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Categorizing Group Opinions Through Fuzzy Similarity Relation

Hayao Miyagi, Yui Miyagi, and Donshik Kang

Abstract—**This paper proposes a procedure for categorizing human's opinions in circumstances of group decision-making. Fuzzy similarity relation is utilized for categorizing opinion matrix whose each element is derived from the difference between two evaluation vectors. Those vectors are obtained from various opinions of decision-makers and the difference of the two is well expressed by the sigmoid function. Then, similar opinions in the group are clustered according to the nature of the fuzzy similarity relation. Especially, the transitive law plays an important role in the clustering. When the similarity relation is not existed in the opinion matrix and any negotiation is required among decision-makers, the final value of nth power of the opinion matrix is proposed as the consensus value. Furthermore, it is shown that the orthogonal condition of the opinion matrix is useful to derive order relations of opinions. Fuzzy opinion matrix enables one to investigate similarity of opinions of group members at a time and to make categories of similar opinions effectively. The proposed procedure gives decision-makers lead a reasonable group decision-making in the context of logical treatment for various opinions due to diversified views or ideas.**

Index Terms—**Fuzzy opinion matrix, categorization of opinions, fuzzy similarity relation, group decision-making**

I. Introduction

OUR lives are the sum of decisions, whether in business or in personal sphere. In order to get a certain target, we may choose the optimal thing out of the alternatives. Especially group decision-making is the important scheme for making definite policy of the society. However, the difficulty of the group decision-making lies on decision maker's perspectives due to various sense of values and lifestyles. Then in constructing group decision support systems, our interests are in the problems how to aggregate various opinions, based on "similarity" and "dissimilarity" and how to make an agreement when opinions are different.

To solve the multiple criteria decision-making problems, Analytic Hierarchy Process (AHP) [1] proposed by Saaty is widely employed. As aggregation procedures [1], [2] in group decision-making, Saaty presented two representative ways. One is the way to aggregate each pairwise comparison matrix at each stage in a hierarchy. This method gives a total group priority at the final stage. The other is individual member establish his own evaluation, respectively, and then, final priority of the whole group is calculated by aggregating all evaluation results.

To get reciprocal relation of symmetric elements in the pairwise comparison matrix, the geometric mean is mainly used in the former [3]-[5], whereas the arithmetic mean aggregation technique is adopted in the latter [6], [7].

The comprehensive aggregation procedure utilizing both arithmetic mean and geometric mean has also appeared in [8], recognizing the importance of the similarity of individual perspective. Then the final group evaluation is led from integrating individual evaluation.

When the decision-making group consists of versatile types of members who have various views and interests, the acquired final evaluation result could be quite different from some decision maker's evaluations. Accordingly, homogeneous group should be clustered, first, as the precondition of aggregating evaluations. In this connection, Zahir [9], Bolloju [10] emphasize the importance of homogeneity of each evaluation when adopting aggregation of each opinion as the group opinion.

Ota et al. [11] also assumed group members have various senses of values and ideas. That method divided group members into some homogeneous subgroups first, then gathering priorities given by subgroups, final priority as the group is decided. In that paper, "cosine" derived from inner product of evaluation vectors is used as the way of expressing similarity. In fact, analyzing the difference in evaluation is significant in order to reach group consensus and must be taken into account in the decision-making process.

In this paper, a procedure for categorizing human's diversified opinions in circumstances of group decision-making is presented. Fuzzy opinion matrix is developed, first. Then, the fuzzy similarity relation is utilized for categorizing opinion matrix. Each element of the matrix is calculated from the difference between two evaluated vectors. The difference of the two is expressed by the sigmoid function that is useful to quantify similarity and dissimilarity. Then, similar opinions in the group are clustered according to the nature of the fuzzy similarity relation. The transitive law plays an important role in the process of clustering. If the similarity relation is not existed partly in the opinion matrix and the unification is mandatory, some negotiation is required among decision-makers. For the case, in this paper, the final value of nth power of the opinion matrix is proposed as the consensus value. In fact, any fuzzy relation matrix has a nature that its transitive closure is transitive. Furthermore, it is shown that the orthogonal condition of the opinion matrix behaves well in making order relations of opinions.

