

Fuzzy Inventory Control System for Uncertain Demand and Supply

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Abstract— A fuzzy inventory control system for a single item continuous control system is proposed in this research. The model can deal with both uncertain demand and availability of supply using fuzzy logic control system. Conventional stochastic inventory model determines only uncertain demand. However, unavailability of supply can happen in many types of manufacturing system. In the proposed Fuzzy Inventory Control (FIC) system, both demand and availability of supply are described by linguistic terms. Then, the developed fuzzy rules are used to extract the fuzzy order quantity and the fuzzy reorder point continuously. The model is more flexible than the conventional approach due to adjustment of both order quantity and reorder point. A time step simulation is used to analyze the results of inventory cost from the FIC system. Inventory costs of both conventional stochastic models and the proposed inventory system of a case study factory are compared based on the same data sets. It found that FIC system can obtain extremely lower cost than the conventional stochastic model.

Index Terms—Fuzzy Logic Control, Inventory Control System, Uncertain Demand and Supply

I. INTRODUCTION

INVENTORY decisions are both high risk and high benefit throughout the supply chain [1]. A shortage or not enough supply can disrupt manufacturing plan or inventory management, overstock on inventory level also barrier to management. The most importance task of inventory management is making trade-off between the minimization of the total cost and maximization of the customer satisfaction. In real cases, these objectives are very difficult to satisfy regarding to the great number of factors involved and unpredictable events such as uncertainties of demand and supply. It is necessary to apply a suitable control system and policy for each type of product [2].

The inventory control defines how often the stock level is reviewed to determine when and how much to order. It is performed on either a continuous or periodic basic types. In a continuous inventory control system, an order is placed for the same constant amount whenever the inventory on hand

decreased to a certain level, whereas in a periodic system, an order is placed for a variable amount after the specific regular time interval [3]. Normally, a continuous system is used for Class A items, which represent a large percentage of the total dollar value of inventory. These inventory levels should be as low as possible, and safety stocks minimized. This requires accurate demand and supply estimations.

Conventional inventory models assume certain or uncertain demand and supply [3]. However, in reality both demand and supply are uncertain due to change of orders, random capacity of suppliers, or unpredictable events. Since some of uncertainties within inventory systems cannot be considered appropriately using concepts of probability theory, fuzzy set theory has been used in modeling of inventory systems since 1980s. Fuzzy set theory, originally introduced by Zadeh [4], provides a framework for considering parameters that are vaguely or unclearly defined or whose values are imprecise or determined based on subjective beliefs of individuals. Some researches applied fuzzy set and fuzzy number to determined uncertainties in demand, order quantity and lead time [5]-[7]. Roy and Maiti [8], [9] solved the classical EOQ problem with a fuzzy goal and fuzzy inventory costs using a fuzzy non-linear programming method. Uncertain supply was determined in using Markov chain [10] and mathematical approach [11]. Both methods determine time to wait before the next order. However, these methods are complicated and difficult to understand.

Another approach to simplify complicated system is to use Fuzzy Logic Control (FLC). FLC has been applied in many applications in industry such as machine control, scheduling and system controls including inventory control [12]-[14]. Yimer and Demirli [14] have presented a fuzzy simulation model using FLC of a single item inventory system with variable demand to evaluate the economic order quantity under uncertain lead time. Babai and Dallery [15] proposed a dynamic inventory control under demand, yield and lead time uncertainties. Kamal and Sculfort [2] presented a fuzzy modeling of inventory control system in uncertain lead time and demand. However, few of these researches consider both uncertainty quantities of demand and supply. Most of them determine uncertain supply by considering time to reorder with the same quantity of economic order. But varying of amount of reorder point has not been in concern. In case of unavailability of supply, increasing of reorder point can be used to protect shortage.

