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Chaos Suppression of a New Variable Structure Chaotic System

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***Abstract*—This paper studies the chaos suppression of a new variable structure chaotic system. A quasi sliding mode control (QSMC) scheme is newly presented. The proposed QSMC not only prevents chattering phenomenon which frequently appears in the conventional sliding mode control systems, but also stabilizes and drives the controlled system into an arbitrary and predictable neighborhood of zero even when the input nonlinearities exist. An example is given to illustrate the effectiveness of the proposed controller design.**

***Index Terms*—Quasi sliding mode control; Chaos suppression; Chattering**

I. Introduction

Chaotic systems exhibit unpredictable behavior, sensitivity to initial conditions and irregular dynamics and it has been found in many physical systems, such as mechanical systems, engineering systems and power converters, etc. Consequently, various studies of effective control methods have been proposed to achieve stabilization of chaotic systems, for instance, optimal control [1], sliding method control [2-3], state feedback control [4, 5] and the backstepping design technique [6, 7], etc. [8]. In numerous control methods, sliding mode control (SMC) is frequently adopted because SMC can offer inherent advantages, such as fast response, good transient performance and insensitive to variation in plant parameters or external disturbances. However, in the traditional SMC systems, ideal sliding mode only exists for infinite frequency switching operation. Consequently, thus control input in actuality is impossible to implement and will cause the undesired chattering phenomenon [8, 9, 10]. Therefore, various methods for suppression of chattering phenomenon have been presented such as in [9, 11-14]. However, those controllers in [9, 11-14] are all under the assumption of linear input. In practice, due to physical limitation, there exist nonlinearities in the control input and it must be taken into account when designing and implementing a control scheme [15]. Furthermore, as mentioned in [16], to directly implement nonlinear chaotic systems with electronic circuits, there exists a major difficulty, that is, the state variables of system occupy a wide dynamic range with values that exceed reasonable power supply limits. Inspired by the aforementioned reasons, this study first introduces a new design parameter into the chaotic systems such that the structure of systems is variable and the dynamic ranges for the system states can be regulated. Then, a QSMC scheme for the chaos control of the considered system is newly proposed. The QSMC prevents chattering phenomenon which frequently appears in the conventional sliding mode control systems. Under the proposed QSMC, the system states can be stabilized and driven into an arbitrary and predictable neighborhood of zero even when the input nonlinearities exist. Last, illustrative simulation results are presented to demonstrate and verify the effectiveness of the proposed QSMC method.

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II. System description and Problem Formulation

In this section, we mainly consider the chaos suppression of a new three-dimensional chaotic system.

A. A New Three-Dimensional Chaotic System with Variable Structure

Recently, a new three-dimensional chaotic system has been introduced as follows [17]

$$\begin{aligned} \dot{x} &= -ax(t) + by(t) \\ \dot{y} &= cx(t) - x(t)z(t) - dy(t) \\ \dot{z} &= x(t)y(t) - e[x(t) + z(t)] \\ [x(0)\ \ y(0)\ \ z(0)]^T &= [x_0\ \ y_0\ \ z_0]^T \end{aligned} \tag{1}$$

where *a*, *b*, *c*, *d*, *e* are parameters of system (1), when *a* = 25.6, *b* = 66.8, *c* = 39.22, *d* = 0.2, *e* = 4, system (1) displays a typical attractor [17],[18]. $[x(t)\ y(t)\ z(t)] \in R^3$ is the state vector, $[x_0\ \ y_0\ \ z_0]^T$ is the initial value vector. However, as shown in [17] [18], chaotic system (1) has a wide dynamic attractor which results in the problem of power saturation. To remove this drawback, we design a variable structure to regulate the state amplitude with a parameter k. The parameter k is defined as

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} k\hat{x} \\ k\hat{y} \\ k\hat{z} \end{bmatrix} \tag{2}$$

then, we have

$$\begin{aligned} k\dot{\hat{x}}(t) &= -ak\hat{x}(t) + bk\hat{y}(t) \\ k\dot{\hat{y}}(t) &= ck\hat{x}(t) - k\hat{x}(t)k\hat{z}(t) - dk\hat{y}(t) \\ k\dot{\hat{z}}(t) &= k\hat{x}(t)k\hat{y}(t) - e[k\hat{x}(t) + k\hat{z}(t)] \end{aligned} \tag{3}$$

