Enhancing the Performance of Fixed-Structure Robust Loop Shaping Control using Genetic Algorithm Approach

Piyapong Olranthichachat and Somyot Kaitwanidvilai

***Abstract*—This paper is proposed an algorithm, *Genetic Algorithm (GA) based fixed-structure H_∞ loop shaping control,* to the problem-solving of conventional H_∞ loop shaping, such as high-order and difficult to be usable in general. In this approach, a structure of controller is specified combine H_∞ loop shaping method is solved by GA algorithm. Additionally, in the proposed technique, the desired performance weighting function, which is defined by using GA. The performance and robustness of the proposed controller are investigated in a pneumatic servo system in comparison with that of the controller designed by conventional H_∞ loop shaping. Results of simulation demonstrate the advantages of simple structure and robustness against plant perturbations and disturbances of the proposed controller. Experiments are performed to verify the effectiveness of the proposed technique.**

***Index Terms*—H_∞ loop shaping, Genetic Algorithm, pneumatic servo system**

I. INTRODUCTION

Over the past several years, high performance control in pneumatic servo system has been investigates area of research. Controlling of these servo systems is controlled in such a way that the controlled piston can move to any target position therefore in previous research works, high performance control techniques [1-3] have been successfully applied to design a controller for pneumatic servo system, such as H_∞ control with Mirror Feedback by Kimura T., Hara S. and Takamori T. in 1996 [1], fuzzy state feedback control by H. Schulte and H. Hahn in 2004 [2], block-oriented approximate feedback linearization by Fulin Xiang and Jan Wikander in 2003 [3], and etc. Moreover in high performance controller, the robust controller has become an interesting for pneumatic servo system because it can be guaranteed in perturbed condition. These controllers mentioned above have high order and complicated to design controllers then produce an effect in generally industrial applications.

To solve a problem, several approaches proposed a fixed-structure cooperate with robust control method [4-6]. To produce an effect, these controllers have simple structure and acceptable controller order. In [4], a fixed-structure with robust H_∞ optimal control problem was solved by using genetic algorithm (GA). The intelligent genetic algorithm to solve the mixed H_2/ H_∞ optimal control problem was proposed in [5]. Moreover the fixed-structure robust control mentioned above, fixed-structure with H_∞ loop shaping control [6] was proposed which is easy to design weighting function in this method more than in [4-5]. H_∞ loop shaping control [7] technique requires only two specified weights, pre-compensator and post-compensator weights, for shaping the nominal plant so that the desired open loop shape is achieved. The fixed structure robust based on searching algorithms such as genetic algorithm, particle swarm optimization technique, tabu-search, etc., can be employed in area of research.

By the advantage mentioned above, the fixed-structure with H_∞ loop shaping control is simple and effective than other robust control techniques because the selection of weighting function is based on the concept of classical loop shaping. However, the weighting function of H_∞ loop shaping in [6-7] is chosen by trail and error method. To overcome this problem, GA based fixed-structure H_∞ loop shaping is proposed to synthesize optimal fixed-structure H_∞ loop shaping controller and weighting function at the same time. In this paper, GA is employed to find the parameters of the weighting functions which can to defined performance of the proposed controller. In addition, the proposed controller is applied in fuzzy model which can be guaranteed robustness in the long-stroke actuator. Simulation results show that the controller designed by the proposed approach has a good performance and robustness as well as simple structure.

The remainder of this paper is organized as follows. Section II covers the pneumatic servo system modeling. In section III, conventional H_∞ loop shaping and the proposed technique are discussed as well as GA algorithm. The design examples and results are demonstrated in section IV. And in section V the paper is summarized.

II. PNEUMATIC SERVO SYSTEM

A Dynamic model of a pneumatic system can be presented with the following state space [8].

$$\begin{aligned} \dot{X} &= AX + Bu \\ y &= CX + Du \end{aligned} \qquad (1)$$

where

This work was supported in part by Faculty of Engineering, King Mongkut's Institute of Technology Ladkrabang, Bangkok, Thailand.

