Proceedings of the International MultiConference of Engineers and Computer Scientists 2012 Vol II,
IMECS 2012, March 14 - 16, 2012, Hong Kong

$\mathcal{H}_\infty$ Tracking Control of LPV Systems over a Communication Network

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***Abstract*—This paper investigates the problem of $\mathcal{H}_\infty$ tracking control design over a communication network that considers network-induced delays, packet losses, and so on. The goal of this paper is to present the method of applying a relaxation technique to improve the $\mathcal{H}_\infty$ tracking performance in such a framework, which allows to fully exploit the information on parameters of LPV systems.**

***Index Terms*—Linear Parameter Varying (LPV), Network Control System (NCS), tracking control.**

I. Introduction

The design of networked control systems for nonlinear systems have been recently received considerable attention in control societies to overcome the spatial limits of the traditional control systems. Thus, abundant literature on networked control systems (NCSs) have been published in the literature: for stabilization problem [3], [4], $\mathcal{H}_\infty$ stabilization problem [2], [5], $\mathcal{H}_\infty$ tracking control problem [1], and so on. In particular, the linear parameter-varying (LPV) approach has been regarded as a way capable of systematically representing nonlinear systems because the well-established linear-system theory including the convex optimization can be still applied to the analysis and synthesis of such systems. However, an important issue in the LPV-model-based approach [6] is how to take advantage of the measurable parameter in the direction of improving the control performance, such as $\mathcal{H}_\infty$ performance under consideration in this paper.

Thus, various attempts have been carried out to achieve the performance improvement (see [7]). However, to the best of our knowledge, there are only handful of results that partially exploit the information on parameters of LPV systems in the process of studying the tracking control problem. Motivated by the above concerns, we discuss a way to address the problem of $\mathcal{H}_\infty$ tracking control design over a communication network that considers network-induced delays, packet losses, and so on. To this end, methodologically, we propose the continuous-time version of the relaxation technique given in [10].

This research was supported by Basic Science Research Program through the National Research Foundation of Korea(NRF) funded by the Ministry of Education, Science and Technology (2010-0009743). This research was supported by the MKE(The Ministry of Knowledge Economy), Korea, under the ITRC(Information Technology Research Center) support program supervised by the NIPA(National IT Industry Promotion Agency) (NIPA-2011-(C-1090-1121-0006)) & (NIPA-2011-(C1090-1111-0011)). This research was supported by World Class University program funded by the Ministry of Education, Science and Technology through the National Research Foundation of Korea (R31-10100).

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The rest of the paper is organized as follows. Section II gives a mathematical description of NCSs and presents useful lemmas. Section III presents the main result of this paper. Furthermore, through a numerical example, Section IV shows the verification of our results. Finally, Section V makes the concluding remarks.

Notation: Throughout this paper, we will adopt standard notions. The notations $X \geq Y$ and $X > Y$ mean that $X - Y$ is positive semi-definite and positive definite, respectively. In symmetric block matrices, $(*)$ is used as an ellipsis for terms that are induced by symmetry.

II. System description and useful lemmas

Consider an LPV system of the following form:

$$\begin{aligned} \dot{x}(t) &= A(\Theta_t)x(t) + B_u(\Theta_t)u(t) + B_w(\Theta_t)w(t) \\ y(t) &= C(\Theta_t)x(t), \end{aligned} \tag{1}$$

where $x(t) \in \mathbb{R}^{n_x}$ and $u(t) \in \mathbb{R}^{n_u}$ denote the state and the control input, respectively; and $w(t) \in \mathbb{R}^{n_w}$ denotes the disturbance input such that $w(t) \in \mathcal{L}_2^+$ and $\theta(t)$ is the time-varying parameter and $y(t)$ is the output. Assume that $\sum_{i=1}^{r} \theta_i(t) = 1$ and $0 \leq a_i \leq \theta_i(t) \leq b_i (i = 1, 2, ..., r)$.

$$A(\Theta_t) = A_0 + \sum_{i=1}^{r} \theta_i(t) A_i, B_u(\Theta_t) = B_{u,0} + \sum_{i=1}^{r} \theta_i(t) B_{u,i}, \tag{2}$$

$$B_w(\Theta_t) = B_{w,0} + \sum_{i=1}^{r} \theta_i(t) B_{w,i}, C(\Theta_t) = C_0 + \sum_{i=1}^{r} \theta_i(t) C_i$$

Connected with (1), we consider the following dynamic system that generates the reference signal $x_r(t) \in \mathbb{R}^{n_x}$:

$$\dot{x}_r(t) = A_r x_r(t) + r(t), \tag{3}$$

where A_r is an asymptotically stable matrix and $r(t)$ denotes the reference input belonging to $\mathcal{L}_2^+$. Here define the tracking error $e(t)$ as $e(t) = x(t) - x_r(t)$ and apply the following control law to (1):

$$u(t) = F(\Theta_t)e(t - d(t)), \tag{4}$$

where $e(t - d(t)) = x(t - d(t)) - x_r(t - d(t))$ and $d_1 \leq d(t) \leq d_2$,

$$F(\Theta_t) = F_0 + \sum_{i=1}^{r} \theta_i(t) F_i.$$

Then the resultant closed-loop system is given by

$$\dot{\zeta}(t) = \tilde{A}(\Theta_t)\zeta(t) + \tilde{A}_d(\Theta_t)\zeta(t - d(t)) + \tilde{B}_w(\Theta_t)\tilde{w}(t), \tag{5}$$

