Job Shop Scheduling with Fixed Delivery Dates

Cheng-Hsiang Liu

***Abstract*—In most classical scheduling models, a job is assumed to deliver to a customer at the instant of a job processing completion. In numerous practical situations, however, multiple delivery dates exist, where the time interval between any two consecutive delivery dates is constant. This type of fixed delivery strategy results in substantial cost savings for transportation and handling. This study focuses on job scheduling in a dynamic job shop environment to minimize the sum of the total due-date cost and the total earliness penalty. This study identified two existing dispatching rules, and proposed six new rules by explicitly considering different due date costs per time unit and/or earliness penalty per time unit of a job. The simulation results show that the proposed dispatching rules are significantly superior to their original counterparts.**

***Index Terms*—Fixed interval delivery, Job shop, Dispatching rule, simulation**

I. Introduction

THE scheduling problem has been extensively investigated over many years. A standard assumption of most classical scheduling literatures is that a job is delivered to a customer at the instant of a job processing completion. However, several practical situations show that they are not adequately represented by this classical assumption regarding delivery dates. In these situations, finished jobs are supplied to the customer by a truck at the earliest of a series of fixed delivery dates, which falls at or after processing completion. As mentioned in Matsuo [6] and Lee and Li [4], this type of fixed interval delivery strategy results from transportation and handling economies that are advantageous to consolidate shipments. Customers prefer their jobs to be shipped as early as possible.

The earliest reference to fixed delivery dates in related literature is by Matsuo [6], who provided an example of 200 jobs that are shipped over a 40-day planning horizon with 1 shipping time per day. The objective was to find an overtime utilization level and a job sequence that jointly provide a good trade-off between overtime costs and penalties for late delivery. Lesaoana [5] also considered problems with fixed delivery dates, and provided algorithms for several different scheduling environments and objectives. Chand et al. [1] considered a single machine scheduling problem with the first-come first-deliver policy, where the shop must assign delivery dates to jobs and find a feasible due date schedule to minimize lead time penalty and earliness costs. They developed a dynamic programming algorithm and several dominance results for the problem. Chhajed [2] considered a single machine scheduling problem with independent lead time for jobs and earliness penalties. He also showed the problem to be NP-hard, even when two delivery dates are present, and provided bounds on the optimal solution for the two delivery dates based on the Lagrangian relaxation of the problem. Lee and Li [4] considered the non-preemptive single machine scheduling problem with multiple delivery dates and showed that this problem is strongly NP-hard for an arbitrary number of delivery dates. They provided a pseudo-polynomial dynamic programming algorithm for when the number of delivery dates is bounded by a given constant. Hall et al. [3] examined the solvability of scheduling problems in which jobs are dispatched to customers only at fixed times. They either provided an efficient algorithm or established that such an algorithm is unlikely to exist for almost all considered problems.

The abovementioned studies on this subject examined only static cases in which all jobs are ready to start at time zero. However, in numerous real systems, this scheduling problem is even more difficult because jobs arrive on a continuous basis, which is why the process is called, dynamic scheduling. This paper considers the dynamic job shop scheduling problem with multiple delivery dates, where the time between two consecutive delivery dates is a given constant. This study focuses on scheduling jobs to minimize the sum of the total due-date cost and the total earliness penalty. This study is perhaps the first of its kind for developing dispatching rules to address the scheduling problem with fixed interval delivery dates. A thorough investigation was performed to observe the performance and their interactions of these decision factors regarding the total cost of jobs.

II. Terminology and Problem Description

To understand the problem and the various rules in this study, the terminology used in this paper is as follows:

N: number of jobs
M_i: total number of operations on job i
A_i: time of arrival of job i on the shop floor
τ: length of each delivery interval
C_i: completion time of job i
$C_{i,j-1}$: completion time of the previous operation (i.e., operation j-1) of job i
P_{ij}: processing time for operation j of job i
T_{now}: time instant at which dispatching decision is made

Manuscript received January 06, 2012; revised January 16, 2012. This work was supported by National Science Council of Taiwan under grant no. NSC 99-2221-E-020-023-MY2.

