Numerical Approximations of Average Run Length for AR(1) on Exponential CUSUM

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Abstract— **We study the Cumulative Sum (CUSUM) procedure when observations are from a first order autoregressive model with exponential white noise. The objective of this paper is to present a numerical integration method for evaluating ARL_0 and ARL_1, where ARL_0 is the average run length when the process is in control and ARL_1 is the average run length when the process is out of control. The integrals in the Integral Equation (IE) method for the CUSUM procedure are approximated by using the Gauss-Legendre rule for numerical integration. The results obtained from the numerical integration method are compared with results obtained from explicit formulae. We have shown that the results obtained from the two methods are in excellent agreement.**

Index Terms— **Cumulative Sum, First order autoregressive, Average Run Length, Exponential distribution, Integral Equation.**

I. Introduction

Statistical Process Control (SPC) is widely used to detect and monitor process changes in many areas such as industrial manufacturing [18], finance and economics [9], computer science and telecommunications [19], [23], epidemiology and surveillance [10], [25], [32] and in other areas of applications. Various SPC charts have been developed and extensively studied, for example, Shewhart [24], Exponentially Weighted Moving Average (EWMA) procedure [22] and Cumulative Sum (CUSUM) chart [21], [14]. A traditional assumption for evaluating the characteristics of SPC charts is that variables are random, independent and identically distributed. However, in practice, observations are not always identically and independently distributed (i.i.d.), for example, in continuous manufacturing processes where most observations are sequentially autocorrelated. Many SPC procedures have been developed to detect changes of mean and dispersion in autocorrelated manufacturing processes (see [1], [2], [4], [13], [16], [20], [23], [29], [30], [31]). However, these authors have used simulations and not analytical methods for determining the ARL. Simulation is commonly used to analyse the characteristics of methods which measure the number of observations that are required in order to decide if a stochastic process has changed from an in-control to an out-of-control state. In the past decade, many approaches have been developed for comparing the performance of SPC charts, for example, the Monte Carlo (MC), IE [7], Markov Chain Approach (MCA) [17] and Martingale approach [26, 27].

In this article, we study the ARLs of the CUSUM procedure when observations are from a first order autoregressive model with exponential white noise. We derive integral equations for the ARLs and then solve the equations numerically using the Gauss-Legendre numerical integration rule. We then compare the results obtained from the numerical integration method with the results from explicit formulae derived in [6].

The outline of the paper is as follows. In sections 2 and 3, we describe the characteristics of the SPC procedures and the properties of the CUSUM chart. In sections 4 and 5, we describe the numerical integral equation approach and show the numerical results. Section 6 contains a discussion and conclusions.

II. The Characteristics of SPC Procedures

In this paper, we discuss the characteristics of SPC procedures based on the assumption that $\xi_1, \xi_2, ..., \xi_n$ are sequentially observed identically distributed independent random variables with an exponential distribution function

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$F(x,\lambda)$. We assume that the parameter λ has the value λ_0 in the in-control state, the value $\lambda \neq \lambda_0$ in an out-of-control state, and that the change occurs at a change-point time $\theta \leq \infty$. We assume that the parameters of the in-control and out-of-control states are known.

A typical method of detecting change points in SPC charts is to define some statistic X_n and a control boundary limit h of X_n such that an alarm signal is given when X_n exceeds h. Typically, a first exit time (τ) over a boundary defined as

$$\tau_h = \inf\{n \geq 0; X_n \geq h\}, \quad (1)$$

is used for the alarm signal.

We define $\mathbb{E}_\theta(\cdot)$ as the expectation under distribution $F(x,\lambda_0)$ that the change-point occurs at time θ from the in-control value λ_0 to an out-of-control value λ. Typical measures for alarm times τ are

$$ARL_0 \approx \mathbb{E}_\infty \tau_h \geq T, \quad (2)$$

where T is given (usually large) and

$$ARL_1 \approx \mathbb{E}_1 \tau_h \leq (\tau | \tau \geq 1), \quad (3)$$

ARL_0 is a measure of the time before a process that is still in-control is signaled as being out-of-control and ARL_1 is a measure of the time before a process that has gone out-of-control is signaled as being out-of-control. The ARL_0 and ARL_1 are two conflicting criteria that must be balanced in control charts.

III. THE CUSUM PROCEDURE

The CUSUM procedure was first proposed by [21] and it has been found to be an effective method for detecting small changes. Its properties have been investigated by many authors (see [3], [5], [14], [28]).

