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Delay-Dependent Control and Stability Analysis for T–S Fuzzy Systems with a Sensor Delay

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***Abstract*—The paper investigates delay-dependent control and stability analysis for T−S fuzzy systems with a sensor delay. Based on a Lyapunov-Krasovskii function (LKF), a less conservative delay-dependent stability condition in the form of parameterized linear matrix inequalities (PLMIs) is established. In the derivation process, Jensen inequality is introduced to estimate the upper bound of the derivative of LKF for T−S fuzzy systems with a sensor delay. Networked-controller in the form of bilinear matrix inequalities (BMIs) is designed. Since BMIs are very complicated and difficult to solve the problem, BMIs are substituted for PLMIs using some mathematical techniques. Sequentially, to fully exploit the convexity of fuzzy weighting functions, we shall replace the derived PLMIs by a finite set of linear matrix inequalities (LMIs) by considering all possible conditions associated with fuzzy weighting functions. Numerical example is provided to illustrate the effectiveness and the benefit of the proposed approach.**

***Index Terms*—T–S fuzzy systems, Sensor delay, Jensen inequality, Parameterized linear matrix inequality (PLMI)**

I. Introduction

REAL phenomena such as biological process, network and mechanics can be represented by dynamic systems with a time-delay as theoretical models. Because of these time delays, the systems frequently cause performance degradation or system instability. Thus, many efforts and attention have been made to solve this problem over the last decades. Particularly, the stability analysis for the complex nonlinear time-delayed systems has received much attention.

Complex nonlinear systems can be represented by a Takagi-Sugeno (T-S) fuzzy system, effectively [1]. There have appeared some stability analysis and controller design of T-S fuzzy systems with a time-delay [2-6]. We can classify the existing stability criteria into two types: delay-independent and delay-dependent criteria. It is well known that the delay-dependent stability criteria are generally less conservative than the delay-independent stability criteria for small delays. Thus, recent efforts have focused on deriving the delay-dependent criteria for the stability analysis of T-S fuzzy systems with a time-delay. As for the delay-dependent criteria, the Lyapunov-Krasovskii functional (LKF) approach is widely used and the free-weighting matrix approach [7-12] is proposed to derive an improved stability criterion with respect to a time delay. However, the bounding techniques for cross-terms and delay terms in the derivative of the LKF frequently lead to conservative results. Thus, there is still a matter for further investigations when estimating the upper bound of the derivative of LKF for T-S fuzzy systems with a time-delay.

Control systems with a sensor delay replace the traditional control system since the computer network technology has been developed rapidly [13-16]. The stability analysis and controller design of these systems also have received much attention recently. The sensors of the system must transmit the data using networks since the direct connection of the sensor and the controller is impossible in Endless hot rolling process and robot-soccer[17]. The system to model these cases can be expressed as a networked control system without controller-actuator delay. So networked channel including a sensor delay is spanned only between the sampler and the controller.

Motivated by the above problem, in this paper, we concentrate on improving the stability conditions and designing sensor-delayed controller for T-S fuzzy systems. To this end, we establish an appropriate LKF and propose some mathematical techniques including Jensen inequality to develop the BMIs for substituted solvable LMIs. On the basis of these approaches, we obtain a less conservative stability condition in the form of parameterized linear matrix inequalities (PLMIs) depend on fuzzy weighting functions. Since solving the PLMIs is equivalent to solving an infinite number of linear matrix inequalities (LMIs), we substitute the derived PLMIs by a finite set of LMIs by considering all possible conditions associated with fuzzy weighting functions [18].

II. Problem Statement

In this paper, all matrices are assumed to have compatible dimensions. The identity matrices and zero matrices are denoted by I and 0, respectively. The notation (*) denotes the symetric block in a symmetric matrix.