Cheng-Hsiang Liu is an Associate Professor of Department of Industrial Management at the National Pingtung University of Science & Technology in Taiwan. He received his M.S. and Ph.D. degrees from the National Chiao Tung University (NCTU) in Taiwan, both in industrial engineering and management. His research areas include data mining, decision support systems, and operation scheduling and production control. (Tel: 886-8-7703202, ext. 7717; E-mail: chliu.uw@gmail.com).

ISBN: 978-988-19251-9-0
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

Z_i: priority index of job i at instant T_{now}
λ: number of completed deliveries
α_i: due date cost per time unit of job i
β_i: earliness penalty per time unit of job i

When N jobs require scheduling, the set of all possible delivery dates is $\{\tau, 2\tau, 3\tau, \ldots\}$. Thus, if $(k-1)\tau < C_i \le k\tau$, the due date cost of job i is $(k\tau - a_i)\times\alpha_i$, and the earliness penalty of job i is $(k\tau - C_i)\times\beta_i$. In this case, job i is delivered at time $k\tau$. The total cost of job i is shown in (1). The objective function Z is shown in (2).

$$TC_i = (k\tau - a_i)\times\alpha_i + (k\tau - C_i)\times\beta_i \tag{1}$$

$$Z = \sum_{i=1}^{N} TC_i \tag{2}$$

III. Identification of Dispatching Rules for Evaluation

This paper first identifies three well-known dispatching rules to minimize total cost, followed by the development of the modified rules. In this study, the selection of dispatching rules is aimed at minimizing the total cost of jobs. Shortening job flow time may decrease the total cost of the jobs. In addition, this research did not use due date-related performance measures. Therefore, this study included two simple and popular rules such as the shortest processing time (SPT) and the first-come first-serve (FCFS) for minimizing job flow time to reduce the total cost of jobs. This study also derived six rules modified from the above mentioned two traditional dispatching rules, denoted as SPT-α, SPT-β, SPT-αβ, FCFS-α, FCFS-β, and FCFS-αβ.

Existing rules

SPT = Priority to process a job at any given machine, which proceeds onto the job queued in front of a machine with a minimum P_{ij} at time T_{now}

FCFS= Priority to process a job at any given machine, which proceeds onto the job queued in front of a machine with a minimum $C_{i,j-1}$ at time T_{now}

Proposed rules

At first, this study modifies the conventional SPT and FCFS rules by considering the due date cost per time unit of a job. The proposed rules aim to minimize the total due date cost of the jobs. The proposed rules are presented below.

Rule 1: SPT-α

This proposed rule seeks to process a job early according to the due date cost per time unit of a job, apart from handling the processing time for a job. The priority index for this proposed rule is as follows (with the job having a minimum Z_i at time T_{now} for selection):

$$Z_i = \frac{P_{ij}}{\alpha_i} \tag{3}$$

Rule 2: FCFS-α

By regarding the due date cost per time unit of a job, the priority index of job i is computed as follows:

$$Z_i = \frac{C_{i,j-1}}{\alpha_i} \tag{4}$$

The job with the minimum value of Z_i at time T_{now} is selected for processing.

The following three rules (SPT-β and FCFS-β) are modifications of the previous rules with a focus on earliness penalty of a job. When the current time is larger than the previous delivery date, the job with higher earliness penalty has a higher probability of being chosen compared to the others for the subsequent job for processing. Otherwise, the job with a smaller earliness penalty has a higher probability of being selected compared to the others. Among these rules, this study defines a ratio (R) of the achieved rate to the next delivery date. Close to 1 indicates that the delivery date is almost due. Close to 0 denotes that one new delivery interval just began. R is computed as follows:

$$R = \frac{T_{now} - \lambda\times\tau}{\tau} \tag{5}$$

Rule 3: SPT-β

The priority index for this proposed rule is as follows:

$$\begin{aligned} &\textit{If } R < 0.5 \textit{ Then}\\ &\quad Z_i = P_{ij}\times\beta_i\\ &\textit{Else}\\ &\quad Z_i = \frac{P_{ij}}{\beta_i}\\ &\textit{End} \end{aligned} \tag{6}$$

The job with the minimum value of Z_i is selected for processing.

