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Fixed Structure Robust 2DOF H-infinity Loop Shaping Control for ACMC Buck Converter using Genetic Algorithm

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***Abstract*–This paper proposes a new technique for designing a robust controller for an ACMC (Average Current Mode Control) buck converter. The proposed technique is based on the concept of 2DOF H_∞ loop shaping control (2DOF HLS) which can be adopted to find the robust controller. However, the structure of this controller is normally complicated with high order because the order of the designed controller depends on the order of the plant and weighting function. This makes the difficulty in the implementation. The proposed technique can overcome this problem by using genetic algorithm (GA) to solve the structure specified 2DOF H-infinity loop shaping control. Simulation results in the ACMC buck converter verify the effectiveness of the proposed controller.**

***Index Terms* – 2DOF H_∞ loop shaping control, genetic algorithm, ACMC buck converter**

I. INTRODUCTION

Step-down DC to DC converter (buck converter) is a power electronic circuit which produces a lower output voltage than input voltage. This converter is an important circuit in several portable electronic devices, and it can be used in many industrial applications, such as computer hardware, dc-motor drives, telecommunication equipment, etc. The controller of this converter is usually designed by an analog circuit which is simple and cheap. In addition, analog circuit is suitable for the circuit with high switching frequency which is normally adopted in DC to DC converter. Designing of a high performance DC to DC converter controller requires the technique which can incorporate both robustness and performance in the controller design. In the previous research works, many techniques can fulfill the above mentioned requirement, e.g. H_2/H_∞, mixed sensitivity, H_∞ loop shaping control, 2DOF H_∞ loop shaping control [1-3], etc. In these techniques, the designing of weighting function and controller is based on the concept of optimal control theory. This paper focuses on the design of 2DOF H_∞ loop shaping controller which is a powerful technique to design a robust controller. In this technique, time-domain specification can be incorporated in the design by specifying a proper model reference, while the frequency domain specification can be incorporated via the design of weighting function.

The controller designed by H_∞ loop shaping control can make the system robust; however, the structure of the resulting controller has high order which is not suitable for practical work. Especially, the robust controller with high order is not feasible for the analog circuit design. To overcome this problem, fixed structure robust loop shaping control is proposed to design the structure specified robust controller; this technique gains more attention because the structure of controller is simple with low order, and it still retains both the robustness and performance. However, time domain specification cannot be directly specified in the design. To enhance the ability of the robust loop shaping control, this paper proposes a new technique which adopts the genetic algorithm [6-7] for synthesizing the optimal parameters of the 2DOF robust controller. Based on the concept of 2DOF control, time and frequency domain specifications can be incorporated in the proposed design. Filter and PID controller are adopted as the specified structure in this paper.

The remainder of this paper is as follows. Section 2 presents the model of ACMC buck converter. Section 3 presents the details of conventional 2DOF H_∞ loop shaping control and the proposed technique. Simulation results are illustrated in Section 4. Finally, Section 5 concludes the paper.

II. CONVERTER MODEL

Current mode control (CMC) is extensively used for controlling the DC to DC converter. In this technique, there are two control loops which are inner current loop and outer voltage loop. The current loop is adopted for controlling the inductor current; the reference command of this loop is obtained from the output of the outer voltage loop. The voltage loop is adopted for controlling the output voltage. The main principle of the control with two feedback loops is that when the output voltage is lower than the command, the converter will increase the inductor current to regulate the output voltage. There are two types of current mode control those are Peak Current Mode Control (PCMC) and Average Current Mode Control (ACMC). The circuit of PCMC is very simple; however, the main disadvantage of this control is poor noise immunity. The ACMC can overcome this problem. The dynamic model of ACMC buck converter can be written as (1) which is the transfer function from the current reference (V_c) to the output voltage (V_0) [4-5]. The typical circuit of ACMC buck converter is shown in Fig. 1.