$$z(t) = \tilde{C}(\Theta_t)\zeta(t), \tag{6}$$

where $\zeta(t) = \mathbf{col}(e(t), x_r(t))$, $\tilde{w}(t) = \mathbf{col}(w(t), r(t))$,

$$\tilde{A}(\Theta_t) = \begin{bmatrix} A(\Theta_t) & A(\Theta_t) - A_r \\ 0 & A_r \end{bmatrix}, \tag{7}$$

$$\tilde{A}_d(\Theta_t) = \begin{bmatrix} B_u(\Theta_t)F(\Theta_t) & 0 \\ 0 & 0 \end{bmatrix}, \tag{8}$$

$$\tilde{B}_w(\Theta_t) = \begin{bmatrix} B_w(\Theta_t) & -I \\ 0 & I \end{bmatrix}, \tilde{C}(\Theta_t) = \begin{bmatrix} C(\Theta_t) & 0 \end{bmatrix}$$

Lemma 2.1: For real matrices X, Y, and $S > 0$ with appropriate dimensions, it is satisfied that $0 \le (X - SY)^T S^{-1}(X - SY)$ and hence the following inequality holds: $Y^T SY \ge X^T Y + Y^T X - X^T S^{-1} X$. Further if $X = \mu I$, then $Y^T SY \ge \mu Y + \mu Y^T - \mu^2 S^{-1}$, where μ is a scalar. On the other hand, if $S < 0$, then it is assured that $Y^T SY \le -\mu Y - \mu Y^T - \mu^2 S^{-1}$

III. Main results

A. PLMI-type condition

Choose a Lyapunov-Krasovskii functional

$$\begin{aligned} V(t) \triangleq & V_1(t) + V_2(t) + V_3(t), \\ V_1(t) = & \zeta(t)^T P\zeta(t), \\ V_2(t) = & \int_{t-d_1}^{t} \zeta^T(\alpha)Q_1\zeta(\alpha)d\alpha + \int_{t-d_2}^{t} \zeta^T(\alpha)Q_2\zeta(\alpha)d\alpha, \\ V_3(t) = & d_1 \int_{-d_1}^{0}\int_{t+\beta}^{t} \dot{\zeta}^T(\alpha)R_1\dot{\zeta}(\alpha)d\alpha d\beta \\ & + d_{21}\int_{-d_2}^{-d_1}\int_{t+\beta}^{t} \dot{\zeta}^T(\alpha)R_2\dot{\zeta}(\alpha)d\alpha d\beta \end{aligned} \tag{9}$$

where P, Q_1, Q_2, R_1, and R_2 are positive definite with appropriate dimensions. Let us define an augmented state $\eta(t) = \mathbf{col}(\zeta(t), \zeta(t-d_1), \zeta(t-d(t)), \zeta(t-d_2), \tilde{w}(t))$ and the corresponding block entry matrices as

$$\begin{aligned} & e_1 \triangleq \begin{bmatrix} I & 0 & 0 & 0 & 0 \end{bmatrix}, e_2 \triangleq \begin{bmatrix} 0 & I & 0 & 0 & 0 \end{bmatrix}, \\ & e_3 \triangleq \begin{bmatrix} 0 & 0 & I & 0 & 0 \end{bmatrix}, e_4 \triangleq \begin{bmatrix} 0 & 0 & 0 & I & 0 \end{bmatrix}, \\ & e_5 \triangleq \begin{bmatrix} 0 & 0 & 0 & 0 & I \end{bmatrix}, \\ & \Phi_t \triangleq \tilde{A}(\Theta_t)e_1 + \tilde{A}_d(\Theta_t)e_3 + \tilde{B}_w(\Theta_t)e_5 \end{aligned}$$

Time derivative of $V_i(t)$

$$\dot{V}_1(t) = \eta^T(t)\mathbf{He}(e_1^T P\Phi_t)\eta(t), \tag{10}$$

$$\dot{V}_2(t) = \eta^T(t)\left(e_1^T(Q_1+Q_2)e_1 - e_2^T Q_1 e_2 - e_4^T Q_2 e_4\right)\eta(t), \tag{11}$$

$$\dot{V}_3(t) = \eta^T(t)\Phi_t^T\left(d_1^2 R_1 + d_{21}^2 R_2\right)\Phi_t\eta(t) + \mathcal{O}, \tag{12}$$

where

$$\begin{aligned} \mathcal{O} = & -d_1\int_{t-d_1}^{t} \dot{\zeta}^T(\alpha)R_1\dot{\zeta}(\alpha)d\alpha - d_{21}\int_{t-d(t)}^{t-d_1} \dot{\zeta}^T(\alpha)R_2\dot{\zeta}(\alpha)d\alpha \\ & - d_{21}\int_{t-d_2}^{t-d(t)} \dot{\zeta}^T(\alpha)R_2\dot{\zeta}(\alpha)d\alpha \end{aligned}$$

$$\dot{V}(t) = \eta^T(t)\Pi_0\eta(t) + \mathcal{O} \tag{13}$$

where

$$\begin{aligned} \Pi_0 = & \mathbf{He}(e_1^T P\Phi_t) + e_1^T(Q_1+Q_2)e_1 - e_2^T Q_1 e_2 - e_4^T Q_2 e_4 \\ & + \Phi_t^T\left(d_1^2 R_1 + d_{21}^2 R_2\right)\Phi_t \end{aligned} \tag{14}$$

