Comparison of Anisotropy of Human Mandible, Human Femora and Human Tibia with Canine Mandible and Canine Femora and With Bovine Femurs

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ABSTRACT - The norm of elastic constant tensor and the norms of the irreducible parts of the elastic constants of human mandible, human femora and human tibia and canine mandible and canine femora and bovine femur , bovine femur haversian and bovine femur plexiform are calculated. The relation of the scalar parts norms and the other parts norms and the anisotropy of these types of bones are presented. The norm ratios are used to study anisotropy of these types of bones.

Index Terms: Mandible; Femora; Femur; Tibia; Anisotropy; Elastic Constants.

I. INTRODUCTION

The decomposition procedure and the decomposition of elastic constant tensor is given in [1] and in the appendix, also the definition of norm concept and the norm ratios and the relationship between the anisotropy and the norm ratios are given in [1] and in the appendix. As the ratio becomes close to one the material becomes more isotropic, and as the ratios and become close to one the material becomes more anisotropic as explained in [1] and in the appendix.

II. CALCULATIONS

By using table 1, and the decomposition of the elastic constant tensor, we have calculated the norms and the norm ratios as shown in table2.

Table 1, Elastic Constants (GPa), [2, 3 4, 5, and 6]

Bone	c_{11}	c_{22}	c_{33}	c_{44}	c_{55}	c_{66}	c_{12}	c_{13}	c_{23}
Human Mandible	18.0	20.2	27.6	6.23	5.61	4.52	9.98	10.1	10.7
Human Femora	19.0	22.2	29.7	6.67	5.67	4.67	9.73	11.9	11.9
Human Tibia	11.6	14.4	22.5	4.91	3.56	2.41	7.95	6.10	6.92
Canine Mandible	15.9	18.8	27.1	4.63	4.12	3.81	8.33	9.79	9.79
Canine Femora	16.2	17.1	15.9	2.51	2.73	2.72	10.9	11.5	11.5
Bovine Femur	14.1	18.4	25.0	7.00	6.30	5.28	6.34	4.84	6.94
Bovine Femur Haversian	21.2	21.0	29.0	6.30	6.30	5.40	11.7	12.7	11.1
Bovine Femur Plexiform	22.4	25.0	35.0	8.20	7.10	6.10	14.0	15.8	13.6

Table 2, the norms and norm ratios

Bone	N_s	N_d	N_n	N	$\frac{N_s}{N}$	$\frac{N_d}{N}$	$\frac{N_n}{N}$
Human Mandible	46.347	7.115	0.902	46.8989	0.9882	0.1517	0.0192
Human Femora	49.968	8.351	1.076	50.6726	0.9861	0.1648	0.0212
Human Tibia	32.952	7.904	2.290	33.9637	0.9702	0.2327	0.0674
Canine Mandible	42.280	8.135	2.889	43.1522	0.9798	0.1885	0.0669
Canine Femora	39.951	0.848	0.544	39.9633	0.9997	0.0212	0.0136
Bovine Femur	37.666	7.804	1.305	38.4876	0.9786	0.2028	0.0339
Bovine Femur Haversian	51.382	6.308	1.097	51.7795	0.9923	0.1218	0.0212
Bovine Femur Plexiform	60.841	9.442	1.539	61.5886	0.9879	0.1533	0.0250

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III. CONCLUSION

We can conclude from table 2, by considering the ratio $\frac{N_s}{N}$ that the bone (Canine Femora) is the most isotropic one and the least anisotropic material because it has the smallest ratios of $\frac{N_d}{N}$ and $\frac{N_n}{N}$ among these types of bones, and the bone (Human Tibia) is the least isotropic one because it has the smallest ratio of $\frac{N_s}{N}$ and the most anisotropic material because it has the highest ratios of $\frac{N_d}{N}$ and $\frac{N_n}{N}$ among these types of bones, and for Human bones mentioned above the most isotropic one is (Human Mandible) and most anisotropic is (Human Tibia) and for Canine bones mentioned above the most isotropic one is (Canine Femora) and most anisotropic is (Canine Mandible) and for Bovine bones mentioned above the most isotropic one is (Bovine Femur Haversian) and most anisotropic is (Bovine Femur), and by considering the value of N we found that the highest value is in the case of the bone (Bovine Femur Plexiform) so we can say that the bone (Bovine Femur Plexiform) elastically is the strongest, and the lowest value of N is in the case of the bone (Human Tibia), so we can say that this type of bone is elastically the least strong.