Therefore, the variable structure system with amplitude regulation can be expressed as

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$$\begin{aligned}&\dot{\hat{x}}(t) = -a\hat{x}(t) + b\hat{y}(t)\\&\dot{\hat{y}}(t) = c\hat{x}(t) - k\cdot\hat{x}(t)\hat{z}(t) - d\hat{y}(t)\\&\dot{\hat{z}}(t) = k\cdot\hat{x}(t)\hat{y}(t) - e[\hat{x}(t) + \hat{z}(t)]\\&[\hat{x}(0)\ \ \hat{y}(0)\ \ \hat{z}(0)]^T = [\hat{x}_0\ \ \hat{y}_0\ \ \hat{z}_0]^T\end{aligned} \tag{4}$$

The chaotic motion of system (4) with $k = 1$ is illustrated in Fig. 1(a)-(d), where the initial condition of $[\hat{x}_0\ \hat{y}_0\ \hat{z}_0]^T = [0.2\ \ 0.3\ \ 0.5]^T$ and the regulated state $\hat{x}$ of system (4) is illustrated in Fig. 1(e) with $k = 1$ (i.e. the original system (1)), $k = 0.5$ and $k = 5$. From Fig. 1(e), it reveals the value of state $\hat{x}$ can be regulated as expected.

B. Problem Formulation

In this section, to control the chaotic system (4) effectively, we introduce a control-input u into the differential equation of state $\hat{y}$. The controlled system can be written as

$$\begin{aligned}&\dot{\hat{x}}(t) = -a\hat{x}(t) + b\hat{y}(t)\\&\dot{\hat{y}}(t) = c\hat{x}(t) - k\cdot\hat{x}(t)\hat{z}(t) - d\hat{y}(t) + \phi\big(u(t)\big)\\&\dot{\hat{z}}(t) = k\cdot\hat{x}(t)\hat{y}(t) - e[\hat{x}(t) + \hat{z}(t)]\end{aligned} \tag{5}$$

where $\phi\big(u(t)\big)$ is a continuous nonlinear function with $\phi(0) = 0$, where $\phi: R \to R$ with the law $u(t) \to \phi\big(u(t)\big)$ and inside sector $[\beta_1\ \ \beta_2]$, i.e.

$$\beta_2 u^2(t) \ge u(t)\phi\big(u(t)\big) \ge \beta_1 u^2(t) \tag{6}$$

where β_1 and β_2 are nonzero positive constants[15]. This study aims to design a QSMC such that the state of chaotic system can be driven to predictable and desired bounds even with nonlinear input, i.e.

$$\lim_{t\to\infty}|\hat{x}| \le \gamma_1,\ \lim_{t\to\infty}|\hat{y}| \le \gamma_2,\ \lim_{t\to\infty}|\hat{z}| \le \gamma_3 \tag{7}$$

where $\gamma_i, i = 1, 2, 3$ are positive predictable constants depending on the parameter chosen in the designed QSMC.

III. Switching Surface Design and Definition of Quasi Sliding Manifold

First, a switching surface is selected as

$$s(t) = \hat{y}(t) + \sigma\hat{x}(t) \tag{8}$$

where $s \in R$ and $\sigma > \frac{-a}{b}$ is a designed constant.

Before continuing to estimate the state bound of $\hat{x}$, we give the definition of quasi sliding manifold as follows.

Definition 1: The system is said to be in the quasi sliding manifold if there exists $t_Q > 0$ and $\delta_Q > 0$ such that any solution of the controlled system (5) satisfies $|s(t)| \le \delta_Q$, for all $t \ge t_Q$.

When the system operates in the quasi sliding manifold, i.e. $|s(t)| \le \delta_Q$ for $t > t_Q$, we have the following quasi sliding mode dynamics

$$\dot{\hat{x}}(t) = -\lambda_1\hat{x}(t) + bs(t) \text{ where } \lambda_1 = a + b\sigma \tag{9}$$

Solving the differential eqn. (9) for $\hat{x}$ when $t \ge t_Q$ results in:

$$\hat{x}(t) = e^{-\lambda_1(t-t_Q)}\hat{x}(t_Q) + \int_{t_Q}^{t} e^{-\lambda_1(t-\tau)}\cdot bs(\tau)d\tau \tag{10}$$