By employing the Jensen inequality [8] and the lower bounds lemma [9], we can obtain a upper bound of $\mathcal{O}$ as follows

$$\begin{aligned} \mathcal{O} \le & -(\int_{t-d_1}^{t}\dot{\zeta}(\alpha)d\alpha)^T R_1(\int_{t-d_1}^{t}\dot{\zeta}(\alpha)d\alpha) \\ & - \frac{1}{\rho_1(t)}(\int_{t-d(t)}^{t-d_1}\dot{\zeta}(\alpha)d\alpha)^T R_2(\int_{t-d(t)}^{t-d_1}\dot{\zeta}(\alpha)d\alpha) \\ & - \frac{1}{\rho_2(t)}(\int_{t-d_2}^{t-d(t)}\dot{\zeta}(\alpha)d\alpha)^T R_2(\int_{t-d_2}^{t-d(t)}\dot{\zeta}(\alpha)d\alpha) \\ = & -\eta^T(t)(e_1-e_2)^T R_1(e_1-e_2)\eta(t) \\ & - \frac{1}{\rho_1(t)}\eta^T(t)(e_2-e_3)^T R_2(e_2-e_3)\eta(t) \\ & - \frac{1}{\rho_2(t)}\eta^T(t)(e_3-e_4)^T R_2(e_3-e_4)\eta(t) \end{aligned} \tag{15}$$

where

$$\begin{aligned} & \rho_1(t) = (d(t)-d_1)/d_{21} \ge 0, \rho_2(t) = (d_2-d(t))/d_{21} \ge 0, \\ & \rho_1(t) + \rho_2(t) = 1 \end{aligned} \tag{16}$$

(15) is represented as the following inequality:

$$\begin{aligned} \mathcal{O} \le & \eta^T(t)\Pi_1\eta(t) \\ & - \begin{bmatrix} \sqrt{\frac{\rho_2}{\rho_1}}(e_2-e_3)\eta(t) \\ \sqrt{\frac{\rho_1}{\rho_2}}(e_3-e_4)\eta(t) \end{bmatrix}^T \Pi_2 \begin{bmatrix} \sqrt{\frac{\rho_2}{\rho_1}}(e_2-e_3)\eta(t) \\ \sqrt{\frac{\rho_1}{\rho_2}}(e_3-e_4)\eta(t) \end{bmatrix} \end{aligned} \tag{17}$$

where

$$\begin{aligned} \Pi_1 = & (e_1-e_2)^T R_1(e_2-e_1) + (e_2-e_3)^T R_2(e_3-e_2) \\ & + (e_3-e_4)^T R_2(e_4-e_3) + \mathbf{He}((e_2-e_3)^T S(e_3-e_4)), \end{aligned} \tag{18}$$

$$\Pi_2 = \begin{bmatrix} R_2 & S \\ S^T & R_2 \end{bmatrix} \tag{19}$$

$\dot{V}(t)$ is upper-bounded by

$$\begin{aligned} \dot{V}(t) \le & \eta^T(t)(\Pi_0 + \Pi_1)\eta(t) \\ & - \begin{bmatrix} \sqrt{\frac{\rho_2}{\rho_1}}(e_2-e_3)\eta(t) \\ \sqrt{\frac{\rho_1}{\rho_2}}(e_3-e_4)\eta(t) \end{bmatrix}^T \Pi_2 \begin{bmatrix} \sqrt{\frac{\rho_2}{\rho_1}}(e_2-e_3)\eta(t) \\ \sqrt{\frac{\rho_1}{\rho_2}}(e_3-e_4)\eta(t) \end{bmatrix} \end{aligned} \tag{20}$$

Note: (Stability criterion in the $\mathcal{H}_\infty$ sense) ($\dot{V}(t) + z^T(t)z(t) - \gamma^2\tilde{w}^T(t)\tilde{w}(t) < 0$)

$$0 > \Pi_0 + \Pi_1 + \Pi_3, 0 \le \Pi_2 \tag{21}$$

where

$$\Pi_3 = e_1^T\tilde{C}^T(\Theta_t)\tilde{C}(\Theta_t)e_1 - \gamma^2 e_5^T e_5 \tag{22}$$

Set $P = \mathbf{diag}(P_1, P_2)$ and $\bar{P} = P^{-1} = \mathbf{diag}(\bar{P}_1, \bar{P}_2)$, where $\bar{P}_1 = P_1^{-1}$ and $\bar{P}_2 = P_2^{-1}$. Then,

$$P\tilde{A}(\Theta_t) = \begin{bmatrix} P_1A(\Theta_t) & P_1A(\Theta_t) - P_1A_r \\ 0 & P_2A_r \end{bmatrix},$$

$$P\tilde{A}_d(\Theta(t)) = \begin{bmatrix} P_1B_u(\Theta_t)\bar{F}(\Theta_t)P_1 & 0 \\ 0 & 0 \end{bmatrix},$$

$$P\tilde{B}_\omega(\Theta_t) = \begin{bmatrix} P_1B_\omega(\Theta_t) & -P_1 \\ 0 & P_2 \end{bmatrix},$$

where $\bar{F}(\Theta_t) = F(\Theta_t)\bar{P}_1$

ISBN: 978-988-19251-9-0
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

$$P\Phi_t = X\bar{A}(\Theta_t)Xe_1 + X\bar{A}_d(\Theta_t)Xe_3 + X\bar{B}_\omega(\Theta_t)e_5 \tag{23}$$

where

$$X = diag(P_1, I), \tag{24}$$

$$\bar{A}(\Theta_t) = \begin{bmatrix} A(\Theta_t)\bar{P}_1 & A(\Theta_t) - A_r \\ 0 & P_2 A_r \end{bmatrix}, \tag{25}$$

$$\bar{A}_d(\Theta_t) = \begin{bmatrix} B_u(\Theta_t)\bar{F}(\Theta_t) & 0 \\ 0 & 0 \end{bmatrix}, \tag{26}$$

$$\bar{B}_\omega(\Theta_t) = \begin{bmatrix} B_\omega(\Theta_t) & -I \\ 0 & P_2 \end{bmatrix}, \tag{27}$$