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ISBN: 978-988-19251-2-1
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

$$A=\begin{bmatrix} 0 & 0 & 0 & -\frac{\gamma AP_{p0}}{V_{p0}} \\ 0 & 0 & 0 & \frac{\gamma AP_{n0}}{V_{n0}} \\ 0 & 0 & 0 & 1 \\ -\frac{A}{M} & -\frac{A}{M} & 0 & -\frac{c}{M} \end{bmatrix}, \quad B=\begin{bmatrix} K\frac{\gamma RTA}{V_{p0}} \\ -K\frac{\gamma RTA}{V_{n0}} \\ 0 \\ 0 \end{bmatrix}$$

$$C=\begin{bmatrix}0 & 0 & 1 & 0\end{bmatrix}, \qquad D=0$$

where $y(s)$ is output (position) , $u(s)$ is input (value voltage) , C is the viscous friction coefficient , M is piston mass , γ is the ratio of specific heat (= 1.4) , R is ideal gas constant , T_S is temperature , G_i is the coefficient of the linearized air mass flow rate , A is areas of piston , V_{po} and V_{no} is the volume of chambers p and n at the operation point , P is pressure in chamber Fig.1. shows the experimental setup of the pneumatic system.

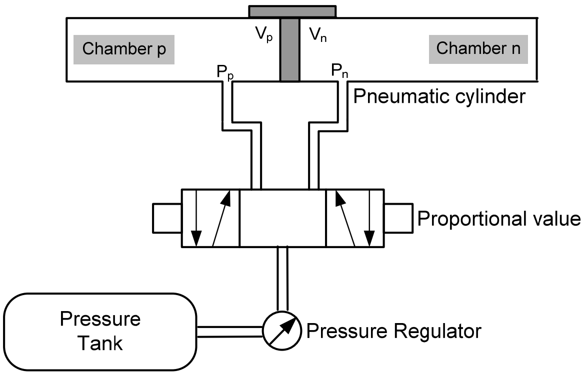


Fig. 1. Experimental setup of the pneumatic servo system.

III. H_∞ Loop Shaping Control and Proposed Technique

A. Standard H_∞ Loop Shaping

H_∞ loop shaping control is an important method to design a robust controller. The basic principle of H_∞ loop shaping design require two weighting functions, pre-compensate and post-compensate the nominal plant (G) to shape the open loop frequency domain. The shaped plant ($G_S = W_1GW_2$) by using a pre-compensator (W_1) and post-compensator (W_2) as shown in Fig.2.

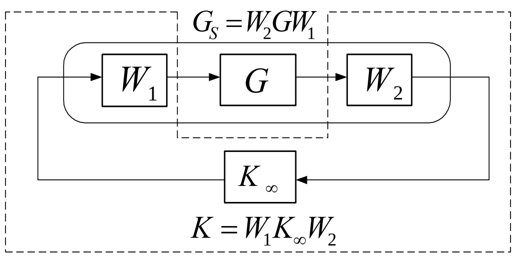


Fig. 2. H_∞ loop shaping design.

In the method, the robust stabilization of the system is formulated as normalized coprime factors. The normalized left coprime factorization of the shaped plant is written as

$$G_\Delta = (M_s + \Delta_{Ms})^{-1}(N_s + \Delta_{Ns}) \tag{2}$$

Where M_S and N_S are normalized denominator and nominator, respectively.

Δ_{Ms} and Δ_{Ns} are uncertainty transfer functions in denominator and nominator factor, respectively.

