An Exact Solution Algorithm for Maximizing the Fleet Availability of an Aircraft Unit Subject to Flight and Maintenance Requirements

Andreas Gavranis, and George Kozanidis

***Abstract*— Flight and Maintenance Planning (FMP) of mission aircraft addresses the question of which available aircraft to fly and for how long, and which grounded aircraft to perform maintenance operations on, in a group of aircraft that comprise a unit. The objective is to achieve maximum fleet availability of the unit over a given planning horizon, while also satisfying certain flight and maintenance requirements. Heuristic approaches that are used in practice to solve the FMP problem often perform poorly, generating solutions that are far from optimum. On the other hand, the more sophisticated mathematical optimization models that have been developed to tackle this problem handle small problems effectively, but tend to be computationally inefficient for larger problems that often arise in practice. In this work, we develop an exact solution algorithm for the FMP Problem, which is capable of identifying the global optimal solution of realistic size problems in very reasonable computational times. Initially, this algorithm obtains a valid upper bound on the optimal objective function value by solving a simplified relaxation of the original problem; then, this value is gradually reduced, until a feasible solution that attains it is identified. The algorithm employs special valid inequalities (cuts), which exclude solutions that do not qualify for optimality from further consideration. The experimental results that we present demonstrate that the proposed solution algorithm is significantly more efficient than a commercial optimization software package that can be used alternatively for the solution of the problem under consideration.**

***Index Terms*— fleet availability; flight and maintenance planning; exact solution algorithm; mixed integer linear programming.**

I. Introduction

FLIGHT and Maintenance Planning (FMP) is an important decision making problem arising at the operation level of numerous types of mission fleets, involving military or fire-fighting aircraft, rescue choppers, etc. The objective is to maximize fleet availability, while also satisfying certain flight and maintenance requirements.

In this work, we develop an exact solution algorithm for the FMP problem, which is capable of identifying the global optimal solution of realistic size problems in very reasonable computational times. This algorithm employs an iterative procedure that considers successive relaxations of the original problem, adding a suitable valid inequality (cut) each time the associated solution does not qualify for optimality. The computational results that we report demonstrate the superiority of the proposed algorithm over a commercial optimization software package that can be used alternatively for the solution of the FMP problem.

The remainder of this paper is structured as follows. In Section II, we summarize the related literature, and in Section III, we present the FMP optimization model. In Section IV, we develop the exact methodology for the solution of this model, and in Section V, we present experimental results demonstrating its computational efficiency. Finally, in Section VI we summarize our conclusions.

II. Literature Review

Numerous problems dealing with aircraft operations have been investigated in the past, both in the commercial, as well as in the military context. In the context of military applications, Safaei et al. [1] develop a mixed integer optimization model to formulate the problem of workforce-constrained maintenance scheduling for a fleet of military aircraft, and use generic optimization software to solve it. The model utilizes a network flow structure in order to simulate the flow of aircraft between missions, the hangar and the repair shop.

The increasing importance of effective military aircraft maintenance was recently also recognized by the Operations Research and Management Science (ORMS) community. The 2006 Franz Edelman INFORMS Award for outstanding operations research and management science practice was bestowed on Warner Robins Air Logistics Center [2]. Working with Realization Technologies and faculty from the University of Tennessee, WR-ALC used an operations research technique called Critical Chain to reduce the number of C-5 aircraft in the depot undergoing repair and overhaul from twelve to seven in just eight months. As a direct consequence, the time required to repair and overhaul the C-5 aircraft was reduced by 33%.

Although FMP is an important decision making problem encountered in several diversified areas, the relevant published research is rather limited. Sgaslik [3] introduces a decision support system for maintenance planning and mission assignment of a helicopter fleet that partitions the master problem into two sub-problems which are formulated as elastic mixed integer programs and solved separately with standard optimization software. Instead of maximizing the

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A. Gavranis is a Doctoral Student and a Research Associate at the Department of Mechanical Engineering of the University of Thessaly, Volos, Greece, phone: +30-24210-74110, e-mail :agavranis@gmail.com
G. Kozanidis is an Assistant Professor at the Department of Mechanical Engineering of the University of Thessaly, Volos, Greece, phone: +30-24210-74057, e-mail :gkoz@mie.uth.gr

fleet availability, the suggested approach minimizes the cost associated with the violation of some of the problem's constraints (e.g., those referring to the required flight time, the maintenance capacity and the flight time of each individual aircraft), while also maintaining a certain lower bound on the fleet availability.