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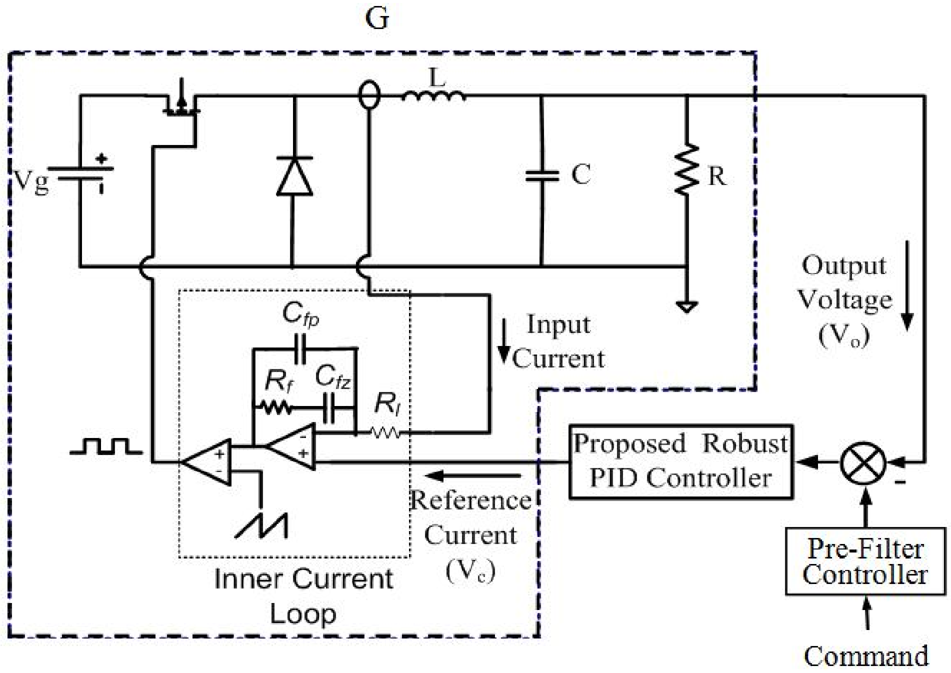

Fig. 1 Typical circuit of ACMC buck converter

$$\frac{V_o(s)}{V_c(s)} = \frac{K_m(1+r_cCs)[G_{CA}+1]G_{dv}(s)}{1+T_c(s)} \tag{1}$$

Where

$$K_m = \frac{1}{V_m} \tag{2}$$

$$G_{dv}(s) = \frac{V_g}{R+Ls+RLCs^2} \tag{3}$$

$$T_c(s) = \frac{R_S K_m V_g[1+RCs][1+G_{CA}]}{R+Ls+RLCs^2} \tag{4}$$

$$G_{CA} = \frac{K_c(1+\frac{s}{\omega_z})}{s(1+\frac{s}{\omega_p})} \tag{5}$$

$$K_C = \frac{1}{R_I(C_{fp}+C_{fz})} \tag{6}$$

$$\omega_z = \frac{1}{R_f C_{fz}} \tag{7}$$

$$\omega_p = \frac{C_{fz}+C_{fp}}{R_f C_{fz} C_{fp}} \tag{8}$$

III. 2DOF H_∞ LOOP SHAPING CONTROL AND PROPOSED TECHNIQUE

A. 2DOF H_∞ Loop Shaping Control

2DOF H_∞ loop shaping control is a robust control technique which the time domain specification can be incorporated in the design. The controllers designed by this approach are feed-forward pre-filter and feed-back controllers. The feed-forward pre-filter controller (K_1) is adopted to control the time domain response of the closed loop system, and the feed-back controller is designed for achieving the desired robust stability and the disturbance rejection requirement. In this technique, only a pre-compensator weight function (W_1) and a reference model (T_{ref}) are needed to be specified; the shaped plant (G_s) is formulated as the normalized co-prime factors, which separate the plant G_s into the normalized nominator and denominator factors (N_s and M_s, respectively). Fig. 2 shows the uncertainty model of the perturbed plant and robust controllers adopted in this approach.