The normalized left co-prime factor can by stabilized by $\|\Delta_{Ns}, \Delta_{Ms}\|_\infty \le \varepsilon$, ε is the uncertainty boundary called stability margin. To determine optimal stability margin (ε_{opt}), there is a unique method as follow [9].

$$\varepsilon_{opt} = (1+\lambda_{\max}(XZ))^{-1/2} \tag{3}$$

where X and Z are the solutions of two Riccati in (4) and (5) respectively, λ_{max} is the maximum eigenvalue.

$$(A-BS^{-1}D^TC)Z + Z(A-BS^{-1}D^TC)^T - ZC^TR^{-1}CZ + BS^{-1}B^T = 0 \quad \ldots(4)$$

$$(A-BS^{-1}D^TC)^TX + X(A-BS^{-1}D^TC) - XBS^{-1}B^TX + C^TR^{-1}C = 0 \quad \ldots(5)$$

where $S = I + D^TD$, $R = I+DD^T$. In this approach, stability margin (ε) is chosen less than the optimal stability margin (ε_{opt}) to synthesize H_∞ loop shaping controller (K_∞) that satisfies.

$$\|T_{zw}\|_\infty = \left\| \begin{bmatrix} I \\ K_\infty \end{bmatrix} (I - G_sK_\infty)^{-1}[I \quad G_s] \right\|_\infty \le \varepsilon^{-1} \tag{6}$$

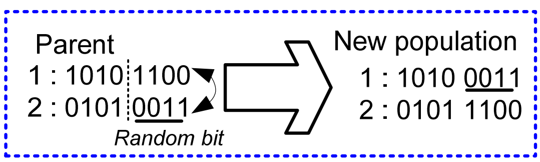
where $\|T_{zw}\|_\infty$ is the infinity norm from the disturbances w to state z. The details of this solving the optimal controller problem in (6) are available in [9]. The usable feedback controller (K) for the nominal plant is determined as follow

$$K = W_1K_\infty W_2 \tag{7}$$

Fig. 2 shows the controller in H_∞ loop shaping control.

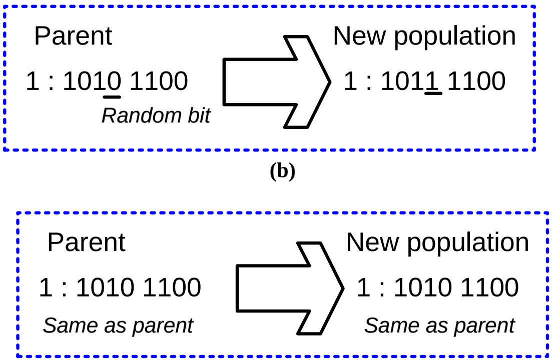
B. Proposed technique

In the proposed technique, genetic algorithm (GA) is applied to design optimal fix structure H_∞ loop shaping controller with weighting function. This algorithm is more adaptable to any complex optimization problem. Moreover, it is requires only upper, lower bounds of solution and GA parameters which is easy to implement. GA is an iterative algorithm which applies the concept of chromosomes. In each iteration, called generation, the new populations are obtained by three genetic operators (crossover, mutation, and reproduction) [10] which are shown in Fig. 3. Genetic operators are evaluated by decoding floating number to binary number, and the fitness values of chromosomes are calculated. Maximum fitness value is selected and kept in the current generation.



ISBN: 978-988-19251-2-1
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

(a)



(b)

(c)

Fig. 3. Genetic operations. (a) Crossover , (b) Mutation and (c) Reproduction.

The proposed algorithm is explained as follows. Assume that the predefined structure controller $K(p)$ has satisfied parameters p and the performance weighting function $W_1(x)$ has satisfied the parameter x. Based on the concept of H_∞ loop shaping, Genetic algorithm is to find the parameters p in the controller $K(p)$ and the parameters x in the weighting function $W_1(x)$ that minimize infinity norm from disturbances w to states z, $\|T_{zw}\|_\infty$ which is subjected to be minimized can be written as.

$$J_{cost} = \gamma = \|T_{zw}\|_\infty = \left\| \begin{bmatrix} I \\ K_\infty \end{bmatrix} (I - G_S K_\infty)^{-1} [I \quad G_s] \right\|_\infty \tag{8}$$