Theorem 1: Let $\mu_1 > 0$, $\mu_2 > 0$ be prescribed. Suppose that there exist a scalar $\gamma > 0$; matrices $\bar{F}$, and $\tilde{S}$; and symmetric positive definite matrices $\bar{P}_1$, P_2, $\tilde{Q}_1$, $\tilde{Q}_2$, $\tilde{R}_1$, and $\tilde{R}_2$ such that

$$0 > \left[\begin{array}{cc|ccc|cc|c} (1,1) & 0 & d_1\bar{A}(\Theta_t) & 0 & d_1\bar{A}_d(\Theta_t) & 0 & d_1\bar{B}_\omega(\Theta_t) & 0 \\ 0 & (2,2) & d_{21}\bar{A}(\Theta_t) & 0 & d_{21}\bar{A}_d(\Theta_t) & 0 & d_{21}\bar{B}_\omega(\Theta_t) & 0 \\ \hline (*) & (*) & (3,3) & \tilde{R}_1 & \bar{A}_d(\Theta_t) & 0 & \bar{B}_\omega(\Theta_t) & \bar{X}\tilde{C}^T(\Theta_t) \\ 0 & 0 & (*) & (4,4) & \tilde{R}_2+\tilde{S} & -\tilde{S} & 0 & 0 \\ (*) & (*) & (*) & (*) & (5,5) & \tilde{R}_2+\tilde{S} & 0 & 0 \\ 0 & 0 & 0 & (*) & (*) & (6,6) & 0 & 0 \\ (*) & (*) & (*) & 0 & 0 & 0 & -\gamma^2 I & 0 \\ \hline 0 & 0 & \tilde{C}(\Theta_t)\bar{X} & 0 & 0 & 0 & 0 & -I \end{array}\right] \tag{28}$$

$$0 \le \begin{bmatrix} \tilde{R}_2 & \tilde{S} \\ \tilde{S}^T & \tilde{R}_2 \end{bmatrix} \tag{29}$$

where

$$\begin{aligned}
&(1,1) = \mu_1^2\tilde{R}_1 + \mathbf{diag}(-2\mu_1\bar{P}_1, -2\mu_1 P_2), \\
&(2,2) = \mu_2^2\tilde{R}_2 + \mathbf{diag}(-2\mu_2\bar{P}_1, -2\mu_2 P_2), \\
&(3,3) = \mathbf{He}(\bar{A}(\Theta_t)) + \tilde{Q}_1 + \tilde{Q}_2 - \tilde{R}_1, \\
&(4,4) = -\tilde{Q}_1 - \tilde{R}_1 - \tilde{R}_2, \\
&(5,5) = -2\tilde{R}_2 - \mathbf{He}(\tilde{S}), (6,6) = -\tilde{Q}_2 - \tilde{R}_2, \\
&\bar{A}(\Theta_t) = \begin{bmatrix} A(\Theta_t)\bar{P}_1 & A(\Theta_t) - A_r \\ 0 & P_2 A_r \end{bmatrix}, \\
&\bar{A}_d(\Theta_t) = \begin{bmatrix} B_u(\Theta_t)\bar{F}(\Theta_t) & 0 \\ 0 & 0 \end{bmatrix}, \bar{B}_\omega(\Theta_t) = \begin{bmatrix} B_\omega(\Theta_t) & -I \\ 0 & P_2 \end{bmatrix}, \\
&\tilde{C}(\Theta_t) = \begin{bmatrix} C(\Theta_t) & 0 \end{bmatrix}
\end{aligned}$$

Then the closed-loop systems (6) is asymptotically stable and satisfies $||z||_2 < \gamma||\tilde{w}||_2$ for all nonzero $\tilde{w}(t) \in \mathcal{L}_2[0,\infty)$ and for any time-varying delay $d(t)$ satisfying $d_1 \le d(t) \le d_2$. Moreover the minimized $\mathcal{H}_\infty$ performance can be achieved by the following optimization problem: $\min\gamma$ subject to (28) and (29). Here the control and observer gain matrices can be reconstructed as

$$F(\Theta_t) = \bar{F}(\Theta_t)\bar{P}_1^{-1}. \tag{30}$$

The $\mathcal{H}_\infty$ stabilization condition is given as follows:

$$0 \le \Pi_2, \tag{31}$$

$$\begin{aligned}
0 &> \Pi_0 + \Pi_1 + \Pi_3 \\
&= \mathbf{He}(e_1^T P\Phi_t) + e_1^T(Q_1+Q_2)e_1 - e_2^T Q_1 e_2 - e_4^T Q_2 e_4 \\
&+ \Phi_t^T\left(d_1^2 R_1 + d_{21}^2 R_2\right)\Phi_t + (e_1-e_2)^T R_1(e_2-e_1) \\
&+ (e_2-e_3)^T R_2(e_3-e_2) + (e_3-e_4)^T R_2(e_4-e_3) \\
&+ \mathbf{He}((e_2-e_3)^T S(e_3-e_4)) + e_1^T\tilde{C}^T(\Theta_t)\tilde{C}(\Theta_t)e_1 \\
&- \gamma^2 e_5^T e_5
\end{aligned} \tag{32}$$