Generally, a nonlinear dynamic system can be represented by the T–S fuzzy systems which expresses the nonlinear system as a weighted sum of linear systems. the ith IF–THEN rule of the fuzzy system:

$$\begin{aligned}\text{Rule } i : &\text{IF } \theta_1(t) \text{ is } M_1^i \text{ and } \cdots \text{ and } \theta_g(t) \text{ is } M_g^i \\ &\text{THEN } \dot{x}(t) = A_i x(t) + B_i u(t), \qquad (1)\end{aligned}$$

where $x(t)$ and $u(t)$ denote state vector and input vector, respectively, A_i and B_i $(i = 1, \cdots, k)$ are constant matrices

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with appropriate dimensions, k is the number of fuzzy rules, M_s^i are the fuzzy sets, $\theta_1(t), \cdots, \theta_g(t)$ are the known premise variables.

The fuzzy system (1) is inferred as

$$\dot{x}(t) = \sum_{i=1}^{k} h_i(\theta(t)) \left(A_i x(t) + B_i u(t)\right), \quad (2)$$

where $M_s^i(\theta_s(t))$ is the membership value of $\theta_s(t)$ in M_s^i,

$$\theta(t) = [\ \theta_1(t)\ \ \theta_2(t)\ \ \cdots\ \ \theta_g(t)\ ]^T,$$
$$h_i(\theta(t)) = \frac{\prod_{s=1}^{g} M_s^i(\theta_s(t))}{\sum_{i=1}^{k}\prod_{s=1}^{g} M_s^i(\theta_s(t))},$$
$$\sum_{i=1}^{k} h_i(\theta(t)) = 1,\ h_i(\theta(t)) \geq 0,\ \forall i.$$

A state feedback T–S fuzzy model based controller:

$$\text{Rule } j : \text{IF } \theta_1(t) \text{ is } M_1^j \text{ and } \cdots \text{ and } \theta_g(t) \text{ is } M_g^j$$
$$\text{THEN } u(t) = K_j x(i_k h), \quad (3)$$
$$t \in \{i_k h + \tau_{i_k},\ k = 1, 2, \ldots\}$$

The output of the controller (4) is given by

$$u(t) = \sum_{j=1}^{k} h_j(\underline{\theta(t)}) K_j x(t - d(t)). \quad (4)$$

The time-varying delay, the sensor delay, $d(t)$ satisfies

$$d_1 \leq d(t) \leq d_2. \quad (5)$$

Combining (1) and (4), the closed-loop system can be obtained

$$\dot{x}(t) = \sum_{i=1}^{k}\sum_{j=1}^{k} h_i(\theta(t)) h_j(\theta(t)) \left[A_i x(t) + B_i K_j x(t - d(t))\right],$$
$$t \in \{i_k h + \tau_{i_k}, i_{k+1} h + \tau_{i_{k+1}}\}. \quad (6)$$

The fuzzy system (6) can be rewritten as follows:

$$\dot{x}(t) = A(\Theta(t)) x(t) + B(\Theta(t)) K(\Theta(t)) x(t - d(t)), \quad (7)$$

where

$$\left[\ A(\Theta(t)) \quad B(\Theta(t))K(\Theta(t))\ \right] \quad (8)$$
$$\triangleq \sum_{i=1}^{k}\sum_{j=1}^{k} h_i(\theta(t)) h_j(\theta(t)) \left[\ A_i \quad B_i K_j\ \right]$$

and $\Theta(t)$ denotes a vector of time-varying fuzzy weighting functions $h_i(\theta(t))$ at time t.

III. Main Result

A. Stability Condition

Choose a The Lyapunov-Krasovskii functional candidate of the following form:

$$V(t) = V_1(t) + V_2(t) + V_3(t), \quad (9)$$
$$V_1(t) = x^T(t) P x(t),$$
$$V_2(t) = \int_{t-d_1}^{t} x^T(\alpha) Q_1 x(\alpha) d\alpha + \int_{t-d_2}^{t-d_1} x^T(\alpha) Q_2 x(\alpha) d\alpha,$$
$$V_3(t) = \int_{-d_1}^{0}\int_{t+\beta}^{t} d_1 \dot{x}^T(\alpha) R_1 \dot{x}(\alpha) d\alpha d\beta \quad (10)$$
$$+ \int_{-d_2}^{-d_1}\int_{t+\beta}^{t} d_{21} \dot{x}^T(\alpha) R_2 \dot{x}(\alpha) d\alpha d\beta,$$