In this paper, FLC is used to treat the uncertainty regarding demand and supply in continuous inventory control system. MATLAB's fuzzy toolbox is used to represent the continuous inventory control system. The demand, supply, order quantity and reorder point are

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described by linguistic terms. The main objective is to evaluate the order quantity and reorder point in each period taking into account the demand and supply uncertainties.

The remainder of this paper is organized as follows. Section 2 is discussed about inventory system. In section 3, the proposed fuzzy control model is presented. Section 4 illustrates numerical examples taken from historical inventory data of a furniture company. Finally, a conclusion is given in the last Section.

II. INVENTORY SYSTEM

A. Inventory management

The purpose of inventory management is to determine the amount of inventory to keep in stock -how much to order and when to replenish. Inventory should be enough to meet customer demand and also be cost effective [13]. There are two basic types of inventory system: a continuous (or fixed order quantity) system and a periodic (or fixed time period) system. A continuous system is normally used with high value items that need to be carefully determined. On the contrary, a periodic system is used with medium to low value items but high volume [4].

In a continuous inventory system, a continual record of inventory level for every item is maintained. Whenever the inventory on hand decreases to a predetermined level, referred to the Reorder point (R), a new order is placed to replenish the stock of inventory. The order that is placed is for a fixed amount that minimizes the total inventory cost. This amount called the Economic Order Quantity (EOQ).

The new continuous inventory control system is proposed in this research.

B. Inventory Costs

Normally, three types of cost are concerned in an inventory model: carrying cost or holding cost, ordering cost and shortage cost.

Carrying cost is the cost of holding items in an inventory. This cost varies with the level of inventory in a stock and occasionally with the length of time an item is held.

Ordering cost is the cost associated with replenishing the stock of inventory being held. This is normally expressed as dollar amount per order or unit. It also depends on order size.

Shortage cost, also referred to stockout, occur when customer demand cannot be met because of insufficient inventory. This shortage may result in a permanent loss of sales. Shortage cost includes loss of profit and loss of goodwill that may become a permanent loss of customers and future sales. Shortage cost has an inverse relationship with carrying cost. As the amount of inventory on hand increases, the carrying cost increases but shortage cost decreases.

C. Inventory Model with Variable Demand

A stochastic inventory model is one of the most fundamental of all inventory models. The importance of the model is that it is still one of the most widely used inventory model in industry, and served as a basis for more sophisticated inventory models. It uses when the uncertainties are treated as randomness and handled by

appealing probability theory. Assuming that, the demand is represented by normal distribution by estimating the average daily demand and its standard deviation.

How much to order can be determined from EOQ that can be calculated by the following equation [2].

$$Q^* = EOQ = \sqrt{\frac{2C_0H\bar{d}(C_h + C_s)}{C_hC_s}}, \quad (1)$$

where

C_o : Ordering cost per time.

C_h : Holding cost per unit per period.

C_s : Shortage cost per unit per period.

\bar{d} : Average weekly demand.

H : Total length of the planning horizon (number of weeks).

The determinant of when to order in a continuous inventory system is the reorder point, the inventory level at which a new order is placed [1]. If demand is uncertain we must add safety stock into the reorder point, the reorder point and the safety stock can be computed by [3]

$$R = \bar{d}L + SS, \quad (2)$$

$$SS = z\sigma_d\sqrt{L}, \quad (3)$$

where

R : Unit of reorder point.

SS : Safety stock.

L : Lead time.

σ_d : The standard deviation of weekly demand.

z : The number of standard deviation corresponding to the service level probability.

III. FUZZY INVENTORY CONTROL MODEL

In the proposed Fuzzy Inventory Control (FIC) model, there are three components; fuzzy inputs, fuzzy outputs and fuzzy rules. Fuzzy logic toolbox of MATLAB is used to construct the FIC model for calculating order quantities and reorder points in any time period. Each element of FIC is shown in fuzzy inference system editor (FIS editor) as shown in Fig.1.