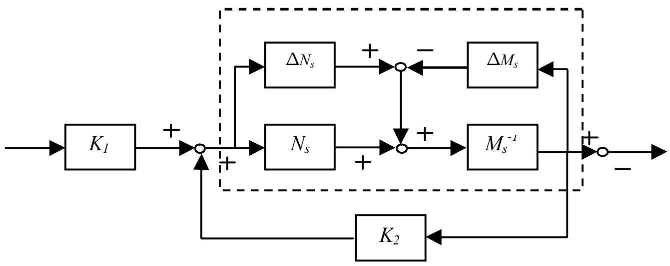

Fig. 2 Co-prime factor robust stabilization problem.

Both the feed-forward pre-filter and feedback controllers (K_1 and K_2) are synthesized by solving the control design problem as shown in Fig. 3.

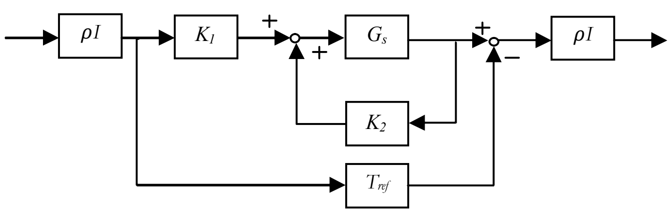

Fig. 3 2DOF H_∞ loop-shaping design problem

Where

T_{ref} is the reference model.

ρ is a scalar value specified by the designer to assign the degree of significance of the time domain specification.

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The shaped plant (G_s) can be written as:

$$G_s = GW_1 = M_s^{-1}N_s \tag{9}$$

Then, the perturbed plant is written as:

$$G_\Delta = (N_s + \Delta_{Ns})(M_s + \Delta_{Ms})^{-1} \tag{10}$$

G_Δ is the shaped plant with uncertainty. Δ_{Ns} is the uncertainty transfer function in the nominator factor and Δ_{Ms} is the uncertainty transfer functions in the denominator factor.

$$| \Delta_{Ns}, \Delta_{Ms} |_\infty \leq \varepsilon \tag{11}$$

Where ε is the uncertainty boundary called stability margin.

Following steps are the procedure to design the 2DOF H_∞ loop shaping controller.

Step1. Specify the pre-compensator weighting function (W_1) for achieving the desired open loop shape.

Step2. Specify T_{ref} which is the desired closed loop transfer function for time domain specifications and select ρ which is a scalar value between 0 and 1. If the designer selects $\rho = 0$, the 2DOF H_∞ loop shaping control becomes the 1DOF H_∞ loop shaping control.

Step3. Find the optimal stability margin (ε_{opt}) by solving the following equation.

$$\gamma_{opt} = \varepsilon_{opt}^{-1} = \left\| \begin{bmatrix} \rho(I - K_{2\infty}G_s)^{-1}K_{1\infty} & K_2(I - G_sK_{2\infty})^{-1}M_s^{-1} \\ \rho(I - G_sK_{2\infty})^{-1}G_sK_{1\infty} & (I - G_sK_{2\infty})^{-1}M_s^{-1} \\ \rho^2[(I - G_sK_{2\infty})^{-1}G_sK_{1\infty} - T_{ref}] & \rho(I - G_sK_{2\infty})^{-1}M_s^{-1} \end{bmatrix} \right\|_\infty \tag{12}$$

The robust stability of the system is measured by the resulting of ε_{opt}. If the $\varepsilon_{opt} < 0.25$ ($\gamma_{opt} > 4$), then go to step 1 to select the new weight function.