Pippin [4] develops a mixed integer linear program (MILP) and a quadratic program for issuing the flight plan of a unit of army helicopters. Both models minimize the cost associated with the deviations of the individual helicopter residual flight times from their target values, but neither of them addresses the decisions related to the maintenance aspect of the problem. A more simplistic model for flight planning of training aircraft was studied by Rosenzweig et al. [5]. The authors solve this model with generic optimization software.

The U.S. Department of the Army has released a Field Manual on Army Aviation Maintenance, which describes a practical "sliding scale scheduling" or "aircraft flowchart" graphical tool for scheduling aircraft for phase/periodic inspection and deciding which aircraft should fly in certain missions [6]. Utilizing this tool, Kozanidis et al. [7] developed a mixed integer nonlinear optimization model and an exact solution algorithm for solving the FMP problem over a single-period planning horizon. In contrast, the solution algorithm that we develop in the current work accommodates a multi-period planning horizon.

Kozanidis [8] proposes a multi-objective MILP model for the FMP problem that maximizes the minimum aircraft and flight time availability of the wing and of the squadrons that comprise it. Ref. [9] complements that work, by developing a single objective optimization model, which adopts one out of these objectives (wing aircraft availability) and incorporates the remaining ones with the introduction of associated constraints. Due to the excess computational effort required for the solution of the aforementioned models, the authors resort to heuristics for solving them [10].

Finally, Cho [11] develops a MILP to model the FMP problem. The proposed formulation generates a daily flight and maintenance plan that distributes the maintenance workload evenly across the planning horizon. The main differences that this model exhibits with respect to the one that we address in the current work, is that it uses different definitions for the objective function and for the flight requirements of the unit. More specifically, that model minimizes the maximum number of aircraft in maintenance at any given time in order to smoothen the variability of the maintenance demand over time. Additionally, it translates the original flight load requirements into sortie assignments, which are successively assigned to the aircraft of the unit.

The author also considers a two-stage formulation, which disaggregates the problem by determining the flight and maintenance decisions in two separate stages. Both the single and the two stage model are solved with generic optimization software, although a discussion that proposes equivalent alternative formulations and outlines the development of potential heuristic solution approaches is also included.

III. Model Formulation

We consider a unit of mission aircraft (typically, a combat wing). In order to retain the readiness of the unit at a high level, the unit command issues at the beginning of each planning horizon associated flight requirements. These requirements (also referred to as *flight load*) determine the total flight time that the aircraft of this unit must fulfill in each time period of the planning horizon.

The fleet availability of a unit is usually expressed in terms of the total number of aircraft that are available to fly (aircraft availability), and in terms of the total residual flight time of all available aircraft (residual flight time availability). The *residual flight time* of an aircraft is defined as the total remaining time that this aircraft can fly before it has to be grounded for a maintenance check. This time is also referred to as *bank time* in the related military literature [6]. The residual flight time of an aircraft is positive if and only if this aircraft is available to fly.

Similarly, we define the *residual maintenance time* of each individual aircraft as the total remaining time that this aircraft needs in order to complete its maintenance service. The residual maintenance time of an aircraft is positive if and only if this aircraft is undergoing a maintenance check (and is therefore not available to fly).

For the maintenance needs of the unit, there exists a station responsible for providing service to its aircraft. This station has certain space (also referred to as *dock space*) and time capacity capabilities. Given the flight requirements of the unit, and the physical constraints that stem from the capacity of the maintenance station, the goal is to issue a flight and maintenance plan for each individual aircraft that maximizes the unit's readiness to respond to external threats, i.e., the fleet availability.

The mathematical model that has been developed for the FMP problem adopts the following mathematical notation:

Sets:

N : set of unit aircraft, indexed by n.