Since the system is in the quasi sliding manifold, one has $|s(t)| \le \delta_Q$. Furthermore, since σ is determined to guarantee $\lambda_1 > 0$, the bound for state $\hat{x}$ is obtained as

$$\begin{aligned}|\hat{x}(t)| &= \left|e^{-\lambda_1(t-t_Q)}\hat{x}(t_Q) + \int_{t_Q}^{t} e^{-\lambda_1(t-\tau)}\cdot b\cdot s(\tau)d\tau\right|\\&\le e^{-\lambda_1(t-t_Q)}\left|\hat{x}(t_Q)\right| + b\cdot\delta_Q\cdot e^{-\lambda_1 t}\int_{t_Q}^{t} e^{\lambda_1\tau}\,d\tau\\&\le e^{-\lambda_1(t-t_Q)}\left|\hat{x}(t_Q)\right| + b\cdot\delta_Q\cdot\frac{1-e^{-\lambda_1(t-t_Q)}}{\lambda_1}\end{aligned} \tag{11}$$

Therefore we have

$$\lim_{t\to\infty}|\hat{x}(t)| \le \gamma_1 = \frac{b}{\lambda_1}\delta_Q \tag{12}$$

In addition, by (8), the bound for $\hat{y}(t)$ can be also obtained as

$$\begin{aligned}\lim_{t\to\infty}|\hat{y}(t)| &= \lim_{t\to\infty}|s(t) - \sigma\hat{x}(t)|\\&\le \lim_{t\to\infty} s(t) + \lim_{t\to\infty}|\sigma||\hat{x}(t)|\\&\le \gamma_2 = \left(1 + \frac{b\sigma}{\lambda_1}\right)\delta_Q\end{aligned} \tag{13}$$

After $|\hat{x}| \le \gamma_1$ and $|\hat{y}| \le \gamma_2$ solving the differential equation of the controlled system for state $\hat{z}$ results in

$$\lim_{t\to\infty}|\hat{z}(t)| \le \gamma_3 = \frac{\gamma_1\gamma_2\cdot k + e\gamma_1}{e} \tag{14}$$

Obviously, from (12)-(14), the bounds of $\gamma_i, i = 1,2,3$ are relative to δ_Q. Therefore, how to control the system with a smaller value of δ_Q is important and the solution is given in the following section.

IV. Sliding Mode Controller Design for Quasi Sliding Manifold

After establishing an appropriate switching surface and estimating the bounds of the system states in the above section, it follows to design a quasi sliding mode controller to ensure the occurrence of the quasi sliding manifold. The continuous controller is proposed as

$$u(t) = -w\eta\frac{s}{|s| + \delta} \tag{15}$$

where $w > \frac{1}{\beta_1}$, $\delta > 0$
and $\eta = |c\hat{x} - \hat{x}\hat{z}\cdot k - d\hat{y} + \sigma(-a\hat{x} + b\hat{y})|$

Theorem 1: Consider the system (5), if this system is controlled by $u(t)$ in (15). Then the system trajectory converges to the quasi sliding manifold with $|s(t)| \le \delta_Q = \frac{\beta_1 w\delta}{\beta_1 w - 1}$.

Proof: Let the Lyapunov function of the system be $V=\frac{1}{2}s^2$, then taking the derivative of V and introducing (5), one has

$$\begin{aligned}\dot{V} &= s\dot{s} = s(\dot{\hat{y}}+\sigma\dot{\hat{x}}) \\ &= s(c\hat{x}-\hat{x}\hat{z}\cdot k-d\hat{y}+\phi(u)+\sigma(-a\hat{x}+b\hat{y})) \\ &\le \eta|s|+s\phi(u)\end{aligned} \quad (16)$$

Furthermore, from eqn. (6) and (15), we have

$$\begin{aligned}u(t)\cdot\phi(u(t)) &= -w\eta\frac{s}{|s|+\delta}\phi(u(t))\ge\beta_1 u^2 \\ &= \beta_1\left(w\eta\frac{s}{|s|+\delta}\right)^2\end{aligned} \quad (17)$$

Thus we have

$$s(t)\phi(u(t))\le-\beta_1 w\eta\frac{s^2}{|s|+\delta} \quad (18)$$

By placing (18) into (16), we get

$$\begin{aligned}\dot{V} &\le \eta|s|-\beta_1 w\eta\frac{s^2}{|s|+\delta} \\ &= \eta|s|-\beta_1 w\eta\left(|s|-\frac{|s|\delta}{|s|+\delta}\right)\end{aligned} \quad (19)$$

