Decoupled Fuzzy Sliding-mode Balance Control of Wheeled Inverted Pendulums Using An 8-bit Microcontroller

Chun-Fei Hsu and Chin-Yi Liu

***Abstract*—A wheeled inverted pendulum (WIP) system is a typical unstable complex nonlinear system widely utilized for educational purposes and control research. The dynamic of a WIP system can be represented as two second-order subsystems which represent the angle of body and the position of wheel. This paper proposes a decoupled fuzzy sliding-mode balance control (DFSBC) system based on a time-varying sliding surface for a WIP system. A decoupled sliding surface which includes the information of two-subsystem is designed to make the state trajectories of both subsystems move toward their sliding surface and then simultaneously approach zeros. The control effort of a WIP system is generated based on the idea that the state can quickly reach the decoupled sliding surface without large overshoot. Moreover, the slope of the decoupled sliding surface is adjusted by a fuzzy system, whose fuzzy rules are constructed based on the idea that the convergence time of the state trajectories can be reduced. Finally, an 8-bit microcontroller-based WIP system is setup. Experimental results show that the proposed DFSBC system can achieve favorable balance control response for the simultaneous control of the angle of body and the position of wheel.**

***Index Terms*—Fuzzy control, sliding-mode control, decoupled sliding surface, microcontroller, wheeled inverted pendulum**

I. Introduction

A wheeled inverted pendulum (WIP) system, like the Segway, has been well recognized as powerful personal transportation vehicles. The kind of transporter can be usually constructed by a synthesis of mechatronics, control techniques, and software. It is a typical unstable complex nonlinear system widely utilized for educational purposes and control research. Motion of a WIP system is governed by under-actuated configuration, i.e., the number of control inputs is less than the number of degrees of freedom to be stabilized [1], which makes it difficult to apply the conventional control approach for controlling Euler-Lagrange systems. The WIP systems are of interest because they have a small footprint and can turn on a dime. The kinematic model of WIP system is insufficient to describe the system behavior and has been proved to be uncontrollable [1]. In fact, balancing of the WIP system is only achieved by considering dynamic effects. Up to now, many studies have attracted a lot of attention of WIP system recently [1-5].

Grasser et al. [1] built a scaled-down prototype of a DSP-controlled two-wheeled vehicle based on the inverted pendulum, and Pathak et al. [2] studied the dynamic equations of the wheeled inverted pendulum by partial feedback linearization. Though the control system used to guarantee stability of the system in [1, 2], developing a highly accurate model of WIP system is complex. Ren et al. [3] proposed a self-tuning PID controller which the controller parameters are tuned automatically to overcome the disturbances and parameter variations. However, the system stability can not be guaranteed. A sliding-mode control method is proposed to be capable of handling both parameter uncertainties and external disturbances [4]. The algorithm of sliding-mode control required the system dynamics; however, it is difficult to be obtained in real-time application. Chiu [5] used an adaptive output recurrent cerebellar model articulation controller. The proposed scheme was implemented in a PC-based experimental system to verify its effectiveness but the design procedure is overly complex.

In the viewpoint of controller design, if the exact model of the controlled plant is well known, there exists an ideal controller to achieve a favorable control performance by possible canceling all the system dynamics [6]. A tradeoff between the stability and accuracy is necessary for the performance of ideal controller. To attack this problem, using human linguistic terms and common sense, several fuzzy controllers have been developed [7, 8]. Fuzzy control uses human-like linguistic terms in the form of IF-THEN rules to capture the nonlinear system dynamics. However, the huge amount of fuzzy rules makes the analysis complex. Some researchers have proposed fuzzy sliding-mode control design methods to reduce the fuzzy control rules [9-11]. By using the sliding surface, the fuzzy sliding-mode control system not only possesses more robustness against parameter variations and external disturbances than fuzzy control, but it also can be easily designed to guarantee the system stability in the Lyapunov sense.