II. Similarity of Opinion Vectors

Conventional well-known methods to express similarity between two nth order vectors are (*a*) "Cosine" derived from inner product, (*b*) Pearson correlation coefficient, and (*c*)

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Deviation pattern similarity. In this section, we investigate the nature of conventional methods in making distinction between "similar" and "dissimilar". Finding drawback of conventional methods, a new method using sigmoid function is presented.

A. Conventional Method

(*a*) *Cosine Method*

Let us consider two vectors as shown in Fig. 1. A similarity expression of two vectors on the multi-dimensional space is "cosine" derived from inner product [12]. If evaluated vectors are unit vectors, then, cosine becomes inner product itself such that

$$\cos\delta = \dot{F}i\,\dot{F}j / |\dot{F}i||\dot{F}j| = \dot{F}i\dot{F}j \quad (1)$$

Then, we have the relations;
(i) *If* $\dot{F}i = \dot{F}j$, *then,* $\cos\delta = 1$,
(ii) *If* $\dot{F}i \perp \dot{F}j$, *then,* $\cos\delta = 0$,
Thus, numbers "1" and "0" imply "similar" and "dissimilar", respectively.

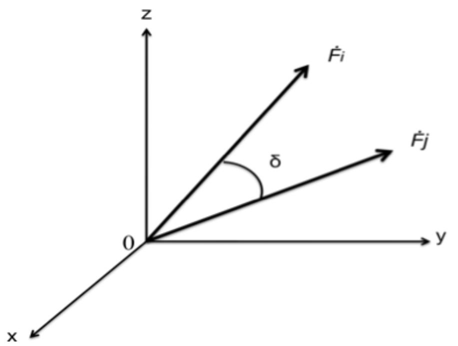

Fig.1. Similarity between two opinion vectors $\dot{F}i$ and $\dot{F}j$

(*b*) *Pearson Correlation Coefficient*

The Pearson correlation coefficient is defined as

$$\cos\delta = (\dot{F}i - \dot{G})\,(\dot{F}j - \dot{Z}) / \,|\dot{F}i - \dot{G}||\dot{F}j - \dot{Z}| \quad (2)$$

where, $\dot{G}$ and $\dot{Z}$ are the average vector whose elements are the average of all elements in $\dot{F}i$ and the average vector whose elements are average of all elements in $\dot{F}j$, respectively. Then we have the relations;
(i) *If* $\cos\delta = 1$, *then,* it is recognized as completely agree.
(ii) *If* $\cos\delta = 0$, *then,* it indicates non-correlation.
(iii) *If* $\cos\delta = -1$, *then,* it is recognized as completely disagree.

(*c*) *Deviation Pattern Similarity*

The Deviation pattern similarity is given as

$$\cos\delta = (\dot{F}i - \dot{E})\,(\dot{F}j - \dot{E}) / \,|\dot{F}i - \dot{E}||\dot{F}j - \dot{E}| \quad (3)$$

where, $\dot{E}$ is the average vector whose kth element is average of kth elements in $\dot{F}i$ and $\dot{F}j$. Then $\cos\delta$ is utilized to give similarity and dissimilarity.

We notice that any method relies on cosine value as similarity. Hence we investigate the nature of cosine function. In Fig. 2, case (1) denotes that the similarity of two vectors equals 1 which implies completely same, whereas dissimilarity is 0 that implies completely different. A problem appears for case (3). In case (3), despite the degree of similarity is given as 0.87, the degree of dissimilarity is 0.5. That is, although both (1) and (2) show well-balanced degree, i.e., 1+0=1 and 0+1=1, case (3) gives the relation 0.87+0.5=1.37>1. It is recognized as a quite awkward expression.