By letting $\bar{R}_1 = X\bar{P}R_1\bar{P}X$ and $\bar{R}_2 = X\bar{P}R_2\bar{P}X$. Then the condition (32) can be converted by (23) into

$$\begin{aligned}
0 >& \mathbf{He}(e_1^T X\bar{A}(\Theta_t)Xe_1 + e_1^T X\bar{A}_d(\Theta_t)Xe_3 + e_1^T X\bar{B}_\omega(\Theta_t)e_5) \\
&+ e_1^T(Q_1+Q_2)e_1 - e_2^T Q_1 e_2 - e_4^T Q_2 e_4 + (X\bar{A}(\Theta_t)Xe_1 \\
&+ X\bar{A}_d(\Theta_t)Xe_3 + X\bar{B}_\omega(\Theta_t)e_5)^T\bar{X}\left(d_1^2\bar{R}_1 + d_{21}^2\bar{R}_2\right)\bar{X} \\
&\times(X\bar{A}(\Theta_t)Xe_1 + X\bar{A}_d(\Theta_t)Xe_3 + X\bar{B}_\omega(\Theta_t)e_5) \\
&+ (e_1-e_2)^T P\bar{X}\bar{R}_1\bar{X}P(e_2-e_1) \\
&+ (e_2-e_3)^T P\bar{X}\bar{R}_2\bar{X}P(e_3-e_2) \\
&+ (e_3-e_4)^T P\bar{X}\bar{R}_2\bar{X}P(e_4-e_3) \\
&+ \mathbf{He}((e_2-e_3)^T S(e_3-e_4)) + e_1^T\tilde{C}^T(\Theta_t)\tilde{C}(\Theta_t)e_1 - \gamma^2 e_5^T e_5
\end{aligned} \tag{33}$$

where $\bar{X} = X^{-1}$. Futher, since $diag(\bar{X},\bar{X},\bar{X},\bar{X},I)e_i^T = e_i^T\bar{X}$, for $i = 1,2,3,4$ and $diag(\bar{X},\bar{X},\bar{X},\bar{X},I)e_5^T = e_5^T$ pre- and post-multiplying both sides of (33) by $diag(\bar{X},\bar{X},\bar{X},\bar{X},I)$ and its transpose yields

$$\begin{aligned}
0 >& \Psi + (\bar{A}(\Theta_t)e_1 + \bar{A}_d(\Theta_t)e_3 + \bar{B}_\omega(\Theta_t)e_5)^T \\
&\times(d_1^2\tilde{R}_1 + d_{21}^2\tilde{R}_2)(\bar{A}(\Theta_t)e_1 + \bar{A}_d(\Theta_t)e_3 + \bar{B}_\omega(\Theta_t)e_5)
\end{aligned} \tag{34}$$

where $\Psi = \mathbf{He}(e_1^T\bar{A}(\Theta_t)e_1 + e_1^T\bar{A}_d(\Theta_t)e_3 + e_1^T\bar{B}_\omega(\Theta_t)e_5) + e_1^T(\tilde{Q}_1+\tilde{Q}_2)e_1 - e_2^T\tilde{Q}_1 e_2 - e_4^T\tilde{Q}_2 e_4 + (e_1-e_2)^T\tilde{R}_1(e_2-e_1) + (e_2-e_3)^T\tilde{R}_2(e_3-e_2) + (e_3-e_4)^T\tilde{R}_2(e_4-e_3) + \mathbf{He}((e_2-e_3)^T\tilde{S}(e_3-e_4)) + e_1^T\bar{X}\tilde{C}^T(\Theta_t)\tilde{C}(\Theta_t)\bar{X}e_1 - \gamma^2 e_5^T e_5$, in which $\tilde{X} = \bar{X}P\bar{X} = \mathbf{diag}(\bar{P}_1, P_2) > 0$, $\tilde{Q}_1 = \bar{X}Q_1\bar{X}$, $\tilde{Q}_2 = \bar{X}Q_2\bar{X}$, $\tilde{R}_1 = \tilde{X}\bar{R}_1\tilde{X}$, $\tilde{R}_2 = \tilde{X}\bar{R}_2\tilde{X}$, and $\tilde{S} = \bar{X}S\bar{X}$. That is, by applying the Schur complement to (34), we can obtain

$$0 > \left[\begin{array}{cc|c} -\bar{R}_1^{-1} & 0 & d_1(\bar{A}(\Theta_t)e_1 + \bar{A}_d(\Theta_t)e_3 + \bar{B}_\omega(\Theta_t)e_5) \\ 0 & -\bar{R}_2^{-1} & d_{21}(\bar{A}(\Theta_t)e_1 + \bar{A}_d(\Theta_t)e_3 + \bar{B}_\omega(\Theta_t)e_5) \\ \hline (*) & (*) & \Psi \end{array}\right] \tag{35}$$