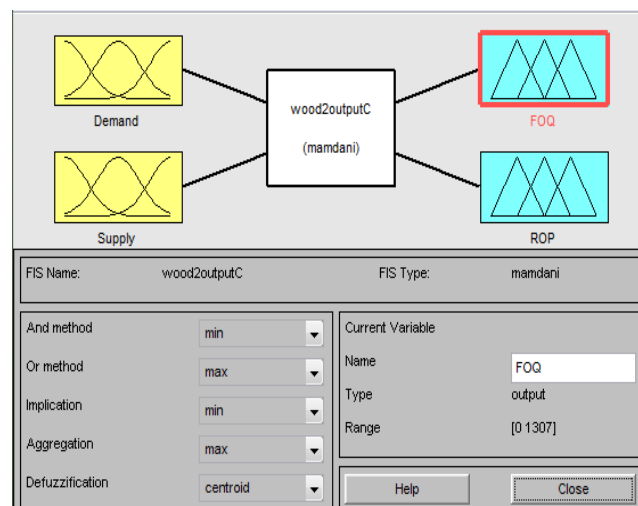


Fig. 1. Fuzzy Inventory Control model in an inference system editor.

Two fuzzy input variables are demand and availability of supply and two output variables are order quantity and reorder point. These variables are represented by linguistic variables. Trapezoidal and triangular functions are used to fuzzify the crisp input and output variables.

A. Fuzzy Inputs

Fuzzy inputs in the fuzzy model are demand and availability of supply which are described by membership functions μ_D and μ_S , respectively. Fuzzy demand and fuzzy availability of supply were determined based on observation and test using normal distributions of historical data. Both of them are assumed to be represented by 3 linguistic values; low, medium, high as shown in Fig. 2 and 3.

The universe of discourse of the demand input space is designed from the real data within the interval $[0, \max(D)]$, where $\max(D)$ is the maximum demand that had been ordered. Demand membership functions in Fig.2 are based on these parameters $(0, \bar{d} - \sigma_d, \bar{d}, \bar{d} + \sigma_d, \max(D))$. The parameters were designed according to characteristics of normal distribution and actual situation of uncertain demand.

Availability of supply is designed based on real data within the interval $[0, \max(S)]$, where $\max(S)$ is maximum availability of supply from the current suppliers of determined planning horizon. Membership functions are shown in Fig.3. The parameters $(0, 0.25\max(S), 0.5\max(S), 0.75\max(S), \max(S))$ are used for supply linguistic values according to normal distribution.

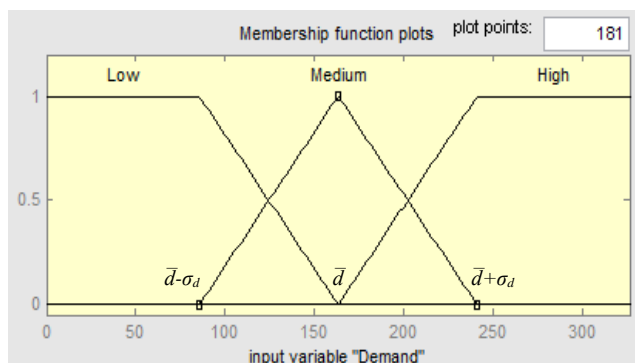


Fig. 2. Demand membership functions.

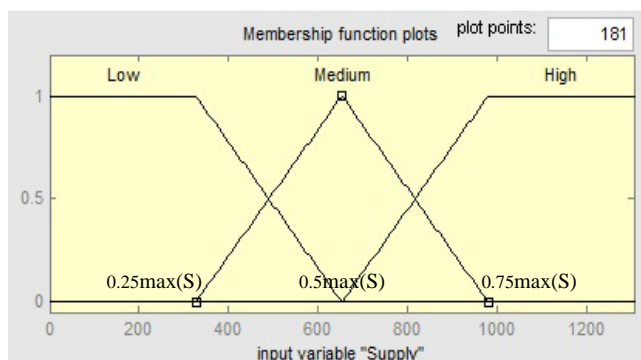


Fig. 3. Availability of supply membership functions.