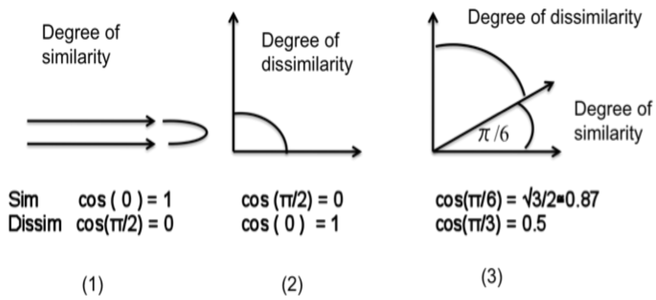

Fig.2. Relation between similarity and dissimilarity

B. Sigmoid Function Method

To improve the drawback of cosine method, we propose sigmoid function method. A sigmoid function that shows similarity and dissimilarity is given in Fig. 3.

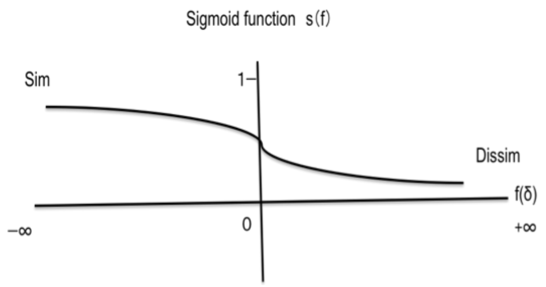

Fig.3. Sigmoid function that shows similarity and dissimilarity

In this paper, sigmoid function s

$$s(f(\delta)) = 1 / (1 + \varepsilon^{f(\delta)}) \quad (4)$$

is utilized to make a mapping

$$s : [-\infty, +\infty] \longrightarrow [1, 0]$$

The role of $f(\delta)$ is a mapping from the angle-based expression to the number between minus infinity and plus infinity, such that

$$f : [0, \pi/2] \longrightarrow [-\infty, +\infty]$$

where

$$f(\delta) = -1 / (\tan 2\delta) \quad (5)$$

As the result, similarity function s with respect to angle δ is obtained as shown in Fig. 4.

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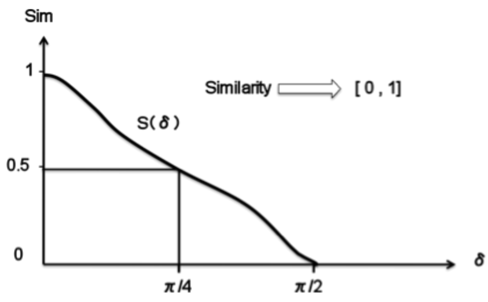

Fig.4. Similarity based on sigmoid function

For this function, we have $s(\pi/6)=0.64$(similarity) and $s(\pi/3)=0.36$(dissimilarity). In fact, this new similarity function gives well-balanced values of similarity against dissimilarity.

III. Fuzzy Similarity Relation

Let us consider the direct product of set X and set Y, such that

$$X \times Y = \{ (x, y) \mid x \in X, y \in Y\}$$

Then fuzzy relation R is characterized by a membership function μ_R as

$$\mu_R : X \times Y \rightarrow [0,1] \quad (6)$$

For this relation, fuzzy relational matrix $\boldsymbol{R}$ is composed as

$$\boldsymbol{R} = [r_{ij}] = [\mu_R (x_i, y_j)], \quad \forall x_i \in X, \forall y_j \in Y \quad (7)$$

Let R and T be relations in XxY and YxZ, respectively. Then the composition of R and T denoted by RoT is defined as follows:

$$RoT \Leftrightarrow \mu_{RoT}(x, z) = \max_y \cdot \min\{ \mu_R(x,y), \mu_T(y,z)\} \quad (8)$$

The next definition about fuzzy similarity relation gives an important role in this paper.

