The Multi-commodity One-to-one Pickup-and-delivery Traveling Salesman Problem with Path Duration Limits

Christian E. M. Plum, David Pisinger, Juan-José Salazar-González and Mikkel M. Sigurd

Abstract—**The design of container shipping networks is an important real world problem, with assets and operational costs in billions of dollars. To guide the optimal deployment of the ships, a single vessel roundtrip is considered by minimizing operational costs and flowing the best paying cargo under commercial constraints. Inspiration for formulation and solution method is taken from the rich research done within pickup and delivery problems. The problem, the multicommodity one-to-one pickup and delivery traveling salesman problem with path duration limits is, to the best of out knowledge, considered for the first time. An arc flow and a path flow model are presented. A Branch and Cut and Price solution method is proposed and implemented.**

Index Terms—**Traveling salesman problem, Liner shipping, Branch-and-Cut-and-Price, shortest path, green logistics.**

I. Introduction

CONTAINER shipping carriers operate worldwide networks consisting of hundreds of vessels and with operating costs of billions of dollars. Developing methods that can improve these network costs and service levels are of huge importance for both the carriers and the customers, indirectly all of us, as most manufactured produce can trace parts of its origin to distant continents.

Research on Liner Shipping Network Design Problems (LSNDP) has so far been limited and relatively isolated. At the same time research in seemingly related problems of various pickup and delivery problems has been abundant. This paper aims to use effective formulations from well studied pickup and delivery problems and adding complexity to attain a closer resemblance with LSNDP problems.

Container shipping networks provide transport of containers from port to port at a fixed schedule, usually weekly, with a predetermined path duration. The networks consist of a number of *services*, a set of similarly sized vessels sailing on a cyclic itinerary of ports. The services meet at certain hub ports where interchange of cargo can happen. Usually the service frequency is weekly, and the roundtrip time is a multiplier of 7, with the same number of vessels assigned, e.g. a 42 days roundtrip service, has 6 vessels assigned, giving a weekly portcall for all ports on the service.

Manuscript received December 8, 2011.
Christian E. M. Plum and David Pisinger {pisinger@man.dtu.dk} are with the Technical University of Denmark, Operations research group at DTU Management, Produktionstorvet, bygning 426, DK-2800 Kgs. Lyngby

Juan-José Salazar-González {jjsalaza@ull.es} are with DEIOC, Facultad de Matemáticas, Universidad de La Laguna, 38721 Tenerife, Spain

Presenting and corresponding author Christian E. M. Plum {christian.edinger.munk.plum@maersk.com} and Mikkel M. Sigurd {mikkel.muhldorff.sigurd@maersk.com} are with Maersk line, Esplanaden 50, DK-1098 København K.

A demand will then be loaded at its origin port to some service, which may bring the demand directly to its destination or unload it at some hub port for transhipment to another service, ultimately bringing the cargo to its destination. For a more general introduction to the economics of liner shipping please refer to Stopford [19] and Notteboom [15].

In this paper we will investigate a limited version of the full LSNDP. This can be interesting as an efficient method for a subset of the LSNDP, which can be extended to cover a larger part of the problem domain later.

The investigated problem will consider the design of a single capacitated service following a simple cyclic rotation where all ports must be visited, a hamiltonian tour.

In its own right this can also be interesting for a network planner designing a single service. Fierce competition often require that path durations are low, while the best paying cargo must be prioritized for the limited capacity. Here it can be part of an important decision support tool.

The problem is to transport a set of demands on a generated roundtrip, where the combined weight of these demands must respect the capacity of all edges. A demand has a maximal path duration which must be respected, alternatively the cargo can be left at port, without being transported. A demand can be partly fulfilled, but must still respect the path duration limit.

This problem will be denoted *the multi-commodity one-to-one pickup-and-delivery traveling salesman problem with path duration limits*, or in short *m-t*-PDTSP, to the best of our knowledge this problem has not been addressed before.