Here, since $\bar{R}_1^{-1} = \tilde{X}\tilde{R}_1^{-1}\tilde{X}$ and $\bar{R}_2^{-1} = \tilde{X}\tilde{R}_2^{-1}\tilde{X}$, it follows from lemma 2.1 that $\bar{R}_1^{-1} \ge 2\mu_1\tilde{X} - \mu_1^2\tilde{R}_1$ and $\bar{R}_2^{-1} \ge 2\mu_2\tilde{X} - \mu_2^2\tilde{R}_2$. In this sense, it is clear that (35) holds if

$$0 > \left[\begin{array}{cc|ccccc} (1,1)' & 0 & d_1\bar{A}(\Theta_t) & 0 & d_1\bar{A}_d(\Theta_t) & 0 & d_1\bar{B}_\omega(\Theta_t) \\ 0 & (2,2)' & d_{21}\bar{A}(\Theta_t) & 0 & d_{21}\bar{A}_d(\Theta_t) & 0 & d_{21}\bar{B}_\omega(\Theta_t) \\ \hline (*) & (*) & (3,3)''' & \tilde{R}_1 & \bar{A}_d(\Theta_t) & 0 & \bar{B}_\omega(\Theta_t) \\ 0 & 0 & (*) & (4,4) & \tilde{R}_2+\tilde{S} & -\tilde{S} & 0 \\ (*) & (*) & (*) & (*) & (5,5) & \tilde{R}_2+\tilde{S} & 0 \\ 0 & 0 & 0 & (*) & (*) & (6,6) & 0 \\ (*) & (*) & (*) & 0 & 0 & 0 & -\gamma^2 I \end{array}\right] \tag{36}$$

ISBN: 978-988-19251-9-0
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

where

$$
\begin{aligned}
(1,1)' &= \mu_1^2\tilde{R}_1 + \mathbf{diag}(-2\mu_1\bar{P}_1, -2\mu_1 P_2),\\
(2,2)' &= \mu_2^2\tilde{R}_2 + \mathbf{diag}(-2\mu_2\bar{P}_1, -2\mu_2 P_2),\\
(3,3)''' &= \mathbf{He}(\bar{A}(\Theta_t)) + \tilde{Q}_1 + \tilde{Q}_2 - \tilde{R}_1 + \bar{X}\tilde{C}^T(\Theta_t)\tilde{C}(\Theta_t)\bar{X},\\
(4,4)' &= -\tilde{Q}_1 - \tilde{R}_1 - \tilde{R}_2,\\
(5,5)' &= -2\tilde{R}_2 - \mathbf{He}(\tilde{S}), (6,6)' = -\tilde{Q}_2 - \tilde{R}_2,
\end{aligned}
$$

There exist a non-convex term in $(3,3)''$. Thus, to deal with the term, we can obtain (28) using the Schur complement. Let us pre- and post-multiply both sides of (31) by $\mathbf{diag}(\bar{X}, \bar{X})$ and its transpose. Then we can obtain

$$
0 \le \begin{bmatrix} \bar{X}R_2\bar{X} & \tilde{S} \\ \tilde{S}^T & \bar{X}R_2\bar{X} \end{bmatrix} \tag{37}
$$

B. LMI-type condition

Another representation for (28) is given as follows:

$$
\begin{aligned}
0 >&\mathcal{L}(\Theta(t))\\
&\triangleq \mathcal{L}_0 + \sum_{i=1}^{r}\theta_i(t)\left(\mathcal{L}_i + \mathcal{L}_i^T\right) + \sum_{i=1}^{r}\theta_i^2(t)\mathcal{L}_{ii}\\
&+\sum_{i=1}^{r}\left(\sum_{j=i+1}^{r}\theta_i(t)\theta_j(t)\mathcal{L}_{ij} + \sum_{j=1}^{i-1}\theta_i(t)\theta_j(t)\mathcal{L}_{ij}^T\right),
\end{aligned} \tag{38}
$$

where

$$
\mathcal{L}_0 \triangleq \left[\begin{array}{cc|cccccc}
(1,1) & 0 & d_1\bar{A}_0 & 0 & d_1\bar{A}_{d,0} & 0 & d_1\bar{B}_{\omega,0} & 0\\
0 & (2,2) & d_{21}\bar{A}_0 & 0 & d_{21}\bar{A}_{d,0} & 0 & d_{21}\bar{B}_{\omega,0} & 0\\
\hline
(*) & (*) & \mathbf{He}(\bar{A}_0)+\tilde{Q}_1+\tilde{Q}_2-\tilde{R}_1 & \tilde{R}_1 & \bar{A}_{d,0} & 0 & \bar{B}_{\omega,0} & \bar{X}\tilde{C}_0^T\\
0 & 0 & (*) & (4,4) & \tilde{R}_2+\tilde{S} & -\tilde{S} & 0 & 0\\
(*) & (*) & (*) & (*) & (5,5) & \tilde{R}_2+\tilde{S} & 0 & 0\\
0 & 0 & 0 & (*) & (*) & (6,6) & 0 & 0\\
(*) & (*) & (*) & 0 & 0 & 0 & -\gamma^2 I & 0\\
\hdashline
0 & 0 & \tilde{C}_0\bar{X} & 0 & 0 & 0 & 0 & -I
\end{array}\right],
$$

$$
\bar{A}_0 = \begin{bmatrix} A_0\bar{P}_1 & A_0 - A_r \\ 0 & P_2A_r \end{bmatrix}, \bar{A}_{d0} = \begin{bmatrix} B_{u,0}\bar{F}_0 & 0 \\ 0 & 0 \end{bmatrix},
$$

$$
\bar{B}_{\omega,0} = \begin{bmatrix} B_{\omega,0} & -I \\ 0 & P_2 \end{bmatrix}, \tilde{C}_0 = \begin{bmatrix} C_0 & 0 \end{bmatrix}
$$