Definition 1

When the following three conditions are satisfied, the relation R on set X is called similarity relation[13],

(1)Reflexivity: $\mu_R(x,x) = 1$; $\boldsymbol{R} \supseteq \boldsymbol{I}$

(2)Symmetry: $\mu_R(x,y) = \mu_R(y,x)$; $\boldsymbol{R} = \boldsymbol{R}^T$,

(3)Transitivity: $\max_y \cdot \min \{ \mu_R(x,y), \mu_R(y,z) \} \leq \mu_R(x,z)$;

$$\boldsymbol{R} o \boldsymbol{R} \subseteq \boldsymbol{R}$$

IV. Possibility of Categorizing Group Opinions

In this section, we investigate the possibility of categorizing opinions. Usually, when two evaluated vectors are presented, the judgment on similarity and dissimilarity is left in decision maker's subjectivity or experience. For presented more than three evaluated vectors, however, the relative consideration is required, because there exist some instances where logical inconsistency yields.

A. Uniting Rule for Three Opinion Vectors

Suppose evaluated vectors given by three persons I, J, K, as shown in Fig. 5. Where, ①, ② or ③ denotes distance between two terminal points, under the assumption $|\dot{F}i|$, $|\dot{F}j|$, $|\dot{F}k|$ are 1.

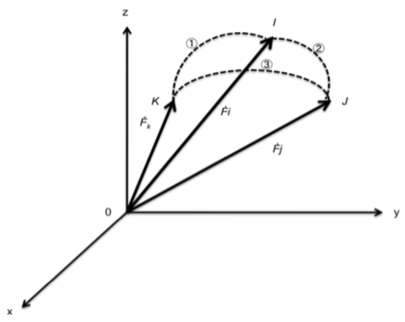

Fig.5. Distances among three evaluated vectors

In order to investigate the uniting rule, let us consider a triangle obtained from Fig. 5, drawing out distances among terminals of $\dot{F}i$, $\dot{F}j$, and $\dot{F}k$. The triangle under the condition ①<②<③ is given in Fig. 6.

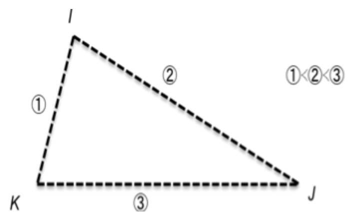

Fig. 6. Triangle composed of three distances

When a conditional value for the similarity is given as distance ρ, we can categorize the situation into following four cases;

1) If ρ < ①, I, J and K are categorized into independent groups, respectively. That means they can not be united no longer.

2) If ① ≤ ρ < ②, I and K belong to a same group, although J is independent.

3) If ② ≤ ρ < ③, logical inconsistency occurs. Such condition implies that I,K and I,J are categorized into same groups, respectively, although J and K can not be united.

4) If ③ ≤ ρ, I,J and K are able to be united, and belong to a same group.

Those cases may be rewritten by using the similarity function $s(\delta)$;$[0, \pi/2] \rightarrow [1,0]$ and its threshold value s^c, where $s^c=1$ implies completely similar and $s^c=0$ implies dissimilar. Thus cases 1) to 4) can be rewritten as fallows:

1) $s^c > s(\delta_{ik}) \rightarrow I,J,K$:independent.

2) $s(\delta_{ik}) \geq s^c > s(\delta_{ji}) \rightarrow$ <u>I,K</u>: same group ,J: independent.

3) $s(\delta_{ji}) \geq s^c > s(\delta_{kj}) \rightarrow$ <u>I,K</u> and <u>I,J</u> :same group, J and K: independent(logical inconsistency).

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4) $s(\delta_{kj}) \geq s^c \rightarrow$ I,J,K: same group.

It is necessary to remove the logical inconsistency part to make reasonable clustering. We can find case 3) is cancelled out by setting $s(\delta_{kj})=s(\delta_{ji})$. Therefore, utilizing cases 2), 3) and 4), we can obtain a uniting rule as shown in Fig. 7.

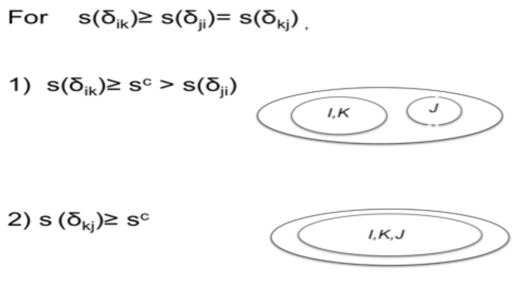

Fig.7. Uniting rule of three vectors

In this way, we can define associability.