APPENDIX

1. ELASTIC CONSTANT TENSOR DECOMPOSITION

The constitutive relation characterizing linear anisotropic solids is the generalized Hook's law [7]:

$$\sigma_{ij} = C_{ijkl}\varepsilon_{kl}\,,\ \varepsilon_{ij} = S_{ijkl}\sigma_{kl} \qquad (1)$$

Where σ_{ij} and ε_{kl} are the symmetric second rank stress and strain tensors, respectively C_{ijkl} is the fourth-rank elastic stiffness tensor (here after we call it elastic constant tensor) and S_{ijkl} is the elastic compliance tensor. There are three index symmetry restrictions on these tensors. These conditions are:

$$C_{ijkl} = C_{jikl}\,,\ C_{ijkl} = C_{ijlk}\,,\ C_{ijkl} = C_{klij} \qquad (2)$$

Which the first equality comes from the symmetry of stress tensor, the second one from the symmetry of strain tensor, and the third one is due to the presence of a deformation potential. In general, a fourth-rank tensor has 81 elements. The index symmetry conditions (2) reduce this number to 81. Consequently, for most asymmetric materials (triclinic symmetry) the elastic constant tensor has 21 independent components. Elastic compliance tensor S_{ijkl} possesses the same symmetry properties as the elastic constant tensor C_{ijkl} and their connection is given by [8]:

$$C_{ijkl}S_{klmn} = \frac{1}{2}\left(\delta_{im}\delta_{jn} + \delta_{in}\delta_{jm}\right) \qquad (3)$$

Where δ_{ij} is the Kronecker delta. The Einstein summation convention over repeated indices is used and indices run from 1 to 3 unless otherwise stated [9].

By applying the symmetry conditions (2) to the decomposition results obtained for a general fourth-rank tensor, the following reduction spectrum for the elastic constant tensor is obtained. It contains two scalars, two deviators, and one-nonor parts:

$$C_{ijkl} = C_{ijkl}^{(0;1)} + C_{ijkl}^{(0;2)} + C_{ijkl}^{(2;1)} + C_{ijkl}^{(2;2)} + C_{ijkl}^{(4;1)} \qquad (4)$$

Where:

$$C_{ijkl}^{(0;1)} = \frac{1}{9}\delta_{ij}\delta_{kl}C_{ppqq}\,, \qquad (5)$$

$$C_{ijkl}^{(0;2)} = \frac{1}{90}\left(3\delta_{ik}\delta_{jl} + 3\delta_{il}\delta_{jk} - 2\delta_{ij}\delta_{kl}\right)\left(3C_{pqpq} - C_{ppqq}\right) \qquad (6)$$

$$C_{ijkl}^{(2;1)} = \frac{1}{5}\left(\delta_{ik}C_{jplp} + \delta_{jk}C_{iplp} + \delta_{il}C_{jpkp} + \delta_{jl}C_{ipkp}\right) - \frac{2}{15}\left(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}\right)C_{pqpq} \qquad (7)$$

$$C_{ijkl}^{(2;2)} = \frac{1}{7}\delta_{ij}\left(5C_{klpp} - 4C_{kplp}\right) + \frac{1}{7}\delta_{kl}\left(5C_{ijpp} - 4C_{ipjp}\right) - \frac{2}{35}\delta_{ik}\left(5C_{jlpp} - 4C_{jplp}\right) - \frac{2}{35}\delta_{jl}\left(5C_{ikpp} - 4C_{ipkp}\right) - \frac{2}{35}\delta_{il}\left(5C_{jkpp} - 4C_{iplp}\right) - \frac{2}{35}\delta_{jk}\left(5C_{ilpp} - 4C_{iplp}\right) + \frac{2}{105}\left(2\delta_{jk}\delta_{il} + 2\delta_{ik}\delta_{jl} - 5\delta_{ij}\delta_{kl}\right)\left(5C_{ppqq} - 4C_{pqpq}\right) \qquad (8)$$

$$C_{ijkl}^{(4;1)} = \frac{1}{3}(C_{ijkl} + C_{ikjl} + C_{iljk}) - \frac{1}{21}\left[\delta_{ij}\left(C_{klpp} + 2C_{kplp}\right) + \delta_{ik}\left(C_{jlpp} + 2C_{jplp}\right) + \delta_{il}\left(C_{jkpp} + 2C_{jpkp}\right) + \delta_{jk}\left(C_{ilpp} + 2C_{iplp}\right)\right.$$