Where $K_\infty = W_1^{-1}(x) K(p) W_2^{-1}$, from final feedback controller in (7) then Assuming that W_1 and W_2 are invertible. In many cases, the weight W_2 is selected as identity matrix I because high performance sensor is used in output feedback can neglect sensor noise effect. The optimization in this problem can be written as

$$\text{Maximize} \quad \left\| \begin{bmatrix} I \\ W_1^{-1}(x) K(p) \end{bmatrix} (I - W_2 G_0 K(p))^{-1} [I \quad G_s] \right\|_\infty^{-1}$$

Subject to :

$$\begin{array}{ll} p_{1,\min} < p_1 < p_{1,\max} & x_{1,\min} < x_1 < x_{1,\max} \\ p_{2,\min} < p_1 < p_{2,\max} \;, & x_{2,\min} < x_2 < x_{2,\max} \;, \\ \vdots & \vdots \\ p_{i,\min} < p_i < p_{i,\max} & x_{i,\min} < x_i < x_{i,\max} \end{array}$$

Constrains in time domain :

$$\begin{aligned} \text{Maximum Overshoot} &< OV \\ \text{Settling Time} &< ST \\ \text{Steady State Error} &< SE \end{aligned}$$

Constrains in frequency domain :

$$\begin{aligned} \text{Bandwidth} &> BW \\ \text{Gain}(\omega_{low\ freq}) &> LG \\ \text{Gain}(\omega_{high\ freq}) &< HG \end{aligned}$$

Where $p_{i,min}$, $x_{i,min}$ is the lower bound values of the parameter p_i and x_i in the parameter vector p and x, respectively.

$p_{i,max}$, $x_{i,max}$ is the upper bound values of the parameter p_i and x_i in the parameter vector p and x, respectively.

OV is the acceptable maximum overshoot.
ST is the acceptable settling time.
SE is the acceptable steady state error.
BW is the bandwidth of the desired loop shape.
LG is the gain in low frequency range of the desired loop shape.
HG is the gain in high frequency range of the desired loop shape.

As shown in the constraints of the above optimization problem, the performance specifications are specified in terms of algebraic or functional inequalities. In this paper, the performance specifications are evaluated by plotting the desired open-loop shape and time domain response of the candidate of controller and weighting function. Thus, the fitness function in the controller synthesis can be written as

$$\text{Fitness (J)} = \begin{cases} \left(\left\| \begin{bmatrix} I \\ W_1^{-1}(x) K(p) \end{bmatrix} (I - W_2 G_0 K(p))^{-1} [I \quad G_s] \right\|_\infty \right)^{-1} & \text{if the constraints are met} \\ 0.0001 & \text{otherwise} \end{cases} \quad \ldots(9)$$

The fitness is set to a small value (in this case is 0.0001) if K does not stabilize the plant.

IV. Simulation and Experimental Results

In this section, the dynamic model of a pneumatic servo system is met by using the system identification. To identify, input signal (value voltage) is fed by Pseudo Binary Random Signal (PBRS) then input signal and output signal (position: meter) are kept and used for the system identification. In this paper, output error (OE) model is applied approximate model's output which error between output signal and model's output is minimized [11]. In our experiments, the sampling time in this pneumatic servo system is 0.011 sec. Fig. 4, the comparison of the simulated model's output and the measured output is shown. The results show that the plant model is accurately approximated by the identified model. The time delay is realized in the plant transfer function by its 2 order Pade's approximation which the time delay is 0.154 sec. From the procedure discussed previously, the identified plant model is found to be

$$\begin{aligned} G_0 &= \frac{0.2347s^2 + 28.28s + 1324}{s^3 + 14.22s^2 + 4858s + 41.4} e^{-0.154s} \\ &\approx \frac{0.2347s^4 + 19.14s^3 + 340.9s^2 - 3.72\times10^4 s + 6.693.72\times10^5}{s^5 + 53.18s^4 + 5918s^3 + 1.96\times10^5 s^2 + 2.46\times10^6 s + 2.09\times10^4} \end{aligned} \quad \ldots(10)$$