This procedure is designed to detect an increase in the mean of an independent and identically distributed (i.i.d.) observed sequence of random variables $\xi_1, \xi_2, ..., \xi_n$. The statistics X_n satisfies the following recursive equation as

$$X_n = (X_{n-1} + \xi_n - a)^+, \quad n = 1,2,..., \quad X_0 = x, \quad (4)$$

where X_n is the CUSUM value of a statistic after n observations, x is an initial value for X_n, $y^+ = \max(0, y)$ and a is a constant. Mazalov and Zhuravlev [19] and George et al. [11] have discussed many cases which lead to this recursive representation. Various modifications of CUSUM algorithms have been given in the literature [14].

In this paper, we consider CUSUM procedures for the case where observations are from a first order autoregressive model with exponential white noise and define:

$$X_n = X_{n-1} + Z_n - a, \qquad n = 1,2,\dots, X_0 = x, \quad (5)$$

where

$$Z_n = \varphi Z_{n-1} + \xi_n, \; -1 < \varphi < 1 \text{ and } \xi_n \sim \exp(\lambda). \quad (6)$$

IV. THE APPROACH FOR EVALUATION OF AVERAGE RUN LENGTH

In this section, we first present the explicit formulae discovered by [6] for ARL and then propose a numerical integral equation approach based on the Gauss-Legendre rule.

A. Explicit Formulae

Busaba et al [6] obtained explicit formulae for the ARL for the CUSUM procedure for a first order autoregressive model with exponential white noise. They used an integral equation approach and derived a Fredholm integral equation of the second type for the ARL_0 and ARL_1. The explicit formulae obtained by solving the integral equations are as follows:

$$ARL_0 = j_0(x) = \left(1 + e^{h+(a-\varphi Z_0)} - h\right)e^h - e^x, \quad x \geq 0 \quad (7)$$

and

$$ARL_1 = j_1(x) = \left(1 + e^{\lambda h + \lambda(a-\varphi Z_0)} - \lambda h\right)e^{\lambda h} - e^{\lambda x}, \quad x \geq 0 \quad (8)$$

where λ is a parameter of the exponential distribution, φ is a smoothing parameter, Z_0 is an initial value of AR(1), h is boundary value and a is reference value.

B. Numerical Integral Equation Approach

This approach was first studied by [8] for approximating the ARL of a Gaussian distribution. He derived and used a Fredholm Integral Equation of the second type. Later, Champ and Rigdon [7] applied this approach to evaluate the ARL for both the CUSUM and EWMA procedures and compared the results with the results obtained from Monte Carlo simulation.

In this paper, we apply the approach to the CUSUM procedure for an AR(1) process. We assume that the system

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is in-control at time n if the CUSUM statistic X_n is in the range $H_L \le X_n \le H_U$ and out-of-control if $X_n > H_U$ or $X_n < H_L$, where H_L is a constant lower bound, $(H_L = 0)$ and H_U is a constant upper bound $(H_U = h)$. We also assume that the system is initially in an in-control state x, i.e. $X_0 = x$ and $0 \le x \le h$. We now define a function $j^{IE}(x)$ as follows:

$$\begin{aligned} j^{IE}(x) &= E_x \tau_h < \infty \\ &= 1 + E_x[I\{0 < X_1 < h\} j(X_1)] + P_x\{X_1 = 0\} j(0) \\ &= 1 + \int_0^h j(y) f(y + a_1 - x) dy + F(a - x) j(0) \qquad (9) \end{aligned}$$

where τ_h is the first exit time defined in (1). Then $j^{IE}(x)$ is the ARL for initial value x.

We now present a numerical scheme for evaluating solutions of the integral equations (9) for the CUSUM procedure which can be written as follows:

$$j^{IE}(x) = 1 + j(0) F(a - x - \varphi Z_0) + \int_0^h j(y) f(a - x - \varphi Z_0 + y) dy, \qquad (10)$$

where $F(x) = 1 - e^{-\lambda x}$ and $f(x) = \frac{dF(x)}{dx} = \lambda e^{-\lambda x}$.