This paper considers a WIP system which uses one 9V DC servomotor to move the body of a WIP system. So the

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considered WIP system only can forward and backward run. The WIP system can be described as two second-order subsystems which represent the angle of body and the position of wheel. Then, a decoupled fuzzy sliding-mode balance control (DFSBC) system is proposed to control the considered WIP system. The proposed DFSBC system uses a decoupled sliding surface including the information of two subsystems to generate a control effort, thus the state trajectories of both subsystems move toward their sliding surface and then simultaneously approach zeros. Moreover, the slope of decoupled sliding surface is adjusted by a fuzzy system, whose fuzzy rules are constructed based on the idea that the convergence time of the state trajectories can be reduced. Since the proposed DFSBC method is easy to implement, an inexpensive 8-bit microcontroller is used to hardware implement. From the experimental results, it shows that the proposed DFSBC system can achieve a favorable balance control response for the simultaneous control of the angle of body and the position of wheel even under an external disturbance.

II. Problem Formulation

The mechanical of the WIP system as shown in Fig. 1 has one dimension of motion. It uses one 9V DC servomotor to move the body of WIP system. This paper considers a LEGO NXT servomotor as shown in Fig. 2. The LEGO NXT servomotor is specific to LEGO Mindstorms NXT set. The DC servomotor uses many gears to get high torque. These gears are not precisely manufactured and assembled; therefore, this results in certain amount of mechanical backlash. The motion equation of a DC servomotor can be simplified as [12-14]

$$J_w\ddot{\theta} + B_w\dot{\theta} = T_e \tag{1}$$

where J_w is the moment of inertia, B_w is the damping coefficient, θ is the rotor position, and T_e denotes the electric torque. The electric torque is defined as

$$T_e = K_t i_w \tag{2}$$

where K_t is the torque constant and i_w is the torque current. The electric equation of a DC servomotor can be simplified as

$$v_w = R_w i_a + K_b\dot{\theta} + L_w\frac{di_w}{dt} \tag{3}$$

where R_w is the DC servomotor resistance, K_b is the back electromotive force coefficient, L_w is the DC servomotor inductance, and v_w is the DC servomotor voltage. Ignoring the DC servomotor's inductance and damping coefficient, the dynamic equation of the DC servomotor can be represented as

$$\ddot{\theta} = -\frac{K_t K_b}{J_w R_w}\dot{\theta} + \frac{K_t}{J_w R_w}u \tag{4}$$

where $u = v_w$ is the control input in voltage.

The WIP system has a single-input multi-output structure where one single input force has to control both the angle of body and the position of wheel at the same time. Figure 1 shows the coordinate of WIP system where θ is the wheel angle, ψ is the body pitch angle, L is the distance of the center of mass from the wheel axle, R is the wheel radius, m and M are the wheel and body weights, respectively, and J_w and J_ψ are the wheel and body pitch inertia moments, respectively. By the Lagrangian method, the motion equation of WIP system including actuator dynamics based on the coordinate system can be derived as follow [1]

$$\mathbf{E}\begin{bmatrix}\ddot{\theta}\\ \ddot{\psi}\end{bmatrix} + \mathbf{F}\begin{bmatrix}\dot{\theta}\\ \dot{\psi}\end{bmatrix} + \mathbf{G}\begin{bmatrix}\theta\\ \psi\end{bmatrix} = \mathbf{H}u \tag{5}$$

where $\mathbf{E}$ denotes the inertia matrix, $\mathbf{F}$ expresses the matrix of centripetal and Coriolis forces, $\mathbf{G}$ is the gravity vector, and $\mathbf{H}$ is the gain vectors. The matrixes can be found as

$$\mathbf{E} = \begin{bmatrix}(2m+M)R^2 + 2J_w + 2J_m & MLR - 2J_m\\ MLR - 2J_m & ML^2 + J_\psi + 2J_m\end{bmatrix} \tag{6}$$

$$\mathbf{F} = \begin{bmatrix}\frac{K_t K_b}{R_m} + f_m + f_w & -\frac{K_t K_b}{R_m} - f_m\\ -\frac{K_t K_b}{R_m} - f_m & \frac{K_t K_b}{R_m} + f_m\end{bmatrix} \tag{7}$$