B. Fuzzy Outputs

Conventional inventory model use fixed value of order quantity and reorder point. However, in real situation of

uncertain demand and availability of supply, fixed values of order quantity and reorder point are not appropriate because materials may not available when they are strongly needed. Moreover, the situation of supply may trend to decrease. Safety of shortage should be carefully determined. So, in the proposed model, two fuzzy outputs are constructed. They are fuzzy order quantity and fuzzy reorder point describe by membership functions μ_Q and μ_R , respectively. Fuzzy order quantity is assumed to be represented by 3 linguistic values; low, medium, high as shown in Fig.4. Reorder point is assumed to be represented by 5 linguistic values; very low, low, medium, high, very high as shown in Fig.5. Linguistic values of order quantity is designed from available of supply because in real situation of uncertain supply order quantity should be in the possible range of supply so the universe of discourse for order quantity output is in the interval $[0, \max(S)]$, where $\max(S)$ is the maximum availability of supply from historical data. The parameters that use to represent 3 linguistic values; low, medium and high are $(0, 0.5\max(S)-R, 0.5\max(S), 0.5\max(S)+R, \max(S))$. The universe of discourse of the reorder point space is the set of real numbers within the interval $[0, 2R]$. Five Linguistic values of reorder point are designed based on reorder point. The parameters $(0, R-SS, R, R+SS, 2R)$ are used.

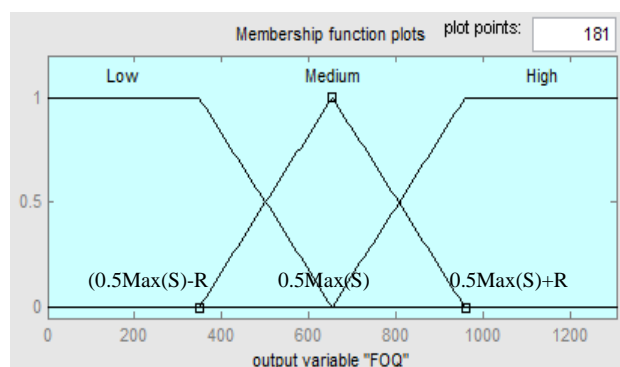


Fig. 4. Order quantity membership functions.

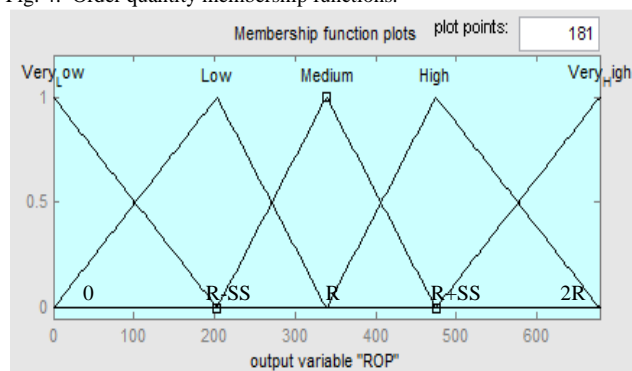


Fig. 5. Reorder point membership functions.

C. Fuzzy Rules

Fuzzy inference type of the proposed system is Mandani. The relationship between demand (x_1), availability of supply (x_2) and order quantity (y_1), reorder point (y_2) are described by the following rules:

- R1: IF (x_1 is 'Low') AND (x_2 is 'Low') THEN (y_1 is 'Medium') AND (y_2 is 'High') ELSE
- R2: IF (x_1 is 'Low') AND (x_2 is 'Medium') THEN (y_1 is 'Low') AND (y_2 is 'Medium') ELSE
- R3: IF (x_1 is 'Low') AND (x_2 is 'High')