For a given quadrature rule for integrals on $[0, h]$, the integral equation can be approximated by

$$j(a_i) \approx 1 + j(a_1) F(a - a_i - \varphi Z_0) + \sum_{k=1}^{m} w_k j(a_k) f(a_k + a - a_i - \varphi Z_0), \quad i = 1, 2, ..., m. \qquad (11)$$

Without loss of generality, we can approximate the integral by a sum of areas of rectangles with bases h/m with heights chosen as the values of $f(a_k)$ at the midpoints of intervals of length h/m beginning at zero, i.e. on the interval $[0, h]$ with the division points $0 \le a_1 \le a_2 \le ... \le a_m \le h$ and weights w_k. Then, we obtain

$$\int_0^h j(y) dy \approx \sum_{k=1}^{m} w_k f(a_k),$$

where $a_k = \frac{h}{m}\left(k - \frac{1}{2}\right), \quad k = 1, 2, ..., m.$

Equation (11) is a system of m linear equations in the m unknowns $j(a_1), j(a_2), ..., j(a_m)$, and it can be written in matrix form as

$$\begin{aligned} J_{m\times1} &= 1_{m\times1} + R_{m\times m} J_{m\times1} \\ (I_m - R_{m\times m}) J_{m\times1} &= 1_{m\times1} \qquad (12) \end{aligned}$$

where

$$J_{m\times1} = \begin{pmatrix} j(a_1) \\ j(a_2) \\ \vdots \\ j(a_m) \end{pmatrix}, \quad 1_{m\times1} = \begin{pmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{pmatrix}$$

$$R_{m\times m} = \begin{pmatrix} F(a - a_1 - \varphi Z_0) + w_1 f(a) & w_2 f(a_2 + a - a_1 - \varphi Z_0) & \cdots & w_m f(a_m + a - a_1 - \varphi Z_0) \\ F(a - a_1 - \varphi Z_0) + w_1 f(a_1 + a - a_2 - \varphi Z_0) & w_2 f(a) & \cdots & w_m f(a_m + a - a_2 - \varphi Z_0) \\ \vdots & \vdots & \ddots & \vdots \\ F(a - a_m - \varphi Z_0) + w_1 f(a_1 + a - a_m - \varphi Z_0) & w_2 f(a_2 + a - a_m - \varphi Z_0) & \cdots & w_m f(a) \end{pmatrix}$$

and $I_m = diag(1, 1, ..., 1)$ is the unit matrix of order m. If there exists $(I_m - R_{m\times m})^{-1}$, then the solution of the matrix equation (12) is as follows:

$$J_{m\times1} = (I_m - R_{m\times m})^{-1} 1_{m\times1}. \qquad (13)$$

To solve this set of equations for the approximate values of $j(a_1), j(a_2), ..., j(a_m)$, we may approximate the function $j^{IE}(x)$ as

$$j^{IE}(x) \approx 1 + j(a_1) F(a - x - \varphi Z_0) + \sum_{k=1}^{m} w_k j(a_k) f(a_k + a - a_i - \varphi Z_0) \qquad (14)$$

with $w_k = \frac{h}{m}$ and $a_k = \frac{h}{m}\left(k - \frac{1}{2}\right)$.

V. Comparison of results

Tables 1 and 2 show a comparison of the approximate values $j^{IE}(x)$ of the ARL obtained from the integral equations with the exact solutions $j_0(x)$ obtained from the explicit formulae for both negative and positive values of the AR(1)s parameter φ.

Table 1: Comparisons of values ARL_0 of $j_0(x)$ from explicit formulas with numerical approximations $j^{IE}(x)$ for $Z_0 = 1$ and φ negative.

h	a		$\varphi=-0.5$		$\varphi=-0.3$	
			$x=1$	$x=3$	$x=1$	$x=3$
3	2	$j_0(x)$	201.803	184.435	157.447	140.080
		$j^{IE}(x)$	201.252	183.937	157.031	139.716
			758.336[1]	704.017	731.364	739.710
	2.5	$j_0(x)$	360.539	343.172	287.410	270.043
		$j^{IE}(x)$	359.509	342.194	286.601	269.286
			742.097	702.832	712.987	711.707
4	2	$j_0(x)$	498.629	481.262	378.059	360.692
		$j^{IE}(x)$	496.785	479.487	376.714	359.416
			772.298	704.204	726.981	716.575
	2.5	$j_0(x)$	930.120	912.753	731.335	713.967
		$j^{IE}(x)$	926.493	909.195	728.529	711.231
			738.01	700.756	733.173	751.457
	3	$j_0(x)$	1641.530	1624.160	1313.790	1296.420
		$j^{IE}(x)$	1634.960	1617.660	1308.570	1291.280
			705.967	705.592	775.808	717.105
5	2	$j_0(x)$	1211.670	1194.300	883.929	866.562
		$j^{IE}(x)$	1205.630	1188.35	879.719	862.438
			706.154	705.203	715.468	721.894
	3	$j_0(x)$	4318.400	4301.030	3427.500	3410.130
		$j^{IE}(x)$	4294.960	4277.680	3409.050	3391.770
			704.891	706.419	723.814	719.773
	4	$j_0(x)$	12763.40	12746.00	10341.60	10324.30
		$j^{IE}(x)$	12692.60	12675.40	10284.50	10267.20
			704.469	705.234	739.195	750.755