$$\mathbf{G} = \begin{bmatrix}0 & 0\\ 0 & -MgL\end{bmatrix} \tag{8}$$

$$\mathbf{H} = \begin{bmatrix}\frac{K_t}{R_m}\\ -\frac{K_t}{R_m}\end{bmatrix} \tag{9}$$

where f_m is the friction coefficient between body and DC servomotor and f_w is the friction coefficient between wheel and floor. The WIP system can be treated as two subsystems A and B, which include the states ($\theta,\dot{\theta}$) and ($\psi,\dot{\psi}$), respectively. The subsystem A is chosen as a primary target while the subsystem B is used as a secondary target. Define the state variable as

$$\mathbf{x} = \left[\theta,\dot{\theta},\psi,\dot{\psi}\right]^T. \tag{10}$$

The motion equations of WIP system can be written as

$$A:\ \ddot{\theta} = f_1(\mathbf{x}) + g_1(\mathbf{x})u \tag{11}$$

$$B:\ \ddot{\psi} = f_2(\mathbf{x}) + g_2(\mathbf{x})u \tag{12}$$

where $f_1(\mathbf{x})$, $f_2(\mathbf{x})$, $g_1(\mathbf{x})$ and $g_2(\mathbf{x})$ are the nonlinear system dynamics. For the controller design, since an exact dynamic WIP model is difficult to develop, controlling a WIP system using conventional control techniques is difficult in real-time applications.

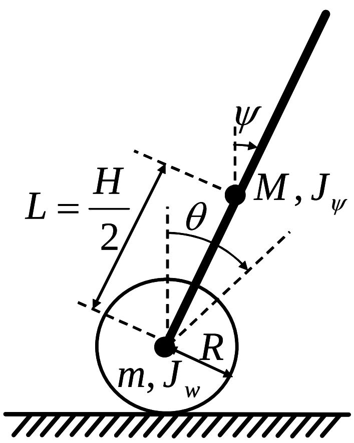

Fig. 1. The coordinate of WIP system.

Fig. 2. The used 9V DC servomotor.

III. DFSBC System Design

Conventional control technologies always require a good understanding of a plant, but the dynamics of a WIP system is difficult to obtain. To attack this problem, this paper proposes a DFSBC system as shown in Fig. 2 to control the whole system states of WIP system to approach to zeros with satisfactory transient responses. Define a decoupled sliding surface as

$$s_d = s_A + \lambda_d s_B \tag{13}$$

where $s_A = \dot{\theta} + \lambda_A \theta$, $s_B = \dot{\psi} + \lambda_B \psi$, and λ_A, λ_B and λ_d are positive constants. The control objective of the DFSBC system is to let the decoupled sliding surface s_d approach zero. In this case, the sliding surface variables s_A and s_B will simultaneously converge to zeros, and then the two subsystems ($\theta, \dot{\theta}$) and ($\psi, \dot{\psi}$) will also converge to zeros simultaneously. From the sliding-mode control viewpoint, the slope of the decoupled sliding surface λ_d will govern the transient responses of the states. The slope of the decoupled sliding surface λ_d will play an important role in governing the interactive transient responses between the states of these two subsystems. Figure 4 shows the region for possible slopes of sliding surfaces in the stable zone of the phase plane (the second and fourth quadrants). If large value of λ_d is available the system will be more stable but the tracking accuracy may be degraded because of a longer reaching time of the representative point to the sliding surface.

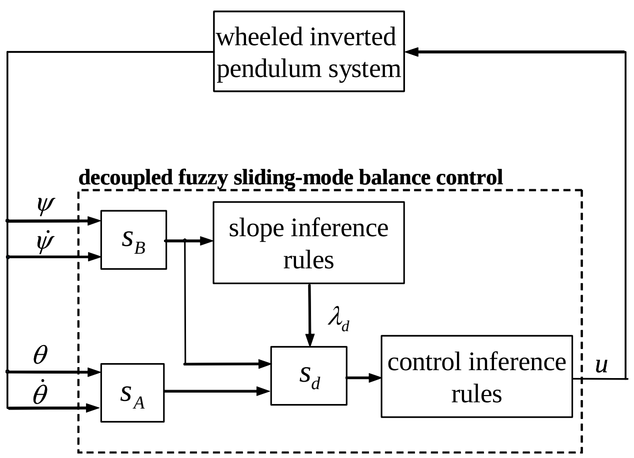

Fig. 3. The black diagram of the proposed DFSBC system.