Definition 2

Under the condition

$$s(\delta_{ik}) \geq s(\delta_{ji}) = s(\delta_{kj}) \tag{9}$$

normalized opinion vectors $\dot{F}_i$, $\dot{F}_j$ and $\dot{F}_k$ are called
(i) "partly associable " , when

$$s(\delta_{ik}) \geq s^c > s(\delta_{ji}) \tag{10}$$

(ii) "completely associable", when

$$s(\delta_{kj}) \geq s^c \tag{11}$$

We can see later that condition (9) plays an important role in verifying fuzzy similarity.

B. Composition of Fuzzy Opinion Matrix

According to the discussion in the former subsection *A*, a fuzzy opinion matrix is defined.

Definition 3

For the normalized opinion vectors, fuzzy relation matrix in which elements are derived from $s(\delta)$ in (4);

$$\boldsymbol{S} = [\, s_{ij} \,],\ \ s_{ij} = s(\delta_{ij}),\ i,j = 1,2,\cdots,n \tag{12}$$

is called opinion matrix.

Here we can derive next theorem referring to *Definitions 2* and *3*.

Theorem 1

For three normalized opinion vectors, opinion matrix $\boldsymbol{S}$ is the similarity relation under the condition

$$s_{ik} \geq s_{ji} = s_{kj} \tag{13}$$

Proof

Conditions for reflexivity and symmetry are clearly satisfied from definition of $s_{ij}=s(\delta_{ij})$. Therefore, we prove transitivity:

$$s_{ik} \geq \bigvee_j (s_{ij} \wedge s_{jk}) \tag{14}$$

A general term $s_{ij} \wedge s_{jk}$ in the right-hand side is rewritten as $s_{ji} \wedge s_{kj}$, because $\boldsymbol{S}$ is symmetric. Hence, using the condition (13), we have

$$s_{ik} \geq s_{ji} \wedge s_{kj} \quad \text{for } j=1,2,3$$

Thus inequality (14) is satisfied.

C. Categorization of Opinions Using Threshold

In actual circumstances, decision-makers have their threshold value in mind. According to the threshold they may judge whether one opinion is similar to others or not. The concept of fuzzy α -cut [14] may be utilized to categorize opinions.

In this subsection, we suppose condition (13) again, to establish α -cut. When α -cut is carried out at (i) $\alpha = a = s^{c1}$; $s^{c1} > s_{ik}, s_{ji}, s_{kj}$, (ii) $\alpha = b = s^{c2}$; $s_{ik} \geq s^{c2} > s_{ji}$, (iii) $\alpha = c = s^{c3}$; $s_{kj} \geq s^{c3}$, three α -level relations are calculated as follows:

$$\boldsymbol{S_a} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad \boldsymbol{S_b} = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad \boldsymbol{S_c} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

These α -cut relations show the equivalence relation. $\boldsymbol{S_a}$ implies that all opinion vectors *I* , *J* and *K* are recognized as dissimilar under the condition; $s^{c1} > s_{ik}, s_{ji}, s_{kj},$. For somewhat weak threshold *b*, opinions *I* and *K* are treated as similar while *J* is dissimilar. $\boldsymbol{S_c}$ insists that all opinions are similar.

V. CATEGORIZATION FROM OPINION MATRIX

When an opinion matrix has the property of fuzzy similarity relation, all opinions can be included in a group, logically. If only some parts satisfy the similarity, decision-makers must decide whether they construct some groups with similar opinions by using satisfying parts or some of them change their opinions according to negotiation etc. That choice depends on decision-makers who concern in the responsibility of establishing the plan.