where P, Q_i, and R_i $(i = 1, 2)$ are symmetric positive definite matrices and $d_{21} = d_2 - d_1$. For later convenience, we define an augmented state $\zeta(t)$ as $\zeta(t) = \left[\ x^T(t) \quad x^T(t-d_1) \quad x^T(t-d(t)) \quad x^T(t-d_2)\ \right]^T$ and $\mathcal{A} = \left[\ A(\Theta(t)) \quad 0 \quad B(\Theta(t))K(\Theta(t)) \quad 0\ \right]$.

The time derivative of $V(t)$:

$$\dot{V}_1(t) = 2\dot{x}^T(t) P x(t), \quad (11)$$
$$\dot{V}_2(t) = x^T(t) Q_1 x(t) - x^T(t-d_1) Q_1 x(t-d_1)$$
$$+ x^T(t-d_1) Q_2 x(t-d_1) - x^T(t-d_2)$$
$$\times Q_2 x(t-d_2),$$
$$\dot{V}_3(t) = \dot{x}^T(t)(d_1^2 R_1 + d_{21}^2 R_2)\dot{x}(t)$$
$$- \int_{t-d_1}^{t} d_1 \dot{x}^T(\alpha) R_1 \dot{x}(\alpha) d\alpha$$
$$- \int_{t-d_2}^{t-d_1} d_{21} \dot{x}^T(\alpha) R_2 \dot{x}(\alpha) d\alpha.$$

Applying Jensen inequality, we can obtain

$$- \int_{t-d_1}^{t} d_1 \dot{x}^T(\alpha) R_1 \dot{x}(\alpha) d\alpha$$
$$\leq - \begin{bmatrix} x(t) \\ x(t-d_1) \end{bmatrix}^T \begin{bmatrix} R_1 & -R_1 \\ -R_1 & R_1 \end{bmatrix} \begin{bmatrix} x(t) \\ x(t-d_1) \end{bmatrix}, \quad (12)$$

$$- \int_{t-d_2}^{t-d_1} d_{21} \dot{x}^T(\alpha) R_2 \dot{x}(\alpha) d\alpha$$
$$= - \int_{t-d(t)}^{t-d_1} d_{21} \dot{x}^T(\alpha) R_2 \dot{x}(\alpha) d\alpha$$
$$- \int_{t-d_2}^{t-d(t)} d_{21} \dot{x}^T(\alpha) R_2 \dot{x}(\alpha) d\alpha$$
$$< - \begin{bmatrix} x(t-d_1) \\ x(t-d(t)) \\ x(t-d_2) \end{bmatrix}^T \begin{bmatrix} R_2 & -R_2 & 0 \\ -R_2 & 2R_2 & -R_2 \\ 0 & -R_2 & R_2 \end{bmatrix} \begin{bmatrix} x(t-d_1) \\ x(t-d(t)) \\ x(t-d_2) \end{bmatrix}. \quad (13)$$

Then $\dot{V}(t)$ can be upper bounded by the following form:

$$\dot{V}(t) \leq \zeta^T(t) \Psi_{11} \zeta(t) + \zeta^T(t) \mathcal{A}^T \left(d_1^2 R_1 + d_{21}^2 R_2\right) \mathcal{A} \zeta(t) \quad (14)$$
$$= \zeta^T(t) \left[\Psi_{11} + \mathcal{A}^T \left(d_1^2 R_1 + d_{21}^2 R_2\right) \mathcal{A}\right] \zeta(t),$$

and $\dot{V}(t) \leq 0$ can be represented by $\psi \leq 0$ using Shur decomposition method, where