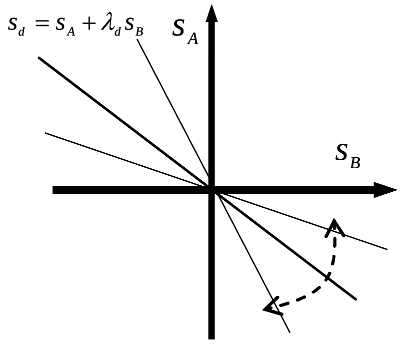

Fig. 4. Slope rotating sliding surface.

The time-varying slope computations of decoupled sliding surface are performed by a function which is derived from the input-output mapping of the one-dimensional fuzzy rule bases. The slope inference rules are expressed as

Rule *i*: If s_B is F_1^i, Then λ_d is α_i (14)

where F_1^i, $i = 1, 2, \cdots, n$ are the labels of the fuzzy sets characterized by the fuzzy membership functions $\mu_{F_1^i}(\cdot)$ and α_i are the singleton slope given in Table 1. This slope inference system is constructed by the idea of decreasing trajectory convergence time. The control inference rules are designed as

Rule *j*: If s_d is F_2^j, Then u is β_j (15)

where F_2^j, $j = 1, 2, \cdots, m$ are the labels of the fuzzy sets characterized by the fuzzy membership functions $\mu_{F_2^j}(\cdot)$ and β_j are the singleton control actions given in Table 2. The control inference system is constructed using the idea that the state can quickly reach the decoupled sliding surface without large overshoot. The fuzzy labels used in this study are negative big (NB), negative medium (NM), negative small (NS), zero (ZO), positive small (PS), positive medium (PM) and positive big (PB). The defuzzification of the output is accomplished by the method of center-of-gravity [8]

$$\lambda_d = (\sum_{i=1}^{m} w_i \times \Theta_i) / \sum_{i=1}^{m} w_i \tag{16}$$

$$u = (\sum_{j=1}^{n} v_j \times \Omega_j) / \sum_{j=1}^{n} v_j \tag{17}$$

where w_i and v_j are the firing weights of the *i*-th and *j*-th rules of Eqs. (14) and (15), respectively. It implies that an important consequence of using the proposed DFSBC method is that the second subsystem is successfully incorporated into the first one.

Table 1. The fuzzy IF-THEN rules of the fuzzy controller.

s_B	NB	NM	NS	ZO	PS	PM	PB
λ_d	17	14	11	10	11	14	17

Table 2. The fuzzy IF-THEN rules of the slope regulator.

s_d	NB	NM	NS	ZO	PS	PM	PB
u	-8	-5	-2	0	2	5	8

IV. Experimental Results

For the control applications, DSP with high-speed analog-to-digital (A/D) converters can be a solution for the real-time control applications. However, the high cost of a

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DSP and the associated hardware restrict its application. On the other hand, a microcontroller can be integrated with some analog peripherals that can compensate for the limitation in computing power while expanding their functionalities at low cost. It would be more suitable for the real-time control applications than DSPs. Based on the advantage, this study proposes an experimental setup using 8-bit microcontroller (MPC82G516) for a WIP system as shown in Fig. 5. The WIP system in this paper has a 9V DC servomotor connected to a gearbox for a wheel and a gyro sensor (ADXRS613) is used to calculate body pitch angle.

The software flowchart of the control algorithm is shown in Fig. 6. In the main program, the initialization parameters and input/output (I/O) setups are to be proceeded. Next, the interrupt interval for the ISR1 with a 1msec sampling rate is set and the interrupt interval for the ISR2 with a 10msec sampling rate is set. An optical encoder built into the NXT DC servomotor. The encoder generates signals in quadrature, and that allows the controller to determine the direction and speed of the servomotor. The gyro offset function takes 100 gyro samples over a time and averages them to get the offset. It also checks the max and min during that time and if the difference is larger than one it rejects the data and gets another set of samples.