A. Liner shipping

Research on Liner Shipping problems has been relatively scarce, for an overview of earlier research on the topic please refer to Christiansen et al. [7] and Christiansen et al. [6]. Since these reviews, the interest in the field has increased and a number of articles has been published, with various approaches and scopes of the LSNDP. The work of Shintani et al. [18] has a detailed description of the problem cost structure and includes consideration of repositioning empty containers. The network design problem considered by Agarwal and Ergun [1] generate multiple services and handles transhipments, a bender's and a column generation based algorithm is implemented. These scale to large instances, but a drawback is that transhipments costs are excluded. The model of Alvarez [2] considers transhipment cost and find solutions for large instances in a heuristical column generation approach. The Branch-and-Cut method of Reinhardt and Pisinger [17] has the first model considering

ISBN: 978-988-19251-9-0
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

both transhipments and allowing for non-simple rotations (with two calls to a single port), smaller instances are solved optimally. The models of Gelareh et al. [9] and Gelareh and Pisinger [10] use a hub location based approach, generating a main service servicing some ports directly and others are feedering to these. The work of Løfstedt et al. [14] describes the domain of LSNDP, discusses the relevant scoping, has a revised model of the problem based on the model of Alvarez [2] and presents a number of benchmark instances for the LSNDP based on real world problems. A novel aggregation of demands was presented in Jepsen et al. [13] giving a new model formulation and decomposition method, though this did not perform well in practice. A heuristic for a short horizon version of the problem is presented by Wang and Meng [20]. This multitude of work on LSNDP shows that interest in these important problems has picked up. Most of this work considers different models and scopes of the problem. The optimal methods can only solve smaller problem instances (10 - 15 ports), and as real world instances are much larger the problem is still open as highlighted by the work of Løfstedt et al. [14].

B. Pickup and Delivery problems

The *m-t*-PDTSP is inspired by the wealth of research done within pickup and delivery problems, Parragh et al. [16] and Berbeglia et al. [4] both give good introductions to these problems, reviews existing literature and proposes classification schemes. In the classification of Parragh et al. [16] the *m-t*-PDTSP is a Single Dial A Ride Problem (SDARP) excluding Time Windows and with the important difference that no depot is required, i.e. cargo can be carried through the depot. In the classification of Berbeglia et al. [4] the *m-t*-PDTSP is a **[1-1|PD|1]**, **1-1** as each commodity has one origin and one destination, **PD** as each vertex must be visited exactly once for combined pickup and delivery and **1** as a single service is generated, but again an important difference from related problems is the lack of depot. The multi-commodity one-to-one pickup-and-delivery traveling salesman problem (*m*-PDTSP) in considered in Hernández-Pérez and Salazar-González [11], the problem is formulated and solution methods based on bender's decomposition are implemented. The *m-t*-PDTSP is an extension of the *m*-PDTSP, with the addition of path duration, no depot or allowing cargo to be carried through it and allowing partial carriage of flow, which has an associated revenue. An often encountered type of subproblem in pickup and delivery problems are Shortest Path Problems with Resource Constraints (SPPRC) which is also found in a decomposition of the *m-t*-PDTSP. For a review on SPPRC problems and algorithms please refer to Irnich and Desaulniers [12].

II. Mathematical Formulation

In the following we introduce the notation, present an arc flow model, followed by a path model and solution method for the *m-t*-PDTSP.

The service must visit all nodes, $i \in V$ exactly once. Directed edges $(i, j) \in A$ exists between all nodes, giving the complete directed graph $G = (V, A)$. Each arch $a \in A$ has a cost to traverse, c_a, representing time charter costs for the vessel, bunker cost for propulsion and portcall costs for visiting the port j. Traversing the arc takes time, t_a, this time will depend on the sailed distance and the speed of the vessel. The service has a capacity Q, which must be respected at all traversed arcs. The generated service can transport the commodities $k \in K$, each commodity k has a source s_k and a destination d_k, $(s_k \neq d_k)$, $(s_k, d_k \in V)$, a volume of cargo $F_k > 0$, a maximal transit time $t_k > 0$ and unit-revenue for transporting, $r_k > 0$. A node i can be the source of one or more commodities, as well as destination for some commodities.