Step4. Select the stability margin and then synthesize the controllers ($K_{1\infty}$, $K_{2\infty}$) that satisfy:

$$\| T_{zw} \|_\infty = \left\| \begin{bmatrix} \rho(I - K_{2\infty}G_s)^{-1}K_{1\infty} & K_2(I - G_sK_{2\infty})^{-1}M_s^{-1} \\ \rho(I - G_sK_{2\infty})^{-1}G_sK_{1\infty} & (I - G_sK_{2\infty})^{-1}M_s^{-1} \\ \rho^2[(I - G_sK_{2\infty})^{-1}G_sK_{1\infty} - T_{ref}] & \rho(I - G_sK_{2\infty})^{-1}M_s^{-1} \end{bmatrix} \right\|_\infty \tag{13}$$

The (1,1) and (2,1) help to limit actuator usage, (1,1) and (1,2) are associated with robust stability optimization, (3,1) is used to model matching and (3,2) is linked to the performance of the loop.

Step5. W_i is designed as the scalar value which is given by

$$W_i = [W_o\ (I - G_s(0)K_2(0))^{-1}G_s(0)K_1(0)]^{-1}T_{ref}(0) \tag{14}$$

Where

$$W_o = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Step6. Final feed-forward pre-filter and feed-back controllers (K_1 and K_2) can be determined by the following equations.

$$K_1 = W_1\, K_{1\infty} W_i \tag{15}$$

$$K_2 = W_1\, K_{2\infty} \tag{16}$$

B. Genetic Algorithm Based Fixed Structure 2DOF H_∞ Loop Shaping Control

The robust controller synthesized by the conventional 2DOF H_∞ loop shaping control is normally complicated with high order. The fixed structure robust controller can solve this problem; thus, this paper proposes the genetic algorithm based fixed structure 2DOF H_∞ loop shaping to solve this problem. This technique can achieve the robustness and performance even the structure specified controller.

Assume that the predefined structures of feed-forward pre-filter $K_1(p_1)$ and feedback controller $K_2(p_2)$ have the satisfied parameters p_1 and p_2, respectively. Parameter of p_1 is synthesized by the proposed method and parameter of p_2 is synthesized by minimize the infinity norm of T_{zw} by using the 1DOF H_∞ loop shaping control method. From (17), $K_2(p_2)$ can be written as:

$$K_2(p_2) = W_1\, K_{2\infty} \tag{17}$$

Then,

$$K_{2\infty} = W_1^{-1}K_2(p_2) \tag{18}$$

By substituting (18) into the infinity norm by using the 1DOF H_∞ loop shaping control, the infinity norm of the transfer function from disturbances to states which is attempted to be minimized can be written as:

$$\|T_{zw}\|_\infty = \left\| \begin{bmatrix} I \\ K_{2\infty} \end{bmatrix} (I + G_sK_{2\infty})^{-1}M_s^{-1} \right\|_\infty \leq \varepsilon^{-1} \tag{19}$$

Then, synthesize $K_1(p_1)$ compare T_{ref} by Evaluate minimize integral of square error (ISE).

In this paper, we adopt the GA for evaluating the optimal solution in any optimization problem. The details of GA can be found in [8]. The proposed method is summarized as follows:

Step1. Shape the singular values of the nominal plant G by W_1. Then evaluate the ε_{opt}. If ε_{opt} is not satisfied ($\varepsilon_{opt} < 0.25$) go to step 1 to redesign the weighting function W_1.

Step2. Design structure of $K_2(p_2)$, as show in (20).

$$K_2(p_2) = Kp + \frac{Ki}{s} + \frac{Kd}{Tds + 1} \tag{20}$$

Step3. Specify the GA parameters such as population size, crossover and mutation probabilities, maximum generation [8], etc for synthesize the maximize stability margin (ε_{opt}) by using the 1DOF H_∞ loop shaping control method.

Step4. Evaluate the fitness function $(J_{cost})^{-1}$ of each chromosome. Select the chromosome with minimum cost function (maximum ε) as the solution of the current generation. For the first generation, set Gen= 1.

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Step5. Increase the *Gen* for a step.