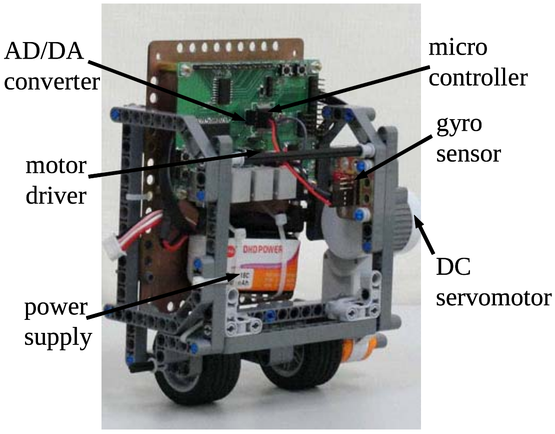

Fig. 5. The experimental setup.

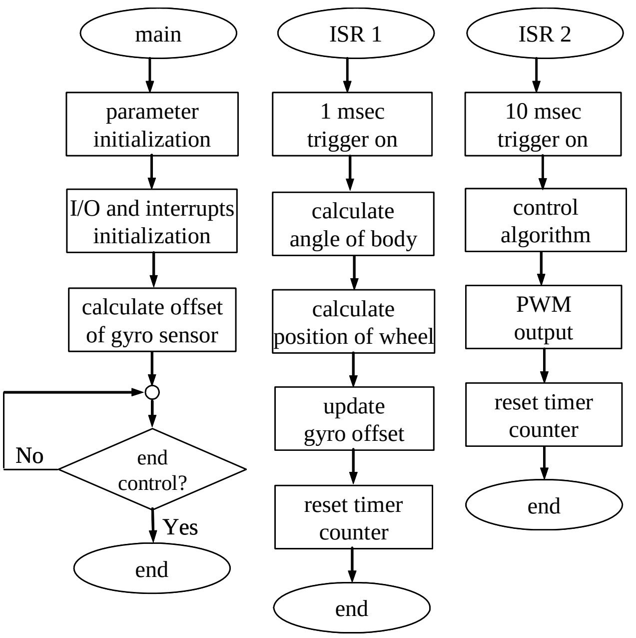

Fig. 6. The software flowchart.

To investigate the effectiveness of the proposed DFSBC system, two scenarios are carried out. Scenario 1 presents stand upright scenario and scenario 2 presents external disturbance scenario. It should be emphasized that the development of the DFSBC system does not need to know the model of WIP system. The parameters in the proposed DFSBC system are selected as $\lambda_A = 0.16$ and $\lambda_B = 1$. The experimental result of scenario 1 is shown in Fig. 7. The angle of body response is shown in Fig. 7(a), the position of wheel response is shown in Fig. 7(b), and the control input is shown in Fig. 7(c). The experimental results show that the angle of body and the position of wheel can be simultaneously controlled to converge to zeros with favorable transient responses. Further, the DFSBC system is applied to the WIP system with external disturbance again. The experimental result of scenario 2 is shown in Fig. 8. The angle of body response is shown in Fig. 8(a), the position of wheel response is shown in Fig. 8(b), and the control input is shown in Fig. 8(c). The experimental results show that the pendulum can stand upright stably and the body can return to the origin point when affected by unknown external disturbances. In summary, the proposed DFSBC method not only exhibits a simpler structure compared with the existing decoupled control methods but also eliminates the need for using fuzzy rules without degradation in performance.

