatively large error. Front spring stiffness, front tire stiffness and mass have relatively small errors.



Figure 12. Percent error of parameters estimation for the full car model (random road input)

VI. CONCLUSIONS

For the root locus analysis and the transfer function analysis, the variation of spring damping and tire stiffness have a big influence on the magnitude of transfer functions between 3 outputs (sprung mass vertical acceleration, sprung mass pitch rate, front tire vertical acceleration) and front road input more than other parameters. This means that sprung damping and tire stiffness have a notable influence on these outputs. During system identification spring damping, tire stiffness and tire mass have relatively small errors than other parameters. Conversely, spring stiffness and sprung mass inertia have a large percent error. This verifies that the spring damping and the tire stiffness is more sensitive than other parameters in full car model. The reason of that is the nonlinear characteristics of the shock absorber and tire. The nonlinear shock absorber model and the nonlinear tire model will be investigated in the future work.

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