Rule 4: FCFS-β

The priority index for this proposed rule is as follows:

$$\begin{aligned} &\textit{If } R < 0.5 \textit{ Then}\\ &\quad Z_i = C_{i,j-1}\times\beta_i\\ &\textit{Else}\\ &\quad Z_i = \frac{C_{i,j-1}}{\beta_i}\\ &\textit{End} \end{aligned} \tag{7}$$

The job with the minimum value of Z_i is selected for processing.

This study also considered the SPT-αβ and FCFS-αβ rules. These rules attempt to yield a minimum value for the sum of the total due date cost and the total earliness penalty. Among the three proposed rules, a set of jobs are classified into two disjointed groups (G_1 and G_2), based on whether the current operation is the final operation.

$$\begin{aligned} &\textit{If the current operation of } J_i \textit{ is not the final operation, Then}\\ &\quad G_1 = G_1 \cup J_i\\ &\textit{Else}\\ &\quad G_2 = G_2 \cup J_i\\ &\textit{End} \end{aligned} \tag{8}$$

Jobs in Set G_1 are sorted according to a predefined criterion dependent on the selected dispatching rule. Jobs in Set G_2 are sorted in ascending order of their β_i values. If a delivery time is distant, a reasonable policy is to process a job belonging to

ISBN: 978-988-19251-9-0
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

Set G_1 beforehand if time remains until the next delivery after all the jobs belonging to Set G_2 are dispatched.

Rule 5: SPT-αβ

Step 1: Jobs in Set G_1 are sorted according to P_{ij}/α_i. Jobs in Set G_2 are sorted in ascending order of their β_i values.

Step 2: Calculate the K value as follows:

$$K = \tau \times (\lambda + 1) - T_{now} - P_{ij}^{(1)} \quad (9)$$

where $P_{ij}^{(1)}$ denotes the processing time of operation j of the first job in Set G_1.

Step 3: A job is chosen for processing according to the following manner:

$$\begin{aligned} & \textit{If } \sum_{i \in G_2} P_i \le K \textit{ Then} \\ & \quad \textit{the first job in } G_1 \textit{ is taken for processing} \\ & \textit{Else} \\ & \quad \textit{the first job in } G_2 \textit{ is taken for processing} \\ & \textit{End} \end{aligned} \quad (10)$$

Rule 6: FCFS-αβ

Step 1: Jobs in Set G_1 are sorted according to P_i/α_i. Jobs in Set G_2 are sorted in ascending order of their β_i values.

Step 2: Calculate the K value as in (9).

Step 3: A job is chosen for processing according to (10).

IV. EXPERIMENTAL DESIGN FOR THE SIMULATION STUDY

The main objective of this study is to evaluate the effectiveness of the modified dispatching rules for reducing the total cost of jobs. To achieve this goal, several experiments were conducted on a simulation model of a hypothetical job shop. This study assumed the presence of ten machines in a job shop. The number of operations for an entering job is randomly sampled in the ranges 2-8, and the corresponding machine visitations are randomly generated among ten machines. No two consecutive operations can be performed on the same machine, and a machine cannot be revisited by a job for a later operation. The processing time P_{ij} is generated based on the normal distribution with an average of 1000 seconds and a standard deviation of 200 seconds. The inter-arrival times of jobs are generated from a negative exponential distribution, which has a mean value chosen to create a certain expected shop utilization rate of 90 %. The model assumptions are as follows.