$$\Psi = \begin{bmatrix} \Psi_{11} & (*) \\ \Psi_{21} & \Psi_{22} \end{bmatrix},$$
$$\Psi_{11} = \begin{bmatrix} A^T(\Theta(t))P + PA(\Theta(t)) + Q_1 - R_1 & (*) & (*) & (*) \\ R_1 & -Q_1 + Q_2 - R_1 - R_2 & (*) & (*) \\ K^T(\Theta(t)) B^T(\Theta(t)) P & R_2 & -2R_2 & (*) \\ 0 & 0 & R_2 & -Q_2 - R_2 \end{bmatrix},$$
$$\Psi_{21} = \begin{bmatrix} d_1 A(\Theta(t)) & 0 & d_1 B(\Theta(t)) K(\Theta(t)) & 0 \\ d_{21} A(\Theta(t)) & 0 & d_{21} B(\Theta(t)) K(\Theta(t)) & 0 \end{bmatrix},$$
$$\Psi_{22} = \mathrm{diag}\{-R_1^{-1}, -R_2^{-1}\}.$$

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Theorem 1: Given d_1, d_2, and $K(\Theta(t))$, the fuzzy system (1) is asymptotically stable if there exist symmetric matrices $P > 0$, $Q_1 > 0$, $Q_2 > 0$, $R_1 > 0$, and $R_2 > 0$ such that the following condition holds:

$$0 > \Psi, \tag{15}$$

where

$$\Psi = \begin{bmatrix} \Psi_{11} & (*) \\ \Psi_{21} & \Psi_{22} \end{bmatrix},$$

$$\Psi_{11} = \begin{bmatrix} A^T(\Theta(t))P + PA(\Theta(t)) + Q_1 - R_1 & (*) & (*) & (*) \\ R_1 & -Q_1 + Q_2 - R_1 - R_2 & (*) & (*) \\ K^T(\Theta(t))B^T(\Theta(t))P & R_2 & -2R_2 & (*) \\ 0 & 0 & R_2 & -Q_2 - R_2 \end{bmatrix},$$

$$\Psi_{21} = \begin{bmatrix} d_1 A(\Theta(t)) & 0 & d_1 B(\Theta(t))K(\Theta(t)) & 0 \\ d_{21} A(\Theta(t)) & 0 & d_{21} B(\Theta(t))K(\Theta(t)) & 0 \end{bmatrix},$$

$$\Psi_{22} = \operatorname{diag}\{-R_1^{-1}, -R_2^{-1}\}.$$

B. Controller Design

To design the controller, we develop the Theorem 1 using pre- and post-multiply both sides of (15) with $\tilde{X} = \operatorname{diag}\{X, X, X, X, I, I\}$ and its transpose. Let us define $X = P^{-1}$, $\tilde{Q}_1 = XQ_1X$, $\tilde{Q}_2 = XQ_2X$, $\tilde{R}_1 = XR_1X$, $\tilde{R}_2 = XR_2X$, and $Y(\Theta(t)) = K(\Theta(t))X$.

$$0 > \tilde{X}\Psi\tilde{X}^T = \Omega,$$

where

$$\Omega = \begin{bmatrix} \Omega_{11} & (*) \\ \Omega_{21} & \Omega_{22} \end{bmatrix},$$

$$\Omega_{11} = \begin{bmatrix} XA^T(\Theta(t)) + A(\Theta(t))X + \tilde{Q}_1 - \tilde{R}_1 & (*) & (*) & (*) \\ \tilde{R}_1 & -\tilde{Q}_1 + \tilde{Q}_2 - \tilde{R}_1 - \tilde{R}_2 & (*) & (*) \\ Y^T(\Theta(t))B^T(\Theta(t)) & \tilde{R}_2 & -2\tilde{R}_2 & (*) \\ 0 & 0 & \tilde{R}_2 & -\tilde{Q}_2 - \tilde{R}_2 \end{bmatrix},$$

$$\Omega_{21} = \begin{bmatrix} d_1 A(\Theta(t))X & 0 & d_1 B(\Theta(t))Y(\Theta(t)) & 0 \\ d_{21} A(\Theta(t))X & 0 & d_{21} B(\Theta(t))Y(\Theta(t)) & 0 \end{bmatrix},$$

$$\Omega_{22} = \operatorname{diag}\{-X\tilde{R}_1^{-1}X, -X\tilde{R}_2^{-1}X\}.$$