ISBN: 978-988-19251-2-1
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

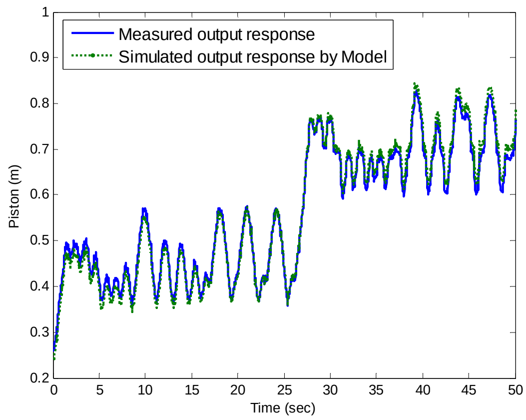


Fig. 4. Comparison between simulated and measured outputs.

A PID with first-order derivative filter is chosen for a proposed fixed-structure robust controller. The controller structure $K(p)$ and weighting function (W_1) are expressed in (11) and (12), respectively. K_p, K_i, K_d τ_d, x_1 and x_2 are parameters to be evaluated by GA.

$$K(p) = K_p + \frac{K_i}{s} + \frac{K_d s}{t_d s + 1} \tag{11}$$

$$W_1(x) = \frac{x_1 s + x_2}{s + 0.001} \tag{12}$$

The weight and controller parameters range are selected as

$$weight\ parameters(x): \begin{array}{l} 0.0001 \le x_1 \le 50 \\ 0.0001 \le x_2 \le 50 \end{array}$$

$$controller\ parameters(p): \begin{array}{l} 0.0001 \le K_p \le 50 \\ 0.0001 \le K_i \le 50 \\ 0.0001 \le K_d \le 50 \\ 0.0001 \le \tau_d \le 50 \end{array}$$

GA parameters and constraint ranges of the performance specification used in this paper are given in Table 1.

Table 1 GA parameters and constraint ranges of the performance specification.

Parameter	value
crossover probability	0.7
mutation probability	0.2
population size	200
maximum generation	30
OV (%)	0.05
SE (%)	0.005
ST (sec)	1
BW (rad/sec)	4
Gain($\omega_{low\ freq}$=0.1)	20

The stability margin of the proposed controller for the genetic search is shown in Fig. 5. As a result, the optimal controller and weighting function are found to be

$$K(p) = 10.5287 + \frac{0.0009}{s} + \frac{0.4119s}{9.7679s + 1} \tag{13}$$

$$W_1 = \frac{11.94s + 0.012}{s + 0.001} \tag{14}$$

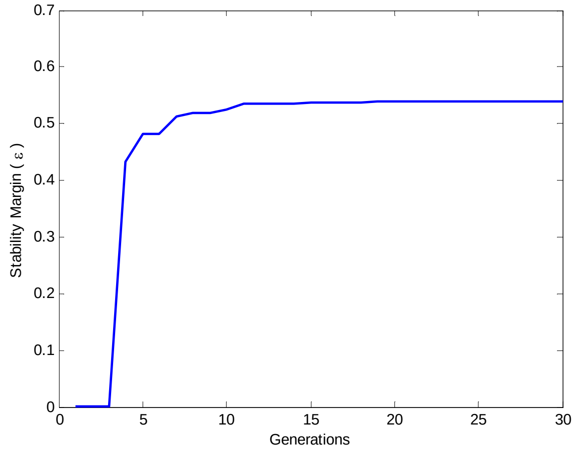


Fig. 5. Convergence of the fitness value.