Since $\frac{s\delta}{|s|+\delta}\le\delta$, we have

$$\begin{aligned}\dot{V} &\le (1-\beta_1 w)\eta|s|-\beta_1 w\eta\delta \\ &= (1-\beta_1 w)\eta(|s|-\frac{\beta_1 w\delta}{\beta_1 w-1})\end{aligned} \quad (20)$$

Since $\beta_1 w>1$ has been chosen in the controller (15), (20) implies that $\dot{V}<0$ whenever $|s(t)|>\delta_Q=\frac{\beta_1 w\delta}{\beta_1 w-1}$. Therefore, $|s|$ will converges to the region of $|s(t)|\le\delta_Q=\frac{\beta_1 w\delta}{\beta_1 w-1}$. Thus the proof is achieved completely.

Remark 1: Since the controller in (15) is continuous, chattering is eliminated.

Remark 2: In fact, δ is a design parameter, therefore, one can select a sufficient small value of δ to make δ_Q and $\gamma_i, i=1,2,3$ arbitrarily bounded in the neighborhood of zero.

V. Numerical Example

In this section, we demonstrate the effectiveness of the proposed QSMC scheme by simulation results. The system parameters are chosen as $k=10$, a = 25.6, b = 66.8, c = 39.22, d = 0.2, e = 4. The initial states are $\hat{x}(0)=0.2, \hat{y}(0)=0.3$ and $\hat{z}(0)=0.5$. For simulation, the nonlinear input is defined as

$$\phi(u(t))=[0.7+0.2\cdot\sin(u(t))]u(t) \quad (21)$$

Based on (6), $\beta_1=0.5$ and $\beta_2=0.9$ can be obtained. Then following the steps in Remark 3, we select $\sigma=1>\frac{-a}{b}$. Therefore, the switching function s is obtained as

$$s(t)=\hat{y}+\hat{x} \quad (22)$$

and the quasi sliding mode controller can be obtained as

$$u(t)=-w\eta\frac{s}{|s|+\delta} \quad (23)$$

with $w=4>\frac{1}{\beta_1}=\frac{1}{0.5}$ and $\delta=0.02$

By Theorem 1 and (12)-(14), we can predict that $|s(t)|\le\delta_Q=0.04$ and the states are bounded by $\gamma_1=0.0289, \gamma_2=0.0689$ and $\gamma_3=0.0339$. The simulation results are shown in Figures 3-4 under the proposed QSMC (23). Figure 2 and Figure 3 show, respectively, the corresponding $s(t)$ and state responses of the controlled system. The continuous QSMC control is shown in Figure 4. Surveying the simulation results, the system state can be bounded by $\gamma_i, i=1,2,3$ calculated above, as we predict. In particular, chattering does not appear due to the continuous control input as shown in Figure 4.

VI. Conclusions

In this paper, we have presented a QSMC scheme for suppressing chaos of a new three-dimensional chaotic system. A design parameter is introduced into the system to regulate the dynamic ranges for the system states. The QSMC prevents chattering phenomenon which frequently appears in the conventional sliding mode control systems. An example is included to illustrate the effectiveness of the proposed QSMC developed in this paper.