Step6. While the current generation is less than the maximum generation, create a new population using GA operators and then go to step 4. If the current generation is the maximum generation, stop.

Step7. Check the performance. If the performance is unsatisfactory, go to step 2 to change structure of controller**.**

Step8. Design structure of $K_1(p_1)$, as show in (21), and specify the reference model, T_{ref}.

$$K_1(p_1) = \frac{1}{K_1 s + 1} \tag{21}$$

Step9. Specify the GA parameters for synthesize $K_1(p_1)$ compare T_{ref} by evaluate minimize integral of square error value (ISE).

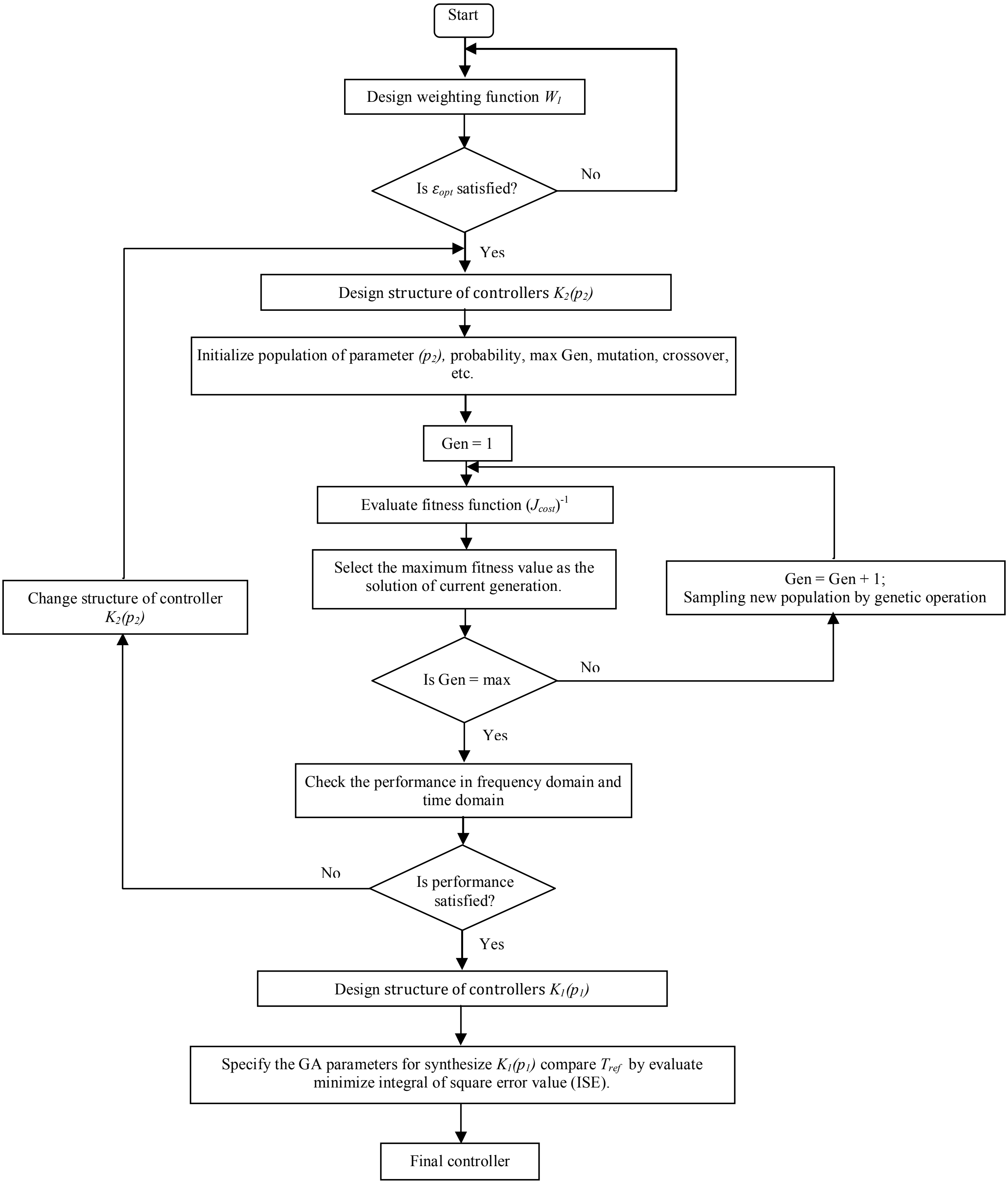

Fig. 4 Flowchart of the proposed design procedure.

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IV. SIMULATION RESULTS

In our study, the ACMC buck converter parameters are given as follows: $R_L = 1.5\Omega$, $V_0 = 10$V, $V_i = 24$V, $L = 100\mu$H, $C = 220\mu$F and $f_{sw} = 100$kHz. The current loop controller is designed by using the technique in [4]. Based on (1) and, the transfer function of plant in the voltage loop can be written as (22). In this paper, both the 2DOF H_∞ loop shaping control and our proposed technique are applied to design the ACMC buck converter controller.

Fig. 5 shows the convergence curve of the solution. As seen in this figure, the optimal stability margin obtained is 0.5834 which is less than the conventional full order2DOF HLS; however, the orders of the proposed controllers