A. Relationship between Categorization and Similarity

If an opinion matrix satisfies similarity relation, logical categorization is possible. In actual situation encountered in our life, however, senseless result may be led, even if similarity relation is satisfied. Satisfying similarity relation means mathematical consistency can be kept but not always fit to our sense of "similar". Hence, as described in Section I IV, a criterion for the judgment or a threshold value s^c becomes important as well as logical satisfaction. For example, suppose an opinion matrix given as

$$\boldsymbol{S_e} = \begin{pmatrix} 1 & 0.8 & 0.2 \\ 0.8 & 1 & 0.2 \\ 0.2 & 0.2 & 1 \end{pmatrix}$$

For the case, possible threshold values are bounded as (i) $\alpha = a = s^{c1}$; $s^{c1} > 0.8, 0.2$, (ii) $\alpha = b = s^{c2}$; $0.8 \geq s^{c2} > 0.2$, (iii) $\alpha = c = s^{c3}$; $0.2 \geq s^{c3}$. If decision makers choose s^c=0.1 as the threshold value, conforming to case (iii), all opinions are united together and recognized as similar. Logical consistency is certainly existed. However, it may be difficult to regard 0.2, 0.8 and 1 as similar, from our usual sense. They may choose rather cases (i) and (ii) than case (iii).

B. Order Relation and Threshold Value

Let us consider the way of giving threshold value. To represent the degree of similarity, five stages of similarity are proposed. Then, these stages are compared to five stages for the degree of importance presented by Saaty [1].

Suppose that three decision-makers *a*, *b* and *c* have given their evaluation results for the alternatives L_1 and L_2 and the evaluated matrix is shown as

$$\boldsymbol{V} = \begin{matrix} a \\ b \\ c \end{matrix} \overset{\begin{matrix} L_1 & L_2 \end{matrix}}{\begin{pmatrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \end{pmatrix}}$$

Where, $\mathbf{x}^T = [x_1, x_2, x_3]$ and $\mathbf{y}^T = [y_1, y_2, y_3]$ are regarded as normalized values. In order to settle an order relation, the product of $\boldsymbol{V}$

$$\boldsymbol{V}^T\boldsymbol{V} = \begin{pmatrix} x_1^2+x_2^2+x_3^2 & x_1y_1+x_2y_2+x_3y_3 \\ y_1x_1+y_2x_2+y_3x_3 & y_1^2+y_2^2+y_3^2 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & \cos\varphi \\ \cos\varphi & 1 \end{pmatrix} \qquad (15)$$

is introduced. Here $\cos\varphi$ is the angle between two vectors $\dot{\mathbf{x}}$ and $\dot{\mathbf{y}}$ as shown in Fig. 8.

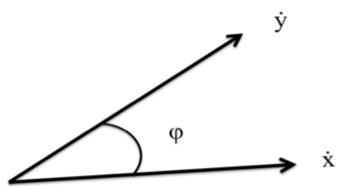

Fig. 8. Interval of two vectors

Equation (15) is well utilized in checking orthogonal nature of two vectors. In fact, when $\cos\varphi = 0$, it means two vectors are orthogonal. On the other hand, $\cos\varphi = 1$ means same vectors.

The representative threshold values are proposed by using five phases φ=0, $\pi/6$, $\pi/4$, $\pi/3$ and $\pi/2$, and defining index Δ;

$$\Delta = |\boldsymbol{V}^T\boldsymbol{V}| = \sin^2\varphi \qquad (16)$$

Then, we make matching the index with quantitative classification presented by Saaty in his analytic hierarchy process. Comparative values are listed in Table 1. Threshold value s($\pi/2$) requires the most strict conditions to unite other opinion, and the choice of s(0) implies all opinions can be united each other.

TABLE 1
THRESHOLD

φ	index Δ	Saaty's index	meaning	threshold
0	0	9	absolute	s(0)
π/6	1/4	7	very strong	s(π/6)
π/4	1/2	5	strong	s(π/4)
π/3	3/4	3	weak	s(π/3)
π/2	1	1	equal	s(π2)

C. Adjustment of Opinions

At the some group decision-making circumstances, there exists a case that some opinions are forced to change, to reach an agreement. Then they may make a great effort to adjust their opinions. Sometimes, a few ideas are neglected or tenaciously negotiated with the rest. Taking the application of proposed method to actual problems into account, adjustment is continued until they reach an agreement without logical inconsistency. Such procedure is substituted by using the convergence matrix of opinion matrix, in this paper.