Since $X > 0$, we have

$$(\tilde{R}_1 - X)\tilde{R}_1^{-1}(\tilde{R}_1 - X) > 0,\ (\tilde{R}_2 - X)\tilde{R}_2^{-1}(\tilde{R}_2 - X) > 0 \tag{16}$$

which are equivalent to

$$-X\tilde{R}_1^{-1}X < \tilde{R}_1 - 2X,\ -X\tilde{R}_2^{-1}X < \tilde{R}_2 - 2X \tag{17}$$

Theorem 2: Given d_1 and d_2, the fuzzy system (1) is asymptotically stable with a feedback gain $K(\Theta(t)) = Y(\Theta(t))X^{-1}$ if there exist symmetric matrices $X > 0$, $\tilde{Q}_1 > 0$, $\tilde{Q}_2 > 0$, $\tilde{R}_1 > 0$, and $\tilde{R}_2 > 0$ such that the following condition holds:

$$0 > \Omega, \tag{18}$$

where

$$\Omega = \begin{bmatrix} \Omega_{11} & (*) \\ \Omega_{21} & \Omega_{22} \end{bmatrix},$$

$$\Omega_{11} = \begin{bmatrix} XA^T(\Theta(t)) + A(\Theta(t))X + \tilde{Q}_1 - \tilde{R}_1 & (*) & (*) & (*) \\ \tilde{R}_1 & -\tilde{Q}_1 + \tilde{Q}_2 - \tilde{R}_1 - \tilde{R}_2 & (*) & (*) \\ Y^T(\Theta(t))B^T(\Theta(t)) & \tilde{R}_2 & -2\tilde{R}_2 & (*) \\ 0 & 0 & \tilde{R}_2 & -\tilde{Q}_2 - \tilde{R}_2 \end{bmatrix},$$

$$\Omega_{21} = \begin{bmatrix} d_1 A(\Theta(t))X & 0 & d_1 B(\Theta(t))Y(\Theta(t)) & 0 \\ d_{21} A(\Theta(t))X & 0 & d_{21} B(\Theta(t))Y(\Theta(t)) & 0 \end{bmatrix},$$

$$\Omega_{22} = \operatorname{diag}\{\tilde{R}_1 - 2X, \tilde{R}_2 - 2X\}.$$

C. Relaxed Condition

Since solving the PLMIs of Theorem 2 is equivalent to solving an infinite number of LMIs, it needs to find a finite number of solvable LMI conditions from PLMIs. The following development gives a relaxed delay-dependent stability condition in the form of a finite set of LMIs.

Another representation for (15):

$$\begin{aligned} 0 &> \mathcal{L}(h(\theta(t))) \\ &\triangleq \mathcal{L}_0 + \sum_{i=1}^{k} h_i(\theta(t))\left(\mathcal{L}_i + \mathcal{L}_i^T\right) + \sum_{i=1}^{k} h_i^2(\theta(t))\mathcal{L}_{ii} \\ &\quad + \sum_{i=1}^{k}\left(\sum_{j=1}^{i-1} h_i(\theta(t))h_j(\theta(t))\mathcal{L}_{ij} \right. \\ &\quad \left. + \sum_{j=i+1}^{k} h_i(\theta(t))h_j(\theta(t))\mathcal{L}_{ij}^T\right) \end{aligned} \tag{19}$$

where

$$\mathcal{L}_0 \triangleq \begin{bmatrix} \tilde{Q}_1 - \tilde{R}_1 & (*) & (*) & (*) & (*) & (*) \\ \tilde{R}_1 & -\tilde{Q}_1 + \tilde{Q}_2 - \tilde{R}_1 - \tilde{R}_2 & (*) & (*) & (*) & (*) \\ 0 & \tilde{R}_2 & -2\tilde{R}_2 & (*) & (*) & (*) \\ 0 & 0 & \tilde{R}_2 & -\tilde{Q}_2 - \tilde{R}_2 & (*) & (*) \\ 0 & 0 & 0 & 0 & \tilde{R}_1 - 2X & (*) \\ 0 & 0 & 0 & 0 & 0 & \tilde{R}_2 - 2X \end{bmatrix},$$