$$
\mathcal{L}_i \triangleq \left[\begin{array}{cc|ccccc:c}
0 & 0 & d_1\bar{A}_i & 0 & d_1\bar{A}_{d,i} & 0 & d_1\bar{B}_{\omega,i} & 0\\
0 & 0 & d_{21}\bar{A}_i & 0 & d_{21}\bar{A}_{d,i} & 0 & d_{21}\bar{B}_{\omega,i} & 0\\
\hline
0 & 0 & \bar{A}_i & 0 & \bar{A}_{d,i} & 0 & \bar{B}_{\omega,i} & \bar{X}\tilde{C}_i^T\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
\hdashline
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{array}\right],
$$

$$
\bar{A}_i = \begin{bmatrix} A_i\bar{P}_1 & A_i - A_r \\ 0 & P_2A_r \end{bmatrix}, \bar{A}_{d,i} = \begin{bmatrix} B_{u,0}\bar{F}_i + B_{u,i}\bar{F}_0 & 0 \\ 0 & 0 \end{bmatrix},
$$

$$
\bar{B}_{\omega,i} = \begin{bmatrix} B_{\omega,i} & -I \\ 0 & P_2 \end{bmatrix}, \tilde{C}_i = \begin{bmatrix} C_i & 0 \end{bmatrix}
$$

$$
\mathcal{L}_{ii} \triangleq \left[\begin{array}{cc|ccccc:c}
0 & 0 & 0 & 0 & d_1\bar{A}_{d,ii} & 0 & 0 & 0\\
0 & 0 & 0 & 0 & d_{21}\bar{A}_{d,ii} & 0 & 0 & 0\\
\hline
0 & 0 & 0 & 0 & \bar{A}_{d,ii} & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
(*) & (*) & (*) & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
\hdashline
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{array}\right],
$$

$$
\bar{A}_{d,ii} = \begin{bmatrix} B_{u,i}\bar{F}_i & 0 \\ 0 & 0 \end{bmatrix}
$$

$$
\mathcal{L}_{ij} \triangleq \left[\begin{array}{cc|ccccc:c}
0 & 0 & 0 & 0 & d_1\bar{A}_{d,ij} & 0 & 0 & 0\\
0 & 0 & 0 & 0 & d_{21}\bar{A}_{d,ij} & 0 & 0 & 0\\
\hline
0 & 0 & 0 & 0 & \bar{A}_{d,ij} & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
(*) & (*) & (*) & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
\hdashline
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{array}\right],
$$

$$
\bar{A}_{d,ii} = \begin{bmatrix} B_{u,i}\bar{F}_j & 0 \\ 0 & 0 \end{bmatrix}
$$

By the $\mathcal{S}$-procedure

$$
0 > \mathcal{L}(\Theta(t)) + \mathcal{N}(\Theta(t)) \tag{39}
$$

where $0 \le \mathcal{N}(\Theta(t))$ is given by $(\sum_{i=1}^{r}\theta_i(t) = 1,\ a_i \le \theta_i(t) \le b_i,\ 0 \le \theta_i(t)\theta_j(t))$

$$
\begin{aligned}
\mathcal{N}(\Theta(t)) =&\mathcal{C}_1 + \mathcal{C}_1^T + \sum_{i=1}^{r}\mathcal{C}_{2i}(\Lambda_i + \Lambda_i^T)\\
&+\sum_{i=1}^{r}\sum_{j=1,j\neq i}^{r}\mathcal{C}_{3ij}(\Sigma_{ij} + \Sigma_{ij}^T),\\
0 =&\mathcal{C}_1 \triangleq \begin{bmatrix} I \\ \theta_1(t)I \\ \vdots \\ \theta_r(t)I \end{bmatrix}^T \begin{bmatrix} I \\ -I \\ \vdots \\ -I \end{bmatrix} \begin{bmatrix} W_0 & W_1 & \cdots & W_r \end{bmatrix} \begin{bmatrix} I \\ \theta_1(t)I \\ \vdots \\ \theta_r(t)I \end{bmatrix},\\
0 \le&\ \mathcal{C}_{2i} \triangleq -\theta_i^2(t) + (a_i + b_i)\theta_i(t) - a_ib_i,\\
0 \le&\ \mathcal{C}_{3ij} \triangleq \theta_i(t)\theta_j(t),
\end{aligned}
$$

for $0 < \Lambda_i + \Lambda_i^T$ and $0 < \Sigma_{ij} + \Sigma_{ij}^T$. With some algebraic manipulations, the constraint $0 \le \mathcal{N}(\Theta(t))$ can be represented as follows:

$$
\begin{aligned}
0 \le&\mathcal{N}(\Theta(t))\\
=&\mathbf{N}_0 + \sum_{i=1}^{r}\theta_i(t)(\mathbf{N}_i + \mathbf{N}_i^T) + \sum_{i=1}^{r}\theta_i^2(t)\mathbf{N}_{ii}\\
&+\sum_{i=1}^{r}\left(\sum_{j=i+1}^{r}\theta_i(t)\theta_j(t)\mathbf{N}_{ij} + \sum_{j=1}^{i-1}\theta_i(t)\theta_j(t)\mathbf{N}_{ij}^T\right),
\end{aligned} \tag{40}
$$

where $\mathbf{N}_0 = W_0 + W_0^T - \sum_{i=1}^{r}a_ib_i(\Lambda_i + \Lambda_i^T)$, $\mathbf{N}_i = (a_i + b_i)\Lambda_i - W_0 + W_i$, $\mathrm{N}_{ii} = -(\Lambda_i + \Lambda_i^T) - (W_i + W_i^T)$, and $\mathrm{N}_{ij} = -(W_i + W_j) + (\Sigma_{ij} + \Sigma_{ji})$. Hence, the condition (39)