- Preemption is prohibited, that is, once the processing of a job has started, stopping it is impossible.
- The machines cannot process more than one job at any time.
- Unlimited buffer exists for WIP.
- Resource storage and machine failure are not considered.
- All the machines are available at zero in the usage time.
- All orders are ready for processing at the beginning of each planning period.
- The transfer time between machines is ignored.

A four-factor full factorial design was employed to conduct a comprehensive study of the effects of the decision factors on the selected performance measures. The factors to be evaluated were the length of each delivery interval, due date cost, earliness penalty, and dispatching rule. Table I lists the parameters. For each of the treatments, ten replications were conducted to minimize variations in the results. The warm-up period for the shop was the time interval from the start of the simulation to the completion of the first 1,000 jobs. Once the simulation reached a steady state, 1,000 jobs from the shop simulation were collected. The performances of the twelve dispatching rules were evaluated regarding the total cost of the jobs.

TABLE I EXPERIMENTAL DESIGN FOR COMPUTATIONAL STUDY

Factors	Levels	Level Description
length of each delivery interval (τ)	2	4-Hour, 8-Hour
due date cost (α)	3	$\alpha_i \in Uniform\,(11, 20)$ $\alpha_i \in Uniform\,(31, 40)$ $\alpha_i \in Uniform\,(51, 60)$
earliness penalty (β)	3	$\beta_i \in Uniform\,(11, 20)$ $\beta_i \in Uniform\,(31, 40)$ $\beta_i \in Uniform\,(51, 60)$
dispatching rule	8	SPT, SPT-α, SPT-β, SPT-αβ FCFS, FCFS-α, FCFS-β, FCFS-αβ

V. RESULTS AND DISCUSSION

The results of the factorial experiment are shown in Tables II-III. Each item in these tables is an average of the ten replications of the experiment. The percentage in tables indicates the percentage difference between the average objective value obtained by the modified rule and the original rule. The key observations from the results listed in these tables are summarized below.

(1) The total cost increases in conjunction with α and/or β levels.

(2) As the τ level lengthens, the total cost increases.

(3) SPT-α is slightly smaller than SPT, when regarding the total cost. FCFS-α seemingly allows a deep cut in the values of total cost compared to FCFS. The effect of addition of α-information to FCFS is significantly larger than the one of addition of α-information to SPT. For τ=4-Hour with and , overall, FCFS-α and SPT-α are superior to their original counterparts (FCFS and SPT) by 15.1 % and 6.9 %, respectively.

(4) The SPT-β and FCFS-β rules do not affect the total cost because a larger total due date cost can diminish the effects of considering β-information on the total cost considerably.

(5) SPT-αβ simultaneously considering α- and β-information is superior to SPT and its counterparts that only consider information related to the α value (SPT-α) or the β value (SPT-β). The same behavior was present in FCFS-based rules.

(6) The proposed SPT-αβ appears to be optimal for the total cost of the jobs under all the tested conditions.

For the combination of α, β, and τ values, the optimal rules of SPT-based and FCFS-based rules (SPT-αβ and FCFS-αβ) were also selected to detect a significant statistical difference in the performances. Because common random-number streams were used to generate ten observations for each combination of factors, the sample observations were not independent. Therefore, using the paired t-test was essential for statistical analysis. Table IV shows the results of the paired t-tests. Each of the approaches are listed in descending order of performance, and grouped into homogeneous subsets labeled with different letters if the difference between the performance means of the two approaches in the subset did not notably exceed the prescribed significance level of 0.05. Based on the compared measurements, the approach with "A" was significantly superior to the approach with "B". Table IV shows that the proposed SPT-αβ can reduce total cost of jobs significantly over FCFS-αβ in all instances.