References

[1] M. T. Yassen, "The optimal control of Chen chaotic dynamical system", Applied Mathematics and Computation, 2002, 131, pp. 171-180.
[2] M. Roopaei, B. R. Sahraei, T. C. Lin, "Adaptive sliding mode control in a novel class of chaotic systems", Communications in Nonlinear Science and Numerical Simulation, Volume 15, Issue 12, December 2010, pp. 4158-4170.
[3] T. Y. Chiang, M. L. Hung, J. J. Yan, Y. S. Yang, J. F. Chang, "Sliding mode control for uncertain unified chaotic systems with input nonlinearity", Chaos Soltions Fractals, 2007, 34, pp. 437-442.
[4] J. H. Park, D. H. J. Won, S. M. Lee, " synchronization of time-delayed chaotic systems", Applied Mathematics and Computation, 2008, 204, pp. 170-177.
[5] G. Chen, X. Dong, "On feedback control of chaotic dynamic systems", Int J Bifurcation Chaos, 1992, 2, pp. 407-411.
[6] T. Yang, L. O. Chua, "Impulsive stabilization for control and synchronization of chaotic systems: theory and application to secure communication", IEEE Trans Circ Syst I, 1997, 44, pp. 976-988.
[7] Y. Yu, S. Zhang, "Controlling uncertain Lu system using backstepping design", Chaos Soltions Fractals, 2003, 15, pp. 897-902.
[8] P. Z. Zhang-Jian, C. Y. Chen, J. J. Yan, S. Y. Ying, H. D. Pei "Quasi Sliding Mode Control of Generalized Lorenz Chaotic Systems", IMECS, Volume 2189, 2011, pp. 913-916.
[9] M. Roopaei, M.Z. Jahromi, "Chattering-free fuzzy sliding mode control in MIMO uncertain systems", Nonlinear Analysis, 2009, 71, pp. 4430-4437.
[10] H. Lee, Vadim I. Utkin, "Chattering suppression methods in sliding mode control systems", Annual Review in Control, 2007, 31, pp.179-188.
[11] X. Zhang, A. Khadra, D. Yang, D. Li, "Unified impulsive fuzzy-model-based controllers for chaotic systems with parameter uncertainties via LMI", Commun Nonlinear Sci Numer Simulat, 2010, 15, pp. 105-114.
[12] M. Roopaei, M.Z. Jahromi, S. Jafari, "Adaptive gain fuzzy sliding mode control for the synchronization of nonlinear chaotic gyros", Chaos, 2009, 19, 013125.

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ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

[13] X.W. Liu, S.M. Zhong, "T-S fuzzy model-based impulsive control of chaotic systems with exponential decay rate", Phys. Lett, 2007, 4, pp. 260-370.
[14] J.E. Slotine, W. Li, "Applied Nonlinear Control, Prentice-Hall", New Jersey, 1991.
[15] K.C. Hsu, "Sliding mode controllers for uncentain systems with input nonlinearity", J. Guidance Control Dynamics, 1998, 21, pp. 666-669.
[16] K.M. Cuomo, A.V. Oppenheim, S.H. Strogatz, "Synchronization of Lorenz-Based Chaotic Circuits with Applications to Communications", IEEE Transactions on circuits and systems-11: analog and digital signal processing 40, 1993, pp. 626-633.
[17] Rui Tang Liang, Li Jing, Fan Bing, Zhai Ming-Yue. "A new three-dimensional chaotic system and its circuit simulation". Phys Lett A, 2009, 58(02) [in Chinese].
[18] F. Zhang, Y. Shu, H. Yang, "Bounds for a new chaotic system and its application in chaos synchronization", Communications in Nonlinear Science and Numerical Simulation, Volume 16, Issue 3, 2011, pp. 1501-1508

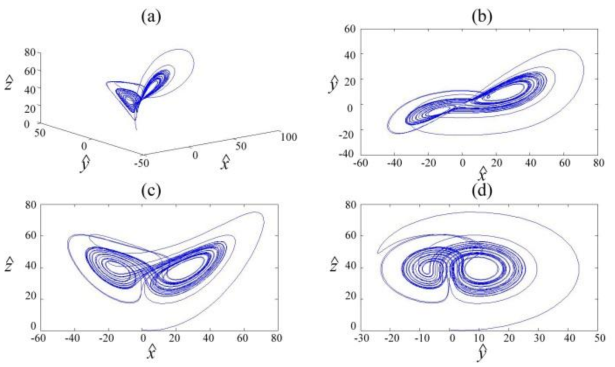

Fig. 1. (a) Trajectories of chaotic system (4) (b) Trajectories projected on the $\hat{x}-\hat{y}$ plane (c) Trajectories projected on the $\hat{x}-\hat{z}$ plane (d) Trajectories projected on the $\hat{y}-\hat{z}$ plane.

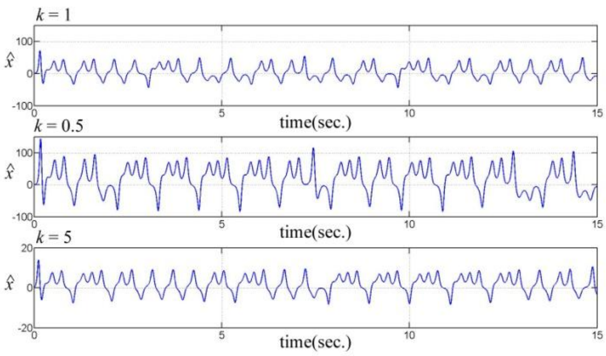

Fig. 1. (e) The state $\hat{x}$ of system (4) with $k=1$, $k=0.5$ and $k=5$.