$$\mathcal{L}_i \triangleq \begin{bmatrix} A_iX & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ d_1A_iX & 0 & 0 & 0 & 0 & 0 \\ d_{21}A_iX & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\mathcal{L}_{ii} \triangleq \begin{bmatrix} 0 & 0 & (*) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ Y_i^T B_i^T & 0 & 0 & 0 & (*) & (*) \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & d_1B_iY_i & 0 & 0 & 0 \\ 0 & 0 & d_{21}B_iY_i & 0 & 0 & 0 \end{bmatrix},$$

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$$\mathcal{L}_{ij} \triangleq \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ Y_i^T B_j^T + Y_j^T B_i^T & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & d_1 B_i Y_j + d_1 B_j Y_i & 0 & 0 & 0 \\ 0 & 0 & d_{21} B_i Y_j + d_{21} B_j Y_i & 0 & 0 & 0 \end{bmatrix}.$$

By the S-procedure

$$0 > \mathcal{L}(h(\theta(t))) + \mathcal{N}(h(\theta(t))), \tag{20}$$

where $0 \le \mathcal{N}(h(\theta(t)))$ is given by ($\sum_{i=1}^{k} h_i(\theta(t)) = 1$, $0 \le h_i(\theta(t)) \le \beta_i$, $0 \le h_i(\theta(t))h_j(\theta(t))$)

$$\mathcal{N}(h(\theta(t))) = \mathcal{C}_1 + \mathcal{C}_1^T + \sum_{i=1}^{k} \mathcal{C}_{2i}(\Lambda_i + \Lambda_i^T) + \sum_{i=1}^{k}\sum_{j=1, j\neq i}^{k} \mathcal{C}_{3ij}(\Sigma_{ij} + \Sigma_{ij}^T),$$

$$0 = \mathcal{C}_1 \triangleq \begin{bmatrix} I \\ h_1(\theta(t)) \\ \vdots \\ h_k(\theta(t)) \end{bmatrix}^T \begin{bmatrix} I \\ -I \\ \vdots \\ -I \end{bmatrix} \begin{bmatrix} W_0 & W_1 & \cdots & W_k \end{bmatrix} \begin{bmatrix} I \\ h_1(\theta(t)) \\ \vdots \\ h_k(\theta(t)) \end{bmatrix},$$

$$0 \le \mathcal{C}_{2i} \triangleq -h_i^2(\theta(t)) + \beta_i h_i(\theta(t)),$$
$$0 \le \mathcal{C}_{3ij} \triangleq h_i(\theta(t))h_j(\theta(t)),$$

for $0 < \Lambda_i + \Lambda_i^T$ and $0 < \Sigma_{ij} + \Sigma_{ij}^T$. With some algebraic manipulations, the constraint $0 \le \mathcal{N}(h(\theta(t)))$ can be represented as follows:

$$\begin{aligned} 0 &\le \mathcal{N}(h(\theta(t))) \\ &= \mathbf{N}_0 + \sum_{i=1}^{k} h_i(\theta(t))(\mathbf{N}_i + \mathbf{N}_i^T) + \sum_{i=1}^{k} h_i^2(\theta(t))\mathbf{N}_{ii} \\ &+ \sum_{i=1}^{k}\left(\sum_{j=1}^{i-1} h_i(\theta(t))h_j(\theta(t))\mathbf{N}_{ij} + \sum_{j=i+1}^{k} h_i(\theta(t))h_j(\theta(t))\mathbf{N}_{ij}^T\right), \end{aligned} \tag{21}$$

where $\mathbf{N}_0 = W_0 + W_0^T$, $\mathbf{N}_i = \beta_i\Lambda_i - W_0 + W_i$, $\mathrm{N}_{ii} = -(\Lambda_i + \Lambda_i^T) - (W_i + W_i^T)$, and $\mathrm{N}_{ij} = -(W_i + W_j) + (\Sigma_{ij} + \Sigma_{ji})$. Hence, the condition (20) becomes