THEN (y_1 is 'Medium') AND (y_2 is 'Low') ELSE
 R4: IF (x_1 is 'Medium') AND (x_2 is 'Low')
 THEN (y_1 is 'Low') AND (y_2 is 'Very High') ELSE
 R5: IF (x_1 is 'Medium') AND (x_2 is 'Medium')
 THEN (y_1 is 'Medium') AND (y_2 is 'High') ELSE
 R6: IF (x_1 is 'Medium') AND (x_3 is 'High')
 THEN (y_1 is 'High') AND (y_2 is 'High') ELSE
 R7: IF (x_1 is 'High') AND (x_2 is 'Low')
 THEN (y_1 is 'Medium') AND (y_2 is 'Very High')
 ELSE
 R8: IF (x_1 is 'High') AND (x_2 is 'Medium')
 THEN (y_1 is 'High') AND (y_2 is 'High') ELSE
 R9: IF (x_1 is 'High') AND (x_2 is 'High')
 THEN (y_1 is 'High') AND (y_2 is 'High')

By taking the max-min compositional operation, the fuzzy reasoning of these rules yields a fuzzy output of fuzzy reasoning. These outputs can be expressed as

$$\mu_{Q_0}(y_1) = (\mu_{D_1}(x_1) \wedge \mu_{S_1}(x_2)) \vee \dots (\mu_{D_n}(x_1) \wedge \mu_{S_n}(x_2)) \quad (4)$$

$$\mu_{R_0}(y_2) = (\mu_{D_1}(x_1) \wedge \mu_{S_1}(x_2)) \vee \dots (\mu_{D_n}(x_1) \wedge \mu_{S_n}(x_2)) \quad (5)$$

where \wedge is the minimum operation and \vee is the maximum operation. D_i , S_i , Q_i , and R_i are fuzzy subsets defined by the corresponding membership functions, i.e., $\mu_{D_i}, \mu_{S_i},$

$$\mu_{Q_i}, \mu_{R_i}$$

Finally, a defuzzification method, called the center-of-gravity method [12], is adopted here to transform the fuzzy inference output into a non-fuzzy value order quantity y_{01} , y_{02} .

$$y_{01} = \frac{\sum y_1(\mu_{Q_0}(y_1))}{\sum \mu_{Q_0}(y_1)} \quad (6)$$

$$y_{02} = \frac{\sum y_2(\mu_{R_0}(y_2))}{\sum \mu_{R_0}(y_2)} \quad (7)$$

In this paper, the non fuzzy value of y_{01} , y_{02} are order quantity and reorder point, respectively. These operations can be done in FIC model which is implemented in Fuzzy Logic Tool Box of MATLAB.

IV. NUMERICAL EXAMPLE

Historical inventory data of a furniture company has been investigated. The company faced the problem of both uncertain demand and unavailability of supply in some periods. The main materials of the company are woods which are uncertain because the quantity of woods depends on environment, rainfall and sources of supply. The company is a make-to-stock manufacturer. Demand is uncertain and randomly fluctuated. Both of demand and availability of supply quantities can be represented by normal distribution. Currently high inventory level is used to protect shortage. Service level that the company wants to guarantee with customers is more than 97%. However, shortage is still raised and total inventory cost is also high.

So, the FIC model is proposed to reduce inventory level

and the total inventory cost. Five sets of data are used to compare between the conventional stochastic model and the FIC model at different service levels. An example of a set of demand and availability of supply data based on normal distribution is shown in Fig 6.