are only 1[th] order and 2[nd] order. The step responses from the proposed controllers and the conventional 2DOF H_∞ loop shaping controllers are shown in Fig. 6. As seen in this figure, the response of the proposed controller is almost the same as the response from the full order controller.

$$G = \frac{3.168\text{x}10^{-17}s^5 + 1.804\text{x}10^{-11}s^4 + 9.234\text{x}10^{-7}s^3 + 0.0059s^2 + 46.98s + 1.132\text{x}10^5}{4.356\text{x}10^{-25}s^7 + 5.143\text{x}10^{-20}s^6 + 4.388\text{x}10^{-15}s^5 + 1.725\text{ x}10^{-10}s^{-4} + 1.563\text{x}10^{-6}s^3 + 0.0111s^2 + 44.41s + 5.659\text{x}10^4} \quad (22)$$

First, we designed the pre-compensator weight function by considering the desired loop shaping. In this case, W_1 is selected as (23). Then, we specified the reference model, T_{ref} as (24). The parameter ρ is set as 0.7.

$$W_1 = \frac{1.5s + 9500}{s + 0.001} \quad (23)$$

$$T_{ref} = \frac{1}{1 + 0.25\text{x}10^{-3}s} \quad (24)$$

Thus, the shaped plant can be written as:

$$G_s = W_1 G$$

$$= \frac{1.5s + 9500}{s + 0.001}\,\frac{3.168\text{x}10^{-17}s^5 + 1.804\text{x}10^{-11}s^4 + 9.234\text{x}10^{-7}s^3 + 0.0059s^2 + 46.98s + 1.132\text{x}10^5}{4.356\text{x}10^{-25}s^7 + 5.143\text{x}10^{-20}s^6 + 4.388\text{x}10^{-15}s^5 + 1.725\text{ x}10^{-10}s^{-4} + 1.563\text{x}10^{-6}s^3 + 0.0111s^2 + 44.41s + 5.659\text{x}10^4} \quad (25)$$

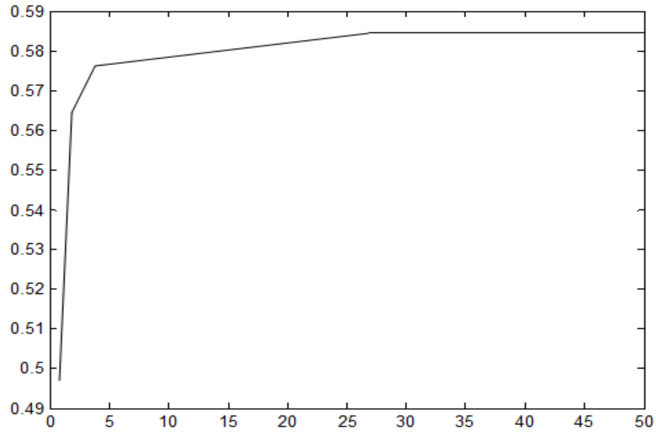

Fig. 5 ε versus generations in GA optimization.