$$\begin{aligned} 0 &> \Gamma_0 + \sum_{i=1}^{k} h_i(\theta(t))(\Gamma_i + \Gamma_i^T) + \sum_{i=1}^{k} h_i^2(\theta(t))\Delta_i \\ &+ \sum_{i=1}^{k}\left(\sum_{j=1}^{i-1} h_i(\theta(t))h_j(\theta(t))\Phi_{ij} + \sum_{j=i+1}^{k} h_i(\theta(t))h_j(\theta(t))\Phi_{ij}^T\right), \end{aligned} \tag{22}$$

where

$$\begin{aligned} \Gamma_0 &= \mathcal{L}_0 + \mathbf{N}_0 = \mathcal{L}_0 + W_0 + W_0^T, \\ \Gamma_i &= \mathcal{L}_i + \mathbf{N}_i = \mathcal{L}_i + \beta_i\Lambda_i - W_0 + W_i, \\ \Delta_i &= \mathcal{L}_{ii} + \mathbf{N}_{ii} = \mathcal{L}_{ii} - (\Lambda_i + \Lambda_i^T) - (W_i + W_i^T), \\ \Phi_{ij} &= \mathcal{L}_{ij} + \mathbf{N}_{ij} = \mathcal{L}_{ij} - (W_i + W_j) + (\Sigma_{ij} + \Sigma_{ji}). \end{aligned}$$

The condition (22) boils down to

$$\begin{aligned} 0 > &\begin{bmatrix} I & h_1(\theta(t))I & \cdots & h_k(\theta(t))I \end{bmatrix} \tilde{\mathcal{L}} \\ &\begin{bmatrix} I & h_1(\theta(t))I & \cdots & h_k(\theta(t))I \end{bmatrix}^T, \end{aligned} \tag{23}$$

where

$$\tilde{\mathcal{L}} \triangleq \left[\begin{array}{c|cccc} \Gamma_0 & (*) & (*) & \cdots & (*) \\ \hline \Gamma_1 & \Delta_1 & (*) & \cdots & (*) \\ \Gamma_2 & \Phi_{21} & \Delta_2 & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & (*) \\ \Gamma_k & \Phi_{k1} & \cdots & \Phi_{k(k-1)} & \Delta_k \end{array}\right], \tag{24}$$

IV. Numerical Example

In this section, The numerical example will be presented to illustrate the approach developed in this paper.

Example: Consider the truck-trailer system [19], that can be represented by the following T–S fuzzy system.

$$\begin{aligned} \text{Rule 1 : IF } \theta(t) &= x_2(t) + a\frac{v\bar{t}}{2L}x_1(t) \\ &+ (1-a)\frac{v\bar{t}}{2L}x_1(t-d(t)) \text{ is about } 0, \\ &\text{THEN } \dot{x}(t) = A_1x(t) + B_1u(t), \\ \text{Rule 2 : IF } \theta(t) &= x_2(t) + a\frac{v\bar{t}}{2L}x_1(t) \\ &+ (1-a)\frac{v\bar{t}}{2L}x_1(t-d(t)) \text{ is about } \pi \textit{ or } -\pi, \\ &\text{THEN } \dot{x}(t) = A_2x(t) + B_2u(t), \end{aligned}$$

where

$$\begin{aligned} A_1 &= \begin{bmatrix} -a\frac{v\bar{t}}{Lt_0} & 0 & 0 \\ a\frac{v\bar{t}}{Lt_0} & 0 & 0 \\ a\frac{v^2\bar{t}^2}{2Lt_0} & \frac{v\bar{t}}{t_0} & 0 \end{bmatrix}, \ B_1 = \begin{bmatrix} \frac{v\bar{t}}{lt_0} \\ 0 \\ 0 \end{bmatrix}, \\ A_2 &= \begin{bmatrix} -a\frac{v\bar{t}}{Lt_0} & 0 & 0 \\ a\frac{v\bar{t}}{Lt_0} & 0 & 0 \\ a\frac{dv^2\bar{t}^2}{2Lt_0} & \frac{dv\bar{t}}{t_0} & 0 \end{bmatrix}, \ B_2 = \begin{bmatrix} \frac{v\bar{t}}{lt_0} \\ 0 \\ 0 \end{bmatrix}. \end{aligned} \tag{25}$$