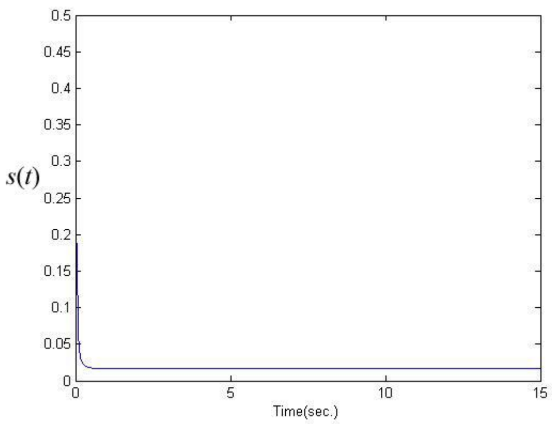

Fig. 2. The time response of switching function s(t).

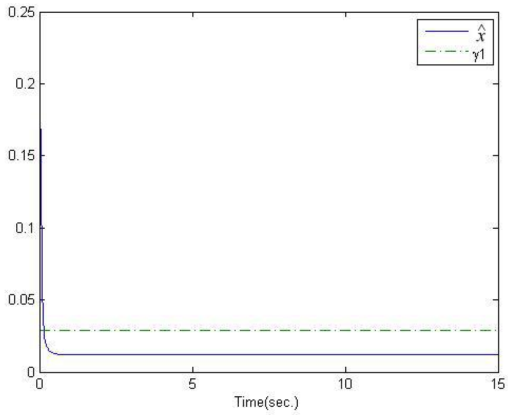

Fig. 3(a). The state response of the controlled system.

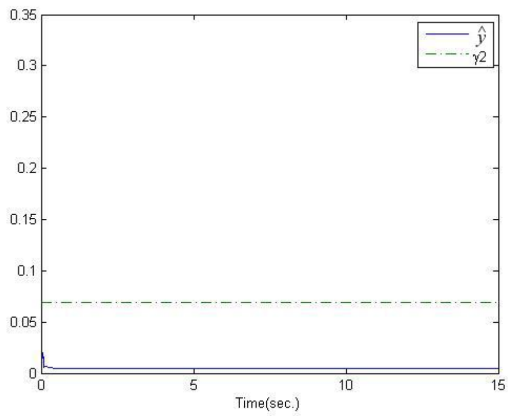

Fig. 3(b). The state response of the controlled system.

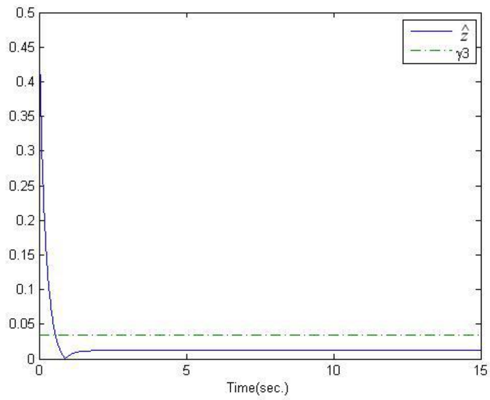

Fig. 3(c). The state response of the controlled system

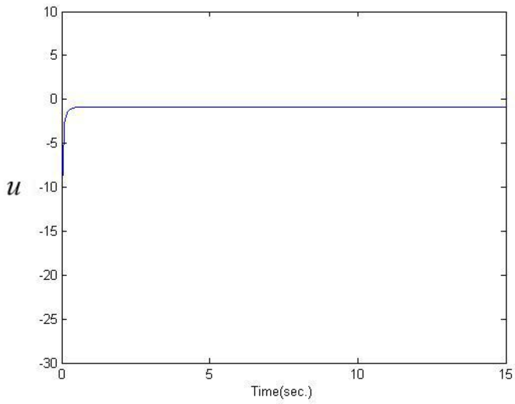

Fig. 4. The time response of continuous QSMC (23)

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