By applying the H_∞ loop shaping method, the optimal stability margin (ε_{opt}) is founded at 0.5853 (γ = 1.7084). This value indicates that the selected weighting function is compatible with the robust stability requirement. The ε = 0.5560 (γ = 1.7983), which is less than the optimal stability margin, is chosen to synthesis the controller. Based on the conventional technique in Section III, the conventional H_∞ loop shaping controller can be synthesized as follows.

$$K(s) = W_1 K_\infty W_2 = \frac{(11.94s + 0.012)}{(s + 0.001)} \times \frac{19.3359\ (s+0.0010)\ (s^2 + 38.96s + 506)\ (s^2 + 13.79s + 4806)}{(s+30.23)\ (s+0.001041)\ (s^2 + 26.29s + 513.5)\ (s^2 + 16.92s + 4777)} \quad \ldots(15)$$

As shown in (15), the controller is 7th order and complicated in structure. Fig. 6 shows the simulation results of the step responses of the proposed controller. Clearly, the settling time and overshoot of the response from proposed controller are close to those of the H_∞ loop shaping controller.

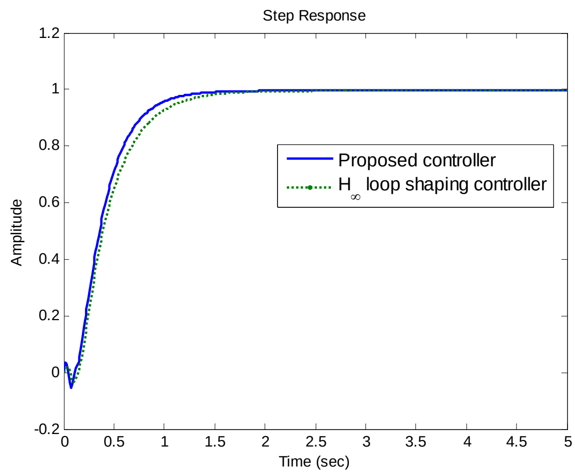


Fig. 6. Responses from the proposed controller and H_∞ loop shaping controller.

Table 2 Identified Local dynamic models of the fuzzy model [12]

position	$G_i(s)$
0.1	$G_{0.1}(s)=\dfrac{0.1194s^2+15.7s+771.9}{s^3+36.7s^2+5237s+25.41}e^{-0.132s}$
0.2	$G_{0.2}(s)=\dfrac{0.2497s^2+32.81s+1614}{s^3+36.65s^2+5234s+84.67}e^{-0.132s}$
0.4	$G_{0.4}(s)=\dfrac{0.1209s^2+16.93s+853.7}{s^3+48.52s^2+5062s+247.4}e^{-0.154s}$
0.6	$G_{0.6}(s)=\dfrac{0.2347s^2+28.28s+1324}{s^3+14.22s^2+4858s+41.4}e^{-0.154s}$
0.8	$G_{0.8}(s)=\dfrac{0.1482s^2+23.12s+1218}{s^3+69.23s^2+4836s+17.72}e^{-0.154s}$
1.0	$G_{1.0}(s)=\dfrac{0.1877s^2+24.32s+1179}{s^3+30.26s^2+4860s+81.56}e^{-0.132s}$
1.1	$G_{1.1}(s)=\dfrac{0.1065s^2+17.16s+909.9}{s^3+73.2s^2+4426s+85.47}e^{-0.132s}$

In the proposed technique, we use the average of stability margin of some specified operating points as our objective function. Firstly, we specify a set of operating points which is used to represent the fuzzy model [12] shown in table 2. In practice, the number of these selected points should be large enough. Average stability margin can be defined as:

$$\varepsilon_{av}=\frac{\sum_{k=0.1}^{1.1}\left\|\begin{bmatrix} I \\ W_1^{-1}(x)K(p)\end{bmatrix}\left(I-W_2G_kK(p)\right)^{-1}[I \quad G_s]\right\|_{\infty}^{-1}}{q} \tag{16}$$

where ε_{av} is stability margin average, q is the number of selected operating points (= 7), The optimal average stability margin (ε_{av}) obtained is 0.4099. Robustness of the proposed controller can be guaranteed robust performance of the global system.