[1] CPU time used

Table 2: Comparisons of values ARL_0 of $j_0(x)$ from explicit formulas with numerical approximations $j^{IE}(x)$ for $Z_0 = 1$ and φ positive.

h	a		$\varphi=0.3$		$\varphi=0.5$	
			$x=1$	$x=3$	$x=1$	$x=3$
3	2	$j_0(x)$	67.058	49.691	47.128	29.761
		$j^{IE}(x)$	66.915	49.670	47.045	29.730
			750.739	751.654	823.623	807.602
	2.5	$j_0(x)$	138.383	121.016	105.524	88.157
		$j^{IE}(x)$	138.024	120.016	105.265	87.950
			726.945	757.416	796.385	818.350
4	2	$j_0(x)$	132.355	114.987	78.179	60.812
		$j^{IE}(x)$	132.025	114.727	78.074	60.776
			736.840	812.609	728.368	733.126
	2.5	$j_0(x)$	326.236	308.869	236.916	219.549
		$j^{IE}(x)$	325.105	308.846	236.154	218.856
			739.928	750.356	741.628	744.515
	3	$j_0(x)$	645.893	628.526	498.629	481.262
		$j^{IE}(x)$	643.440	626.142	496.785	479.780
			776.526	785.418	941.606	763.859
5	2	$j_0(x)$	216.035	198.668	68.771	51.404
		$j^{IE}(x)$	215.564	198.283	69.124	51.843
			788.242	710.912	779.397	782.751
	3	$j_0(x)$	1611.980	1594.610	1211.670	1194.300
		$j^{IE}(x)$	1603.690	1586.410	1205.630	1188.350
			736.949	798.195	721.255	722.440
	4	$j_0(x)$	5406.540	5389.170	4318.400	4301.030
		$j^{IE}(x)$	5377.010	5359.730	4294.960	4277.680
			843.388	871.219	726.263	711.099

It can be seen from Tables 1 to 4, which the analytical explicit solutions are in good agreement with the results obtained from the numerical integral equation approach with 500 nodes in the integration rule. The computational times of the numerical integral equation approach take approximately 15 minutes while the results obtained from the explicit formula take less than 1 second which is much less than the former.

Table 3: Comparisons of values ARL_1 of $j_1(x)$ from explicit formulas with numerical approximations $j^{IE}(x)$ for $Z_0 = 1, \lambda = 2$ and φ negative.

h	a		$\varphi=-0.5$		$\varphi=-0.3$	
			$x=1$	$x=3$	$x=1$	$x=3$
3	2	$j_1(x)$	11.753	8.920	10.264	7.432
		$j^{IE}(x)$	11.742	8.913	10.256	7.427
			740.942	745.279	766.870	768.726
	2.5	$j_1(x)$	16.196	13.363	14.284	11.452
		$j^{IE}(x)$	16.178	13.350	14.270	11.441
			740.194	739.179	754.686	752.439
4	2	$j_1(x)$	16.753	13.920	14.298	11.465
		$j^{IE}(x)$	16.734	13.907	14.284	11.457
			742.455	745.794	760.162	761.768
	2.5	$j_1(x)$	24.077	21.245	20.926	18.093
		$j^{IE}(x)$	24.044	21.217	20.899	18.072
			740.818	739.227	750.677	753.172
	3	$j_1(x)$	33.483	30.650	29.437	26.604
		$j^{IE}(x)$	33.431	30.604	29.393	26.565
			742.721	744.752	761.675	769.959
5	2	$j_1(x)$	22.599	19.766	18.552	15.720
		$j^{IE}(x)$	22.572	19.764	18.536	15.710
			742.222	743.922	746.512	751.129
	3	$j_1(x)$	50.183	47.350	43.512	40.679
		$j^{IE}(x)$	50.088	47.262	43.433	40.607
			744.983	743.532	757.572	801.813
	4	$j_1(x)$	95.662	92.829	84.663	81.830
		$j^{IE}(x)$	95.452	92.626	84.481	81.655
			743.860	763.422	763.328	760.832