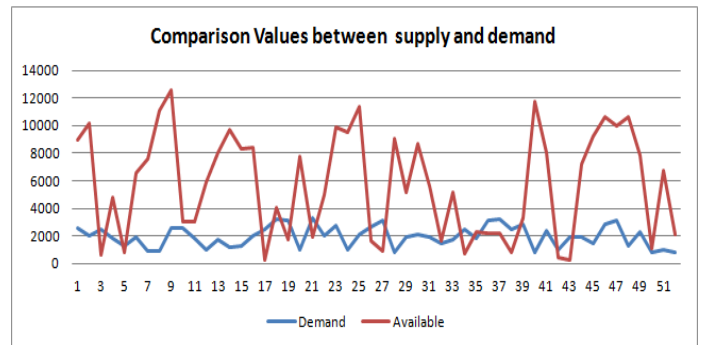


Fig. 6. Quantities of demand and availability of supply in 52 weeks.

From Fig. 6, it can be seen that the availability of materials is extremely fluctuated comparing with demand which may cause unavailability in some period.

In order to test the performance of the proposed FIC system, the conventional stochastic model of EOQ was used to compare. Five data sets that generate from distributions of historical data were generated. The distribution of historical demand is a normal distribution with mean 1,970 units and standard deviation 772 units. Supply is also uncertain with a normal distribution with mean and standard deviation of availability of supply is 5,600 and 3,816 units, respectively. Ordering cost, holding cost and shortage per unit per period of the case study factory are 100 BAHT per order and 0.05 BAHT per unit per period and 59 BAHT per unit per period, respectively.

EOQ and reorder point and safety stock of the stochastic model can be calculated using Eq. (1)-(3). Then, holding cost, ordering cost, shortage cost and total cost of 52 weeks for each data set can be calculated.

Next, a time step simulation procedure for experiment the FIC model in the same environment is analyzed. The simulation operates in loop according to steps described:

1. Generate random number corresponding to the weekly demand and supply based on normal distribution of historical data (use the same data set as conventional stochastic model).
2. Continuous review and update the inventory status balance.
3. If the inventory level is less than the reorder point, a new order quantity and reorder point are generated by fuzzy logic system according to demand and availability of supply at that time.
4. Update the inventory level and repeat from step 2.

The comparison results of costs for 5 data sets at service level 97%, 99% and 99.9% are shown in Fig. 7- Fig. 10.

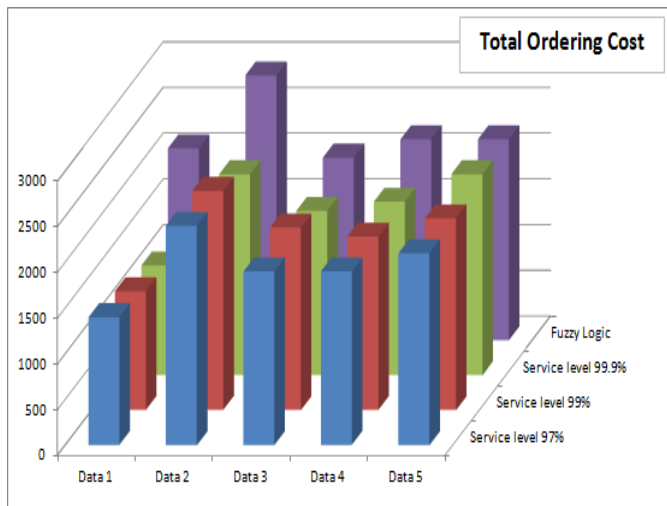


Fig. 7. Comparison of ordering cost of 5 data sets between stochastic EOQ models at service level 97%, 99%, 99.9% and the FIC model.