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$$+\delta_{jl}\left(C_{ikpp}+2C_{ipkp}\right)$$
$$+\delta_{kl}\left(C_{ijpp}+2C_{ipjp}\right)+$$
$$\frac{1}{105}\left[\left(\delta_{ij}\delta_{kl}+\delta_{ik}\delta_{jl}+\delta_{il}\delta_{jk}\right)\left(C_{ppqq}+2C_{pqpq}\right)\right] \quad (9)$$

These parts are orthonormal to each other. Using Voigt's notation [7] for C_{ijkl}, can be expressed in 6 by 6 reduced matrix notation, where the matrix coefficients $c_{\mu\lambda}$ are connected with the tensor components C_{ijkl} by the recalculation rules:

$$c_{\mu\lambda}=C_{ijkl}\,; \qquad (ij\leftrightarrow\mu=1,....,6,kl\leftrightarrow\lambda=1,....,6)$$

That is:

$$11\leftrightarrow1,22\leftrightarrow2,33\leftrightarrow3,23=32\leftrightarrow4$$
$$31=13\leftrightarrow5,12=21\leftrightarrow6.$$

2. THE NORM CONCEPT

Generalizing the concept of the modulus of a vector, norm of a Cartesian tensor (or the modulus of a tensor) is defined as the square root of the contracted product over all indices with itself:

$$N=\|T\|=\left\{T_{ijkl..........}.T_{ijkl..........}\right\}^{1/2}$$

Denoting rank-n Cartesian $T_{ijkl..........}$, by T_n, the square of the norm is expressed as [10]:

$$N^2=\|T\|^2=\sum_{j,q}\left\|T^{(j;q)}\right\|^2=\sum_{(n)}T_{(n)}T_{(n)}=\sum_{(n),j,q}T_{(n)}^{(j;q)}T_{(n)}^{(j,q)}$$

This definition is consistent with the reduction of the tensor in tensor in Cartesian formulation when all the irreducible parts are embedded in the original rank-n tensor space.

Since the norm of a Cartesian tensor is an invariant quantity, we suggest the following:

Rule1. The norm of a Cartesian tensor may be used as a criterion for representing and comparing the overall effect of a certain property of the same or different symmetry. The larger the norm value, the more effective the property is.

It is known that the anisotropy of the materials, i.e., the symmetry group of the material and the anisotropy of the measured property depicted in the same materials may be quite different. Obviously, the property, tensor must show, at least, the symmetry of the material. For example, a property, which is measured in a material, can almost be isotropic but the material symmetry group itself may have very few symmetry elements. We know that, for isotropic materials, the elastic compliance tensor has two irreducible parts, i.e., two scalar parts, so the norm of the elastic compliance tensor for isotropic materials depends only on the norm of the scalar parts, i.e. $N=N_s$, Hence, the ratio $\frac{N_s}{N}=1$ for isotropic materials. For anisotropic materials, the elastic constant tensor additionally contains two deviator parts and one nonor part, so we can define $\frac{N_d}{N}$ for the deviator irreducible parts and $\frac{N_n}{N}$ for nonor parts. Generalizing this to irreducible tensors up to rank four, we can define the following norm ratios: $\frac{N_s}{N}$ for scalar parts, $\frac{N_v}{N}$ for vector parts, $\frac{N_d}{N}$ for deviator parts, $\frac{N_{sc}}{N}$ for septor parts, and $\frac{N_n}{N}$ for nonor parts. Norm ratios of different irreducible parts represent the anisotropy of that particular irreducible part they can also be used to

asses the anisotropy degree of a material property as a whole, we suggest the following two more rules:

Rule 2. When N_s is dominating among norms of irreducible parts: the closer the norm ratio $\frac{N_s}{N}$ is to one, the closer the material property is isotropic.

Rule3. When N_s is not dominating or not present, norms of the other irreducible parts can be used as a criterion. But in this case the situation is reverse; the

larger the norm ratio value we have, the more anisotropic the material property is.

The square of the norm of the elastic stiffness tensor (elastic constant tensor) $\boldsymbol{C_{mn}}$ is:

$$\|N\|^2=\sum_{mn}\left(C_{mn}^{(0;1)}\right)^2+\sum_{mn}\left(C_{mn}^{(0;2)}\right)^2$$
$$+2\sum_{mn}\left(C_{mn}^{(0;1)}.C_{mn}^{(0;2)}\right)+\sum_{mn}\left(C_{mn}^{(2;1)}\right)^2+\sum\left(C_{mn}^{(2;2)}\right)^2$$
$$+2\sum_{mn}\left(C_{mn}^{(2;1)}.C_{mn}^{(2;2)}\right)+\sum_{mn}\left(C_{mn}^{(4;1)}\right)^2 \quad (10)$$

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