When the transitivity of classification matrix is not satisfied, and opinion matrix $\boldsymbol{S}$ does not satisfy similarity relation, we utilize next property [13]:

Property 1

For any fuzzy relation $\boldsymbol{S}$, the transitive closure of $\boldsymbol{S}$, denoted by $\boldsymbol{S}^*$, is transitive:

$$\boldsymbol{S}^* = \boldsymbol{S} \cup \boldsymbol{S}^2 \cup \cdots \cup \boldsymbol{S}^n \qquad (17)$$

If some opinion is breaking consistency of group opinion without logical transitivity, the result of convergence matrix $\boldsymbol{S}^*$ may give a useful suggestion in the negotiating environment.

VI. FLOW OF PROCEDURE FOR CATEGORIZING

Fig. 9 shows algorithm for categorizing opinions. Procedure at each step is as follows:

Step 1: Obtain opinion vectors $\dot{F}_1, \dot{F}_2, \cdots, \dot{F}_n$, from each decision-maker by AHP etc.
Step 2: Develop classification matrix $\boldsymbol{S}$.
Step 3: Check if $\boldsymbol{S}$ has the nature of similarity relation. If yes, go to Step 4. Otherwise, go to Step 5.
Step 4: Do logical clustering processing.

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Step 5: Check if they need modification of opinions.
If yes, go to Step 7. Otherwise, go to Step 6.
Step 6: If only a part of the group satisfies transitivity, make a clustering by itself.
Step 7: Based on convergence result $\boldsymbol{R}^*$, modify some decision-maker's opinion. Reach an agreement.
Step 8: Do level cut based on threshold s^{c}.
Step 9: Aggregate opinions. Group decision-making is established.

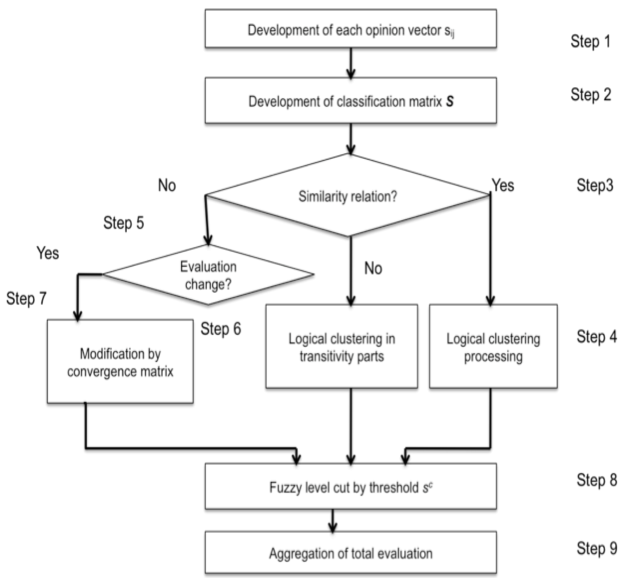

Fig.9. Flow chart of categorization

Ⅶ. Conclusion

In this paper, a categorization method in the decision-making circumstance was presented. The method investigates differences of evaluations obtained from decision-makers through fuzzy similarity relation. The nature of transitivity plays the important role in aggregating opinions. A group member in modern society has a tendency to recognize certain problem from his own standpoint and perspective, having each unique sense of views and criteria. Hence, a logical approach has to be established. We proposed a criterion that determines the degree of similarity, first. Sigmoid function combined with tangent function was found to be effective in expressing “similar” and “dissimilar”.

When establishing a consensus of decision-maker's opinion, we presented a practical measure that indicated interval of opinions. In fact, the threshold depending on human's sense of values or the value system of society is important to judge similarity. Interval index utilizing orthogonal nature of evaluation matrix was proposed dividing the interval into five stages.

Theoretical analysis with fuzzy relation can extract opinions that break consistency of whole opinions. Finding the transitive closure of any fuzzy relation has transitive nature, it was proposed as a consensus value after negotiation or discussion. The proposed method makes us possible to examine and regulate diverse opinions and ideas in the context of group decision-making scenario.

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