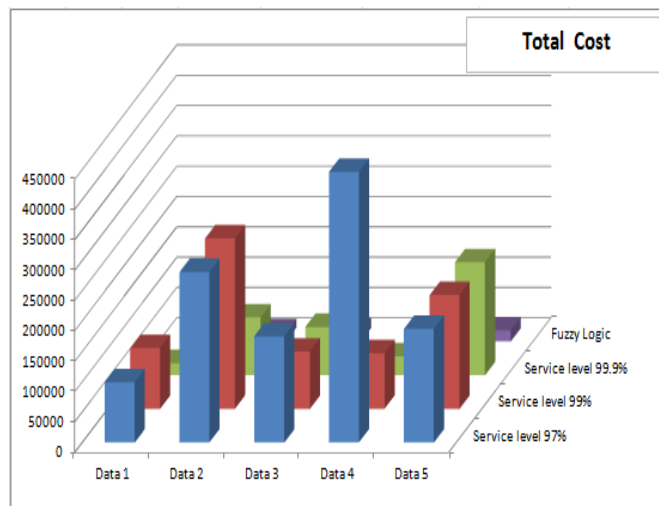


Fig. 10 Comparison of total cost of 5 data sets between stochastic EOQ models at service level 97%, 99%, 99.9% and FIC model.

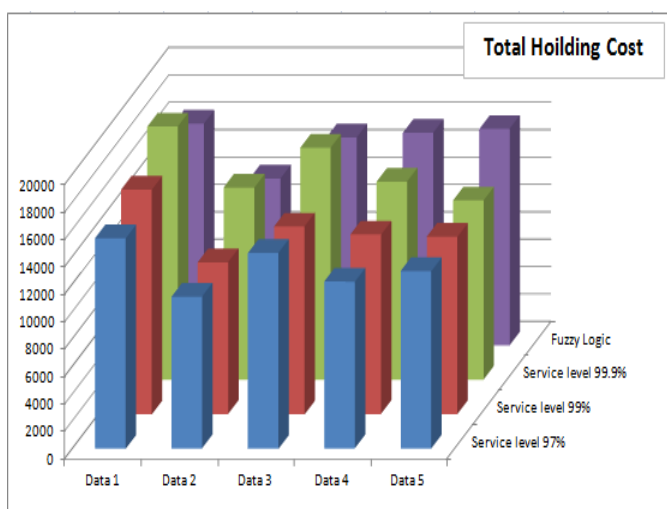


Fig. 8. Comparison of holding cost of 5 data set between stochastic models at service level 97%, 99%, 99.9% and FIC model.

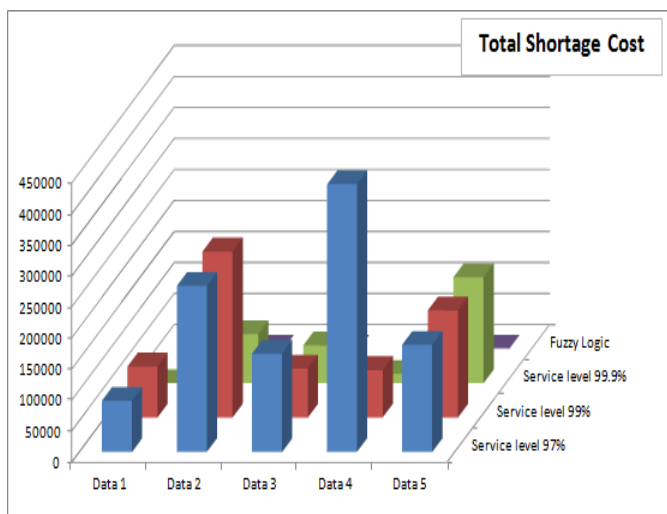


Fig. 9. Comparison of shortage cost of 5 data sets between stochastic EOQ models at service level 97%, 99%, 99.9% and FIC model.

Fig. 7 shown that ordering cost of stochastic models at service level 97%, 99% and 99.9% have lower cost than the FIC model due to less frequency of orders. Fig. 8 shown that holding cost of stochastic models at service level 97%, 99% and 99.9% have lower cost than FIC model. It means that the FIC model has higher average inventory level than stochastic EOQ models. Fig. 9 shown that shortage cost of stochastic models at service level 97%, 99% and 99.9% have higher cost than the FIC model because inventory level of the FIC model is higher than stochastic EOQ models due to consideration of availability of supply. Stochastic models of EOQ cannot handle the extremely fluctuated of availability of supply even the high service levels are considered so shortage cost is raised while the FIC model has no shortage cost. This situation occurs because in the stochastic model of EOQ, availability of supply is assumed to be certain so it cannot react effectively when supply is unavailable. While the FIC model can control both reorder point and order quantity to the appropriate level when they are needed. So, FIC model is more flexibility than stochastic models at any service level. Finally, the FIC model has lower total inventory cost than stochastic models as shown in Fig.10.