The model parameters are given as l=2.8, L=5.5, v=-1.0, $\bar{t} = 2.0$, $t_0 = 0.5$ and $d = \frac{10t_0}{\pi}$. The fuzzy weighting functions are employed

$$\begin{aligned} h_1(\Theta(t)) &= \left(1 - \frac{1}{1 + exp(-3(\Theta(t) - 0.5\pi))}\right) \\ &\left(1 - \frac{1}{1 + exp(-3(\Theta(t) + 0.5\pi))}\right), \\ h_2(\Theta(t)) &= 1 - h_1(\Theta(t)). \end{aligned} \tag{26}$$

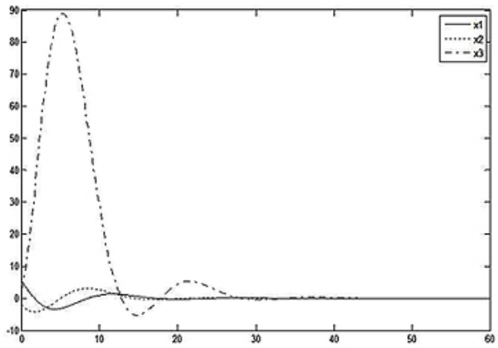

Fig. 1. The response of the states

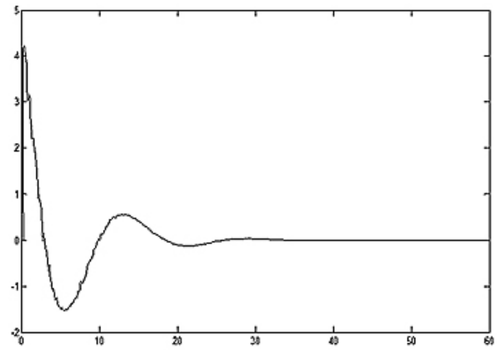

Fig. 2. Control input curve

Applying the LMIs of above relaxed condition, we get the maximal allowable sensor delay $\sigma = 0.34$ and the feedback gain matrices which are feasible

$$K_1 = \left[\begin{array}{ccc} 0.6984 & -0.3023 & 0.0077 \end{array}\right], \qquad (27)$$
$$K_2 = \left[\begin{array}{ccc} 0.7010 & -0.3200 & 0.0082 \end{array}\right].$$

We used the fuzzy controller with a sensor delay $u(t) = h_1K_1x(t-d(t))+h_2K_2x(t-d(t))$ under the initial condition x(0)=[5 -3 2] to control the original dynamic nonlinear system. The Figure 4.1 and the Figure 4.2 show the response of the states and the control input curve which can be obtained by simulation, respectively. Although the entire system has a sensor delay, these simulation results clearly represent that the closed-loop system is asymptotically stable.

V. Conclusion

This paper investigated stability analysis and delay-dependent control for T-S fuzzy systems with a sensor delay. We developed robust stabilization for nonlinear systems with a sensor delay based on T-S fuzzy model approach. To make our delay-dependent stability conditions, we introduced a LKF and proposed some mathematical techniques including Jensen inequality. BMIs was substituted for PLMIs to solve the control problem. And then, we considered all possible conditions suitable for fully exploiting the convexity of fuzzy weighting functions during the process of replacing the derived PLMIs by a finite set of LMIs. Finally the results of the computer simulation showed that the states and the control input of the closed-loop system are asymptotically stable.

The disadvantage of the simulation result is that the maximum allowable sensor delay is relatively small. To obtain a less conservative stability condition with respect to a sensor delay is challenging work.

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