A. Arc Flow Formulation

The problem is to find a maximal profit set of paths in G for a set of commodities k such that containers can be moved from origin to destination, in at most time t_k. All the paths should be placed on a hamiltonian tour, where each arc has some cost and traversal time.

Let x_a be a binary variable set if the service travels on arc $a \in A$. Let f_a^k be the flow of commodity k on arc a, binary variable x_a^k is set iff commodity k uses arc a. The problem can then be formulated as:

$$\min \sum_{a \in A} c_a x_a - \sum_{k \in K} r^k f_{s_k}^k \tag{1}$$

subject to

$$x(\delta^+(i)) = x(\delta^-(i)) \qquad \forall i \in V \tag{2}$$

$$x(\delta^+(i)) = 1 \qquad \forall i \in V \tag{3}$$

$$x(\delta^+(S)) \geq 1 \qquad \forall S \subset V \tag{4}$$

$$\sum_{a \in \delta^+(i)} f_a^k - \sum_{a \in \delta^-(i)} f_a^k \begin{cases} \leq q_k & \text{if } i = s_k, \\ = 0 & otherwise \end{cases} \qquad \forall i \in V, k \in K \tag{5}$$

$$\sum_{k \in K} f_a^k \leq Q x_a \qquad \forall a \in A \tag{6}$$

$$f_a^k \leq F_k x_a^k \qquad \forall a \in A, k \in K \tag{7}$$

$$\sum_{a \in A} x_a^k t_a \leq t_k \qquad \forall k \in K \tag{8}$$

$$f_a^k \geq 0 \qquad \forall a \in A, k \in K \tag{9}$$

$$x_a \in \{0, 1\} \qquad \forall a \in A \tag{10}$$

$$x_a^k \in \{0, 1\} \qquad \forall a \in A, k \in K \tag{11}$$

Where the objective minimizes the cost of the traversed arcs subtracted the revenue of flowed cargo. Constraint (2) that all nodes have the same number of ingoing and outgoing opened edges, where $\delta^+(i)$ and $\delta^-(i)$ denotes the set of ingoing, respectively outgoing, arcs to node i. Each node must be visited exactly once (3). Constraints (4) are subtour elimination constraints that ensure that the service connects all nodes in a single rotation. The conservation of flow is ensured by (5), and the capacity is enforced by (6). We can not transport more cargo than available, is ensured by (7) which also set variables x_a^k. The path duration is ensured by (8).

ISBN: 978-988-19251-9-0
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

B. Path Flow Formulation

The arc flow model given by (1) - (11) is Dantzig - Wolfe decomposed on the variables f_a^k. Let P_k be the set of all feasible paths from s_k to d_k, satisfying constraint (5), (7) and (8), this set has an exponential number of elements. Each path $p \in P_k$ is denoted as a set of arcs, i.e. $p \subset A$, let $\sum_{a \in p} t_a = t_p$ be the transit time of this path. Let λ_p be a non negative real variable, being the volume of flow using path p. The *m-t*-PDTSP can then be formulated as:

$$\min \sum_{a \in A} c_a x_a - \sum_{k \in K} r^k \sum_{p \in P_k} \lambda_p \tag{12}$$

subject to

$$x(\delta^+(i)) = x(\delta^-(i)) \qquad \forall i \in V \tag{13}$$
$$x(\delta^+(i)) = 1 \qquad \forall i \in V \tag{14}$$
$$x(\delta^+(S)) \geq 1 \qquad \forall S \subset V \tag{15}$$
$$\sum_{k \in K} \sum_{p \in P_k : a \in p} \lambda_p \leq Q x_a \qquad \forall a \in A \tag{16}$$
$$\sum_{p \in P_k} \lambda_p \leq F_k \qquad \forall k \in K \tag{17}$$
$$x_a \in \{0, 1\} \qquad \forall a \in A \tag{18}$$
$$\lambda_p \geq 0 \qquad \forall k \in K \tag{19}$$

The objective minimizes the costs of chosen arcs subtracted the revenue of flowed cargo. Constraints (13), (14) and (15) gives the hamiltonian tour. The capacity is enforced by (16). Convexity constraints (17) ensures that at most the available flow is transported.