ISBN: 978-988-19251-9-0
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

becomes

$$
\begin{aligned}
0 >&\Gamma_0 + \sum_{i=1}^{r} \theta_i(t)(\Gamma_i + \Gamma_i^T) + \sum_{i=1}^{r} \theta_i^2(t)\Delta_i \\
&+ \sum_{i=1}^{r} \left(\sum_{j=i+1}^{r} \theta_i(t)\theta_j(t)\Phi_{ij} + \sum_{j=1}^{i-1} \theta_i(t)\theta_j(t)\Phi_{ij}^T \right),
\end{aligned} \tag{41}
$$

where

$$
\begin{aligned}
\Gamma_0 &= \mathcal{L}_0 + \mathbf{N}_0 = \mathcal{L}_0 + W_0 + W_0^T - \sum_{i=1}^{r} a_i b_i (\Lambda_i + \Lambda_i^T), \\
\Gamma_i &= \mathcal{L}_i + \mathbf{N}_i = \mathcal{L}_i + (a_i + b_i)\Lambda_i - W_0 + W_i, \\
\Delta_i &= \mathcal{L}_{ii} + \mathbf{N}_{ii} = \mathcal{L}_{ii} - (\Lambda_i + \Lambda_i^T) - (W_i + W_i^T), \\
\Phi_{ij} &= \mathcal{L}_{ij} + \mathbf{N}_{ij} = \mathcal{L}_{ij} - (W_i + W_j) + (\Sigma_{ij} + \Sigma_{ji})
\end{aligned}
$$

The condition (41) boils down to

$$
0 > \left[I \quad \theta_1(t)I \quad \cdots \quad \theta_r(t)I\right] \tilde{\mathcal{L}} \left[I \quad \theta_1(t)I \quad \cdots \quad \theta_r(t)I\right]^T, \tag{42}
$$

where

$$
\tilde{\mathcal{L}} \triangleq \left[\begin{array}{c|cccc}
\Gamma_0 & \Gamma_1 & \Gamma_2 & \cdots & \Gamma_r \\
\hline
(*) & \Delta_1 & \Phi_{12} & \cdots & \Phi_{1r} \\
(*) & (*) & \Delta_2 & \ddots & \vdots \\
\vdots & \vdots & \ddots & \ddots & \Phi_{(r-1)r} \\
(*) & (*) & \cdots & (*) & \Delta_r
\end{array}\right], \tag{43}
$$

IV. Numerical Example

In this section, Consider the following plant [11]:

$$
\begin{aligned}
\dot{x}_1(t) &= x_2(t) \\
\dot{x}_2(t) &= -x_1^3(t) - 0.1x_2(t) + 12\cos t + u(t)
\end{aligned} \tag{44}
$$

The LPV representation of the above system is as followings:

$$
\begin{aligned}
A_1 &= \begin{bmatrix} 0 & 1 \\ 0 & -0.1 \end{bmatrix}, A_2 = \begin{bmatrix} 0 & 1 \\ -25 & -0.1 \end{bmatrix}, \\
B_{u,0} &= \begin{bmatrix} 0 \\ 1 \end{bmatrix}, B_{\omega,0} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, A_r = \begin{bmatrix} 0 & 1 \\ -3 & -2 \end{bmatrix}, \\
A_0 &= 0, C_0 = 0, B_{u,i} = 0, B_{\omega,i} = 0 (i = 1, 2, ..., r) \\
\theta_1(t) &= 1 - \frac{x_1^2(t)}{25}, \theta_2(t) = \frac{x_1^2(t)}{25}, \\
\mu_1 &= \mu_2 = 1, a_1 = a_2 = 0, b_1 = b_2 = 1, \\
x_1(t) &\in [-5,\ 5], r(t) = \begin{bmatrix} 0 \\ 4\sin t \end{bmatrix}, \omega(t) = 12\cos t \\
x(0) &= [2 \ \ -1]^T, x_r(0) = [-0.5 \ \ 1]^T
\end{aligned}
$$

Fig. 1 displays the first state behaviors. The simulation result shows that the $\mathcal{H}_\infty$ tracking controller on the LPV systems over a communication network has a good performance to track the reference signal.

V. Conclusion

Our future work is directed to proposing a method of addressing the practical case where the parameters in the LPV system and control part are asynchronous/mismatched each other.

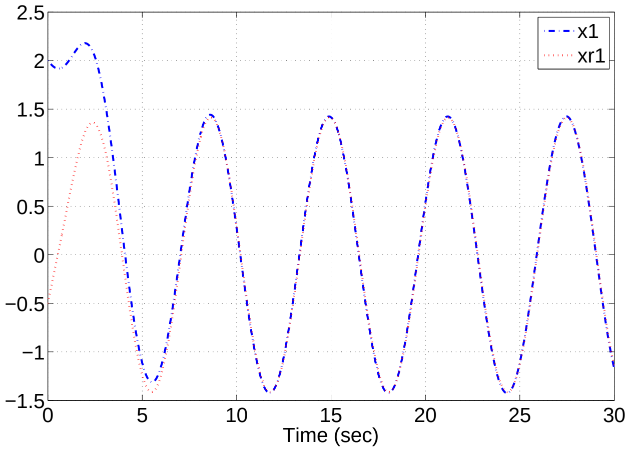

Fig. 1. Simulation Results

Acknowledgment

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ISBN: 978-988-19251-9-0
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)