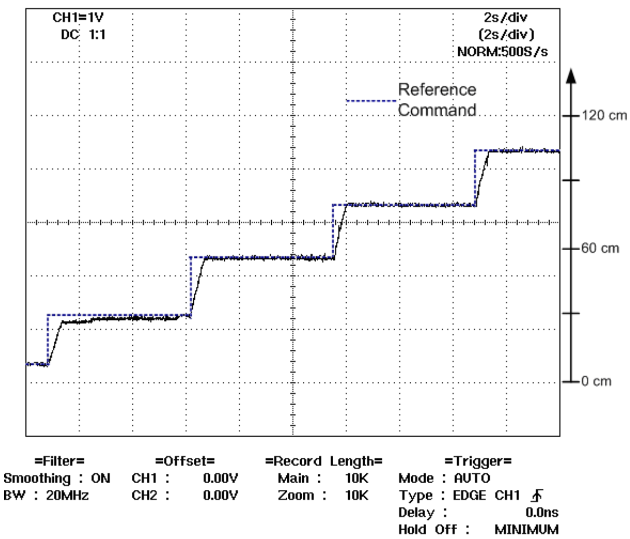


Fig. 7. Experimental results (nominal plant) with proposed controller.

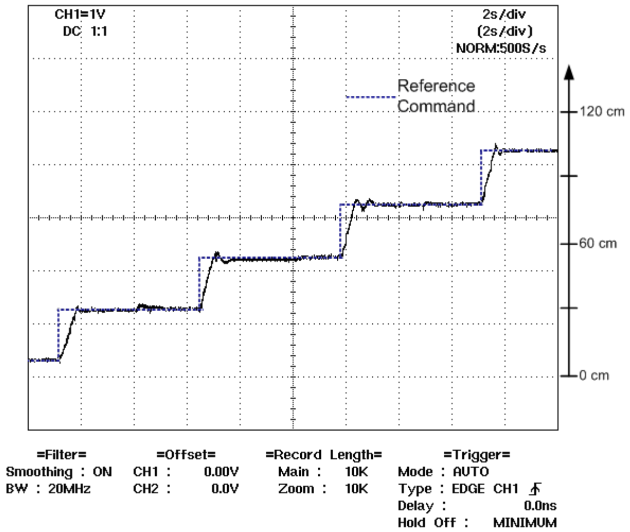


Fig. 8. Experimental results (load mass = 7kg, supply pressure = 500 kPa) with proposed controller.

Experimental results of the proposed controller are shown in Fig. 7 and 8. Fig. 7 shows the output responses at nominal plant of the proposed controller. Settling time (about 1 second) and rise time of the responses from this controller is almost the same as the simulation results. There is no overshoot and steady state error in the responses. To verify the robustness of the proposed technique, we changed the supply pressure of the pneumatic system (400 kPa → 500 kPa) and load mass (5kg → 7kg), equivalent to parameter variations in the plant. The step response of the perturbed plant of the proposed controller is almost the same as that in the nominal plant which is shown in Fig. 8. Clearly, the performance of the proposed controller can guarantee the robust performance when parameters in the plant are changed.

V. Conclusions

The application of structure specified and H_∞ loop shaping method to the design of a fixed-structure H_∞ loop shaping controller for a pneumatic servo system is proposed. The genetic algorithm (GA) can applied in this problem which can not be solved by the classical mathematic. A proposed controller is obtained by GA when fitness function is defined by important variable, stability margin (ε). This variable is used to indicate robustness and performance of the proposed controller. Form the simulation and experiment results, the simple structure and robustness against plant perturbations and disturbance of the proposed controller can be achieved. In this paper, a fuzzy model assures that the proposed technique is guarantee as long as all range actuator.

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ISBN: 978-988-19251-2-1
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)