TABLE II PERFORMANCE OF RULES WITH RESPECT TO TOTAL COST OF THE JOBS (τ=4-Hour)

α	β	Dispatching rule							
		SPT	SPT-α	SPT-β	SPT-αβ	FCFS	FCFS-α	FCFS-β	FCFS-αβ
11~20	**11~20**	182606 (--)	169999 (6.9%)	186484 (-2.1%)	151658 (16.9%)	210746 (--)	178826 (15.1%)	208511 (1.1%)	160387 (23.9%)
11~20	**31~40**	222637 (--)	210222 (5.6%)	222510 (0.1%)	173723 (22.0%)	250420 (--)	218869 (12.6%)	247001 (1.4%)	182705 (27.0%)
11~20	**51~60**	262668 (--)	250444 (4.7%)	260257 (0.9%)	195789 (25.5%)	290094 (--)	258912 (10.7%)	287036 (1.1%)	205023 (29.3%)
31~40	**11~20**	377822 (--)	372405 (1.4%)	388655 (-2.9%)	344096 (8.9%)	443600 (--)	400535 (9.7%)	438958 (1.0%)	371146 (16.3%)
31~40	**31~40**	417852 (--)	412277 (1.3%)	418387 (-0.1%)	366189 (12.4%)	483275 (--)	440574 (8.8%)	476150 (1.5%)	393389 (18.6%)
31~40	**51~60**	457883 (--)	452148 (1.3%)	453769 (0.9%)	388282 (15.2%)	522949 (--)	480612 (8.1%)	517039 (1.1%)	415632 (20.5%)
51~60	**11~20**	573037 (--)	567479 (1.0%)	590826 (-3.1%)	542088 (5.4%)	676455 (--)	630411 (6.8%)	669404 (1.0%)	581518 (14.0%)
51~60	**31~40**	613067 (--)	607220 (1.0%)	614264 (-0.2%)	564536 (7.9%)	716129 (--)	670458 (6.4%)	705299 (1.5%)	603626 (15.7%)
51~60	**51~60**	653098 (--)	646962 (0.9%)	647281 (0.9%)	586984 (10.1%)	755803 (--)	710506 (6.0%)	747041 (1.2%)	625733 (17.2%)

TABLE III PERFORMANCE OF RULES WITH RESPECT TO TOTAL COST OF THE JOBS (τ=8-Hour)

α	β	Dispatching rule							
		SPT	SPT-α	SPT-β	SPT-αβ	FCFS	FCFS-α	FCFS-β	FCFS-αβ
11~20	**11~20**	244666 (--)	231704 (5.3%)	244234 (0.2%)	196020 (19.9%)	272856 (--)	240521 (11.9%)	261053 (4.3%)	202310 (25.9%)
11~20	**31~40**	324752 (--)	311750 (4.0%)	323689 (0.3%)	240929 (25.8%)	352594 (--)	320284 (9.2%)	339300 (3.8%)	246923 (30.0%)
11~20	**51~60**	404839 (--)	391797 (3.2%)	404600 (0.1%)	285838 (29.4%)	432333 (--)	400047 (7.5%)	416390 (3.7%)	291535 (32.6%)
31~40	**11~20**	479937 (--)	473280 (1.4%)	484588.(-1.0%)	423446 (11.8%)	545775 (--)	503334 (7.8%)	525384 (3.7%)	443274 (18.8%)
31~40	**31~40**	560023 (--)	552735 (1.3%)	560574 (-0.1%)	467802 (16.5%)	625513 (--)	583684 (6.7%)	603959 (3.4%)	488483 (21.9%)
31~40	**51~60**	640110 (--)	632190 (1.2%)	641516 (-0.2%)	512158 (20.0%)	705251 (--)	664035 (5.8%)	681044 (3.4%)	533692 (24.3%)
51~60	**11~20**	715208 (--)	708876 (0.9%)	724941 (-1.4%)	640846 (10.4%)	818693 (--)	772278 (5.7%)	789716 (3.5%)	686223 (16.2%)
51~60	**31~40**	795295 (--)	788450 (0.9%)	797459 (-0.3%)	685677 (13.8%)	898431 (--)	852277 (5.1%)	868619 (3.3%)	730844 (18.7%)
51~60	**51~60**	875381 (--)	868023 (0.8%)	878433 (-0.3%)	730509 (16.5%)	978170 (--)	932277 (4.7%)	945697 (3.3%)	775465 (20.7%)