Table 4: Comparisons of values ARL_1 of $j_1(x)$ from explicit formulas with numerical approximations $j^{IE}(x)$ for $Z_0 = 1, \lambda = 2$ and φ positive.

h	a		$\varphi=0.3$		$\varphi=0.5$	
			$x=1$	$x=3$	$x=1$	$x=3$
3	2	$j_1(x)$	6.596	3.763	5.598	2.765
		$j^{IE}(x)$	6.593	3.7464	5.597	2.768
			809.333	776.635	776.713	778.242
	2.5	$j_1(x)$	9.574	6.741	8.293	5.465
		$j^{IE}(x)$	9.567	6.738	8.287	5.458
			739.632	743.673	739.227	740.635
4	2	$j_1(x)$	8.250	5.417	6.605	3.772
		$j^{IE}(x)$	8.248	5.421	6.606	3.779
			881.608	879.081	862.904	867.459
	2.5	$j_1(x)$	13.160	10.328	11.048	8.215
		$j^{IE}(x)$	13.149	10.321	11.040	8.213
			796.993	773.781	747.635	752.091
	3	$j_1(x)$	19.465	16.632	16.753	13.920
		$j^{IE}(x)$	19.441	16.613	16.734	13.906
			761.207	756.636	763.312	768.524
5	2	$j_1(x)$	8.588	5.743	5.868	3.035
		$j^{IE}(x)$	8.589	5.763	5.884	3.058
			755.091	758.664	753.641	754.203
	3	$j_1(x)$	27.071	24.238	22.599	19.766
		$j^{IE}(x)$	27.033	24.207	22.572	19.747
			753.282	783.609	755.341	755.201
	4	$j_1(x)$	57.556	54.723	50.183	47.350
		$j^{IE}(x)$	54.442	54.616	50.088	47.262
			840.331	846.633	756.340	753.501

We also compare the values of $j_0(x)$ and $j_1(x)$ obtained from the explicit formulae and the numerical approximations for varying values of the parameter λ. We assume that $h = 3$, $a = 2$ and the parameter of AR(1), $(\varphi = 0.3, -0.3)$. We found that the numerical results obtained from the IE approach have similar accuracy to the results obtained from the explicit formulae.

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Table 5: Comparisons of values $j_0(x)$ and $j_1(x)$ from explicit formulas with numerical approximations $j^{IE}(x)$ for $h = 3, a = 2, \varphi = -0.3.$

λ	$x=1$		$x=3$	
	$j(x)$	$j^{IE}(x)$	$j(x)$	$j^{IE}(x)$
1.00	157.447	157.031	140.080	139.716
1.01	148.987	148.599	132.181	131.843
1.05	120.724	120.426	105.904	105.648
1.07	109.318	109.056	95.358	95.135
1.10	94.846	94.628	82.037	81.853
1.20	62.245	62.119	52.364	52.262
2	10.265	10.256	7.432	7.427
2.5	6.175	6.172	4.347	4.356
3	4.456	4.454	3.133	3.133
3.5	3.552	3.551	2.526	2.526
4	3.007	3.007	2.174	2.174

Table 6: Comparisons of values $j_0(x)$ and $j_1(x)$ from explicit formulas with numerical approximations, $j^{IE}(x)$ for $h = 3, a = 2, \varphi = 0.3.$

λ	$x=1$		$x=3$	
	$j(x)$	$j^{IE}(x)$	$j(x)$	$j^{IE}(x)$
1.00	67.058	66.915	49.691	49.600
1.01	63.840	63.707	47.034	46.951
1.05	52.971	52.869	38.151	38.091
1.07	48.525	48.434	34.565	34.513
1.10	42.823	42.747	30.014	29.973
1.20	29.658	29.614	19.777	19.757
2	6.596	6.593	8.250	8.248
2.5	4.398	4.398	2.569	2.569
3	3.395	3.394	2.072	2.073
3.5	2.836	2.836	1.810	1.810
4	2.483	2.483	1.650	1.650

VI. CONCLUSIONS

The ARL_0 and ARL_1 for the CUSUM procedure when observations are AR(1) with exponential white noise have been evaluated by two methods based on the integral equation approach. In one method, the integral equations have been solved by numerical methods. In the second method, explicit formulas have been obtained for the solutions. We have shown that the results obtained from the two methods are in excellent agreement.

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