Cost saving of each pair of a stochastic model at each service level and FIC model is also calculated as shown in Table I-III. These results confirm that the FIC model is better than the current stochastic models.

TABLE I
COMPARISON OF TOTAL COST OF THE STOCHASTIC EOQ MODEL AT SERVICE LEVEL 97% WITH FIC MODEL

Data set	Stochastic Model	Fuzzy Logic Model	Saving cost
1	฿ 99,182.55	฿ 18,295.90	81.55 %
2	฿ 281,296.20	฿ 15,081.80	94.64 %
3	฿ 174,671.80	฿ 17,183.00	90.16 %
4	฿ 446,035.55	฿ 17,730.60	96.02 %
5	฿ 187,643.25	฿ 18,005.05	90.40 %
Average	฿ 237,765.87	฿ 17,259.27	90.55%

TABLE II

COMPARISON OF TOTAL COST OF THE STOCHASTIC EOQ MODEL AT SERVICE LEVEL 99% WITH FIC MODEL

Data set	Stochastic Model	Fuzzy Logic Model	Saving cost
1	B 100,121.90	B 18,295.90	81.73 %
2	B 281,296.20	B 15,081.80	94.64 %
3	B 94,347.50	B 17,183.00	81.79 %
4	B 91,545.75	B 17,730.60	80.63 %
5	B 187,643.25	B 18,005.05	90.40 %
Average	B 150,990.92	B 17,259.27	85.84%

TABLE III

COMPARISON OF TOTAL COST OF THE STOCHASTIC EOQ MODEL AT SERVICE LEVEL 99.9% WITH FIC MODEL

Data set	Stochastic Model	Fuzzy Logic Model	Saving cost
1	B 19,689.75	B 18,295.90	07.08 %
2	B 94,996.55	B 15,081.80	84.12 %
3	B 79,337.75	B 17,183.00	78.34 %
4	B 31,055.90	B 17,730.60	42.91 %
5	B 185,742.85	B 18,005.05	90.31 %
Average	B 82,164.56	B 17,259.27	60.55 %

The FLC model can averagely save 90.55%, 85.84%, 60.55% when comparing with stochastic models of EOQ at service level 97%, 99% and 99.9% respectively. The saving cost is increased when service level is increased because shortage is reduced. Higher the order quantity and reorder point is better for this case study. However, the FIC model can save more money than all stochastic models as shown in the results.

V. CONCLUSION

In this work a Fuzzy Inventory Control (FIC) model based on continuous inventory control system was developed. Both demand and supply uncertainties were considered in the model. Fuzzy Logic Tool Box of MATLAB was used to implement the model. Demand and availability of supply are inputs and order quantity and reorder point are outputs of the system. Linguistic values were used for both inputs and outputs. Fuzzy Rules were constructed according to the historical experience. Five data sets of inputs were used to evaluate the FIC model comparing with stochastic models at different service levels. The obtained results clearly show that FIC model can extremely save the total inventory cost. Moreover, there was no shortage in any sets of tested data of FLC model which means that it provides customer satisfaction although demand and available of supply are uncertain. The FLC model is more flexible than the stochastic model because both order quantity and reorder point are reevaluated continuously. So, the model can adjust quicker than the stochastic model.

Fuzzy logic model allows a user to modify or readjust parameters easily when the situation has been changed. So, it is possible to extend learning method to the FLC model.

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