ISBN: 978-988-19251-9-0
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

TABLE IV RESULTS OF THE PAIRED *t*-TEST FOR SPT-αβ AND FCFS-αβ

α	β	Dispatching rule	τ=4-Hour	Dispatching rule	τ=8-Hour
11~20	11~20	SPT-αβ	A	SPT-αβ	A
		FCFS-αβ	B	FCFS-αβ	B
11~20	31~40	SPT-αβ	A	SPT-αβ	A
		FCFS-αβ	B	FCFS-αβ	B
11~20	51~60	SPT-αβ	A	SPT-αβ	A
		FCFS-αβ	B	FCFS-αβ	B
31~40	11~20	SPT-αβ	A	SPT-αβ	A
		FCFS-αβ	B	FCFS-αβ	B
31~40	31~40	SPT-αβ	A	SPT-αβ	A
		FCFS-αβ	B	FCFS-αβ	B
31~40	51~60	SPT-αβ	A	SPT-αβ	A
		FCFS-αβ	B	FCFS-αβ	B
51~60	11~20	SPT-αβ	A	SPT-αβ	A
		FCFS-αβ	B	FCFS-αβ	B
51~60	31~40	SPT-αβ	A	SPT-αβ	A
		FCFS-αβ	B	FCFS-αβ	B
51~60	51~60	SPT-αβ	A	SPT-αβ	A
		FCFS-αβ	B	FCFS-αβ	B

VI. Conclusions

Most research on the development of dispatching rules for job shop scheduling assumed that a job is delivered to a customer at the instant of a job processing completion. In addition, these dispatching rules assume that the due date cost per time unit and earliness penalty per time unit of a given job are the same, though these costs may differ from between jobs in practice. This study is perhaps the first of its kind for developing dispatching rules to address the scheduling problem with fixed interval deliveries and different due date costs per time unit and earliness penalty per time unit of jobs. This study identified two existing rules apart from the six new dispatching rules for performance analysis in job shops. The experimental results indicate that the rules that include information related to the due date cost per time unit and earliness penalty per time unit of a job are excellent for minimizing the total cost of jobs. Overall, the proposed rule SPT-αβ proved to be highly effective for minimizing the total cost of jobs. This research is ongoing to develop dispatching rules for more complex systems. For example, the length of a delivery interval is not fixed. Another promising avenue for research would be the development of heuristic methods to solve problems that jointly consider production and delivery, with the condition that each job may require a different amount of space during transport.

References

[1] S. Chand, D. Chhajed, and R. Traub, “A single-machine scheduling model with fixed-interval deliveries,” *Production and Operations Management*, vol. 3, np. 4, pp. 296-307, 1994.

[2] D. Chhajed, “A fixed interval due-date scheduling problem with earliness and due-date costs,” *European Journal of Operational Research*, vol. 84, pp. 385-401, 1995.

[3] N. G. Hall, M. Lesaoana, and C. N. Potts, “Scheduling with fixed delivery dates,” *Operations Research*, vol. 49, no. 1, pp. 134-144, 2001.

[4] C.-Y. Lee and C.-L. Li, “On the fixed interval due-date scheduling problem,” *Discrete Applied Mathematics*, vol. 68, pp. 101-117, 1996.

[5] M. Lesaoana, “Scheduling with fixed delivery dates,” Ph.D. dissertation, University of Southampton, U.K, 1991.

[6] H. Matsuo, “The weighted total tardiness problem with fixed shipping times and overtime utilization,” *Operation Research*, vol. 36, pp. 293-307, 1988.

ISBN: 978-988-19251-9-0
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)