The feed-forward pre-filter, feedback controllers and W_i are synthesized by applying the 2DOF H_∞ loop shaping method. The resulting controllers are shown in (26) and (27). In this case, W_i is found to be at 3.8479 and the stability margin (ε) is found to be at 0.5568 ($\gamma_{opt} = 1.7960$). Both controllers have 9[th] orders. The structures of feed-forward pre-filter and feedback controllers are complicated with high order; thus, it is difficult to implement them on the ACMC buck converter.

$$K_1(p_1) = W_1 K_{1\infty} Wi \qquad (9^{th}\text{ order}) \quad (26)$$

And

$$K_2(p_2) = W_1 K_{2\infty} \qquad (9^{th}\text{ order}) \quad (27)$$

Next, our proposed technique was adopted to design the robust controller. 1[st] order filter and PID controller are adopted as the pre-specified structure of the controller in the proposed design. When running the GA for 50 generations, the optimal controllers, K_1 and K_2, are obtained as (28) and (29). The stability margin (ε) obtained is 0.5834 (γ_{opt} = 1.7141)

$$K_1(p_1) = \frac{1}{1.64\text{x}10^{-4}s + 1} \quad (28)$$

$$K_2(p_2) = 1.1894 + \frac{6.93\text{x}10^3}{s} + \frac{1.5277}{6.0522s + 1} \quad (29)$$

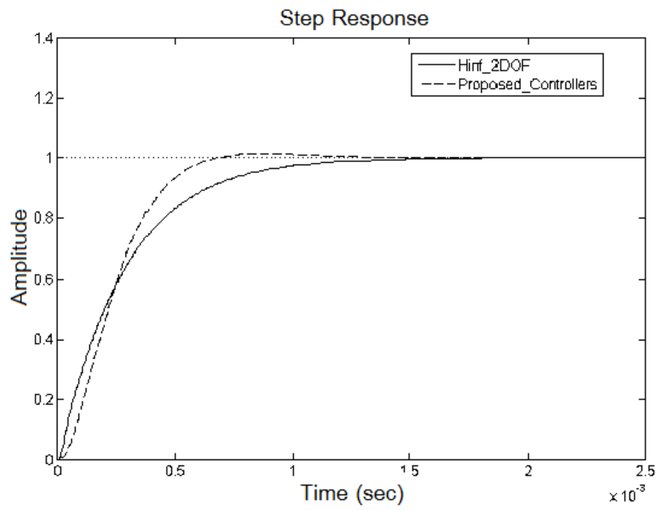

Fig. 6 Step responses of each controller

Table I shows the main performance indexes obtained from the proposed technique and the 2DOF H_∞ loop shaping control technique.

Table I Comparison results of each controller

	Step responses results			
	Rise Time (ms)	Settling time (ms)	Over shoot (%)	Stability margin (ε)
H_inf_2DOF	0.596	1.07	0.05	0.5568
Proposed_Controllers	0.376	0.59	1.25	0.5834

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V. CONCLUSIONS

In this paper, the proposed technique, the fixed structure robust 2DOF controller using Genetic Algorithm method, can be designed for the ACMC buck converter. This technique can overcome the problem of high order of the conventional 2DOF H_∞ loop shaping method. Time domain responses concern the effectiveness of the proposed controller in terms of fast settling time, low maximum overshoot and no steady-state error. As seen in the simulation results, the proposed technique can be applied to design the robust 2DOF controllers for the ACMC buck converter.

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