The exponential subtour elimination constraints (15) are relaxed initially and inserted as *lazy constraints* when violated. A lower bound on the optimal value of this model can be attained by solving the LP-relaxation, where the integrality constraints (18) are replaced with constraints $0 \leq x_a \leq 1 \; \forall \; a \; \in \; A$.

This LP-relaxation is solved using a cut and price algorithm, due to the exponential number of variables λ_p. A restricted master problem is obtained by considering a subset $\bar{P} \subseteq P$ of paths. Additional columns of negative reduced costs are generated by solving a pricing subproblem. Let $\pi_a, free$ be the dual variables for the capacity constraints (16) and let $\theta_k \leq 0$ be dual variables for the convexity constraints (17). Then the pricing problem becomes:

$$\text{Min: } \sum_{a \in A} \pi_a x_a^k - \theta_k - r^k \tag{20}$$

Subject to constraints (5), (7) and (8).

C. Subtour Elimination Constraints

When the LP-relaxation of the path flow formulation has been solved it is checked if any violated subtours can be separated, this is done solving a min cut problem. The violated cuts are added and the LP-relaxation is resolved. When no additional violated subtours can be found, negative reduced costs columns are generated.

D. Pricing Problem

The pricing problem is an Elementary Shortest Path Problem with Resource Constraints (ESPPRC), which is strongly NP-hard as described in Irnich and Desaulniers [12]. The path must have the lowest cost given by arc weights π_a, while respecting path durations $\sum_{a \in A} x_a^k t_a \leq t_k$. As the pricing is working on the dual variables of π_a, negative cycles are likely and the subproblem algorithm must ensure the elementary property.

E. Branching

When all violated cuts and negative reduced costs columns have been added to the current node, and non binary variables x_a exists, branching is commenced. Binary branching is used, by selecting the most fractional x_a and adding constraints $x_a \leq 0$, $x_a \geq 1$ to the two branching children respectively.

III. Computational Results

The algorithm has been implemented using third party libraries were possible. The **COIN-OR DIP** (Galati and Ralphs [8]) framework has been used to implement the Branch-and-Cut-and-Price method, using **CPLEX 12.1** as LP solver. **boost**'s graph library (boost [5]) has an implementation of SPPRC, which has been used heuristically to solve the ESPPRC, solved as a MIP problem to prove optimality. **concorde** (Applegate et al. [3]) has an efficient implementation of a min cut algorithm and **boost** also finds connected components, to check if we have a feasible solution. The implementation has been run on a 3 GB Ram, Intel i5 2.53 GHz using a single core. An initial implementation can solve instances up to 16 nodes and 16 commodities in less than 1 hour, but further work is being done to refine the implementation and better results are expected. At the time of the conference we aim to present detailed computational results for the method.

IV. Conclusion and Further work

This work use effective formulations from well studied pickup and delivery problems on LSNDP by considering a multicommodity one-to-one capacitated pickup and delivery problem, with the extension of path duration time limits. Additionally the solution does not consider any depot, which gives a harder problem. A arc flow model as well as a path flow model and a solution method for this has been proposed. The solution method has been implemented and preliminary results are promising although further work with the implementation is required. Further detail can be included to capture the rich problems faced in liner shipping network design. This could be to: include berth windows, as a carrier will often have a limited number of berth hours available at some port. Allow multiple port calls to some ports, a port call both in- and outbound on a service, e.g. the service would be a non-simple cycle. Allow omissions of ports, i.e. relax hamiltonian tour requirement, so we only visit a port if the demand to / from the port can justify this. Allow multiple edges for the same port-port combination, with different cost and speed, resembling a vessel sailing faster or slower. Enforcing weekly frequency of the service. And ultimately allowing several services to be generated, which may even tranship cargo in between each other.

ISBN: 978-988-19251-9-0
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

ACKNOWLEDGMENT

The authors would like to thank Berit Dangaard Brouer for invaluable comments, Matthew Galati for his help using **COIN-OR DIP** and The Danish Council for Strategic Research for support for the ENERPLAN project.

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