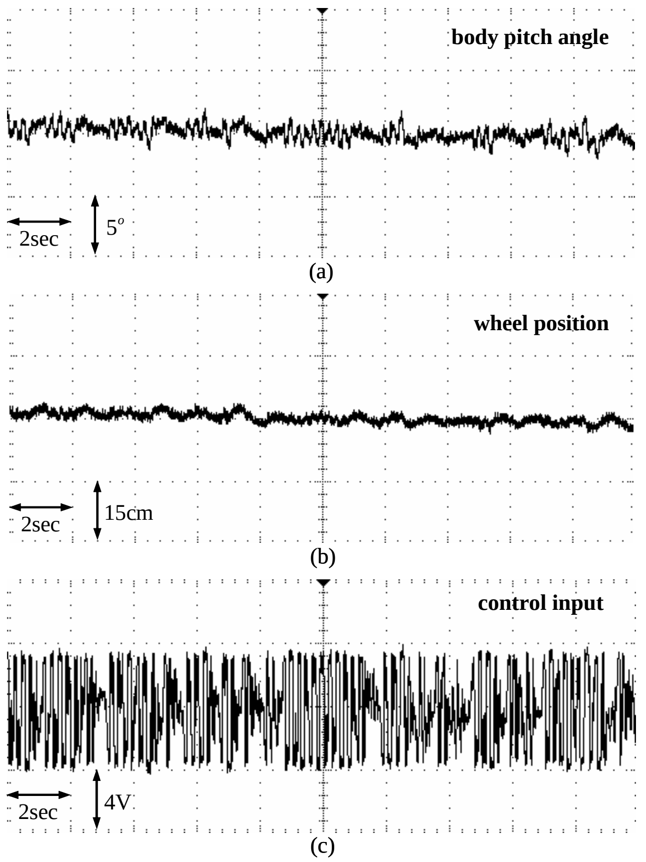

Fig. 7. The experimental results of scenario 1.

V. Conclusions

This paper focuses on adopting a self dynamic balancing control strategy for a wheeled inverted pendulums (WIP) system. The system dynamics of WIP system is described as two second-order subsystems which represent the angle of body and position of wheel. In the proposed decoupled fuzzy sliding-mode balance control (DFSBC) scheme, a decoupled sliding surface which includes information of two subsystems is defined to generate a control effort to make the state trajectories of both subsystems move toward their sliding surface and then simultaneously approach zeros. Moreover, the slope of decoupled sliding surface is adjusted by a fuzzy system, whose fuzzy rules are constructed based on the idea that the convergence time of the state trajectories can be reduced. Finally, an 8-bit microcontroller is used to implement the experimental setup. Experimental results show that the pendulum can stand upright stably and the body can return to the origin point when affected by unknown external disturbances by the proposed DFSBC system.

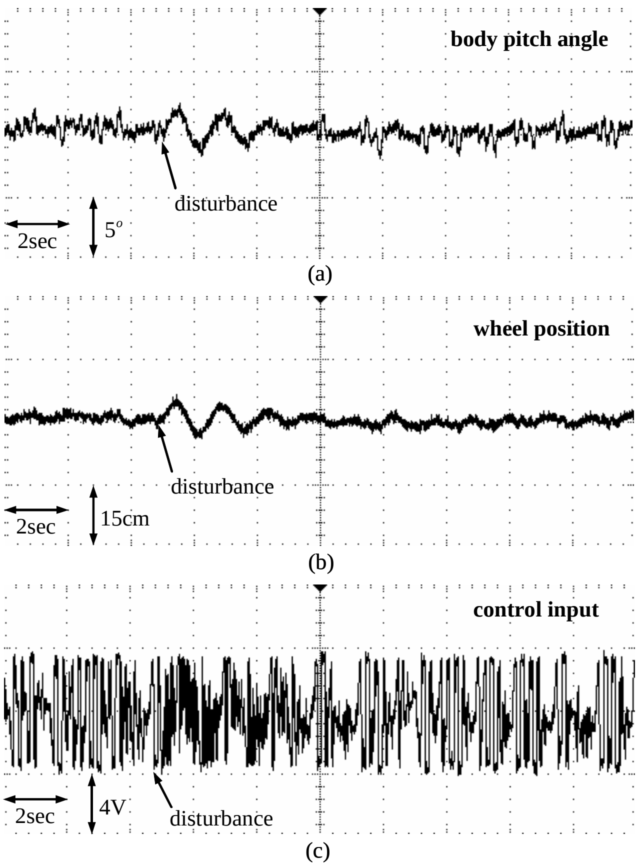

Fig. 8. The experimental results of scenario 2.

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