Parameters:

T : length of the planning horizon, indexed by t,

S_t : flight load in period t,

B_t : time capacity of the maintenance station in period t,

C : space capacity of the maintenance station,

Y : residual flight time of an aircraft immediately after it exits the maintenance station (*phase interval*),

G : residual maintenance time of an aircraft immediately after it enters the maintenance station,

$A1_n$: state (0/1) of aircraft n at the first period of the planning horizon,

$Y1_n$: residual flight time of aircraft n at the first period of the planning horizon,

$G1_n$: residual maintenance time of aircraft n at the first period of the planning horizon,

X_{max} : maximum flight time of an available aircraft in a single time period,

Y_{min} : lower bound on the residual flight time of an available aircraft,

G_{min} : lower bound on the residual maintenance time of a grounded aircraft,

K : a sufficiently large number.

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Decision Variables:

$a_{n,t}$: binary decision variable that takes the value 1 if aircraft n is available in period t, and 0 otherwise,

$y_{n,t}$: residual flight time of aircraft n at the beginning of period t,

$x_{n,t}$: flight time of aircraft n in period t,

$g_{n,t}$: residual maintenance time of aircraft n at the beginning of period t,

$h_{n,t}$: maintenance time of aircraft n in period t,

$d_{n,t}$: binary decision variable that takes the value 1 if aircraft n exits the maintenance station at the beginning of period t, and 0 otherwise,

$f_{n,t}$: binary decision variable that takes the value 1 if aircraft n enters the maintenance station at the beginning of period t, and 0 otherwise,

q_t, $p_{n,t}$, $r_{n,t}$: auxiliary binary decision variables.

The proposed FMP model (Kozanidis, 2009; Kozanidis et al. 2010) is a mixed integer linear program, formulated as follows:

$$Max \sum_{t=2}^{T+1}\sum_{n=1}^{|N|} y_{nt} \quad (1)$$

$$\text{s.t. } y_{n,t+1} = y_{n,t} - x_{n,t} + Yd_{n,t+1},\ n = 1,\dots,|N|,\ t = 1,..,T \quad (2)$$

$$d_{n,t+1} \geq a_{n,t+1} - a_{n,t},\ n = 1,\dots,|N|,\ t = 1,..,T \quad (3)$$

$$a_{n,t+1} - a_{n,t} + 1.1\left(1-d_{n,t+1}\right) \geq 0.1,\ n = 1,\dots,|N|,\ t = 1,..,T \quad (4)$$

$$g_{n,t+1} = g_{n,t} - h_{n,t} + Gf_{n,t+1},\ n = 1,\dots,|N|,\ t = 1,..,T \quad (5)$$

$$f_{n,t+1} \geq a_{n,t} - a_{n,t+1},\ n = 1,\dots,|N|,\ t = 1,..,T \quad (6)$$

$$a_{n,t} - a_{n,t+1} + 1.1\left(1-f_{n,t+1}\right) \geq 0.1,\ n = 1,\dots,|N|,\ t = 1,..,T \quad (7)$$

$$\sum_{n=1}^{|N|} x_{n,t} = S_t,\ t = 1,...,T \quad (8)$$

$$\sum_{n=1}^{|N|} h_{n,t} \leq B_t,\ t = 1,...,T \quad (9)$$

$$\sum_{n=1}^{|N|} (1-a_{n,t}) \leq C,\ t = 2,...,T+1 \quad (10)$$

$$B_t \leq \sum_{n=1}^{|N|} h_{n,t} + K(1-q_t),\ t = 1,...,T \quad (11)$$

$$\sum_{n=1}^{|N|} g_{n,t} \leq \sum_{n=1}^{|N|} h_{n,t} + Kq_t,\ t = 1,...,T \quad (12)$$

$$y_{n,t} + Kp_{n,t} \leq K, n = 1,\dots,|N|,\ t = 1,..,T \quad (13)$$

$$a_{n,t+1} \leq \left(y_{n,t} - x_{n,t}\right)K + Kp_{n,t},\ n = 1,\dots,|N|,\ t = 1,..,T \quad (14)$$

$$g_{n,t} + Kr_{n,t} \leq K, n = 1,\dots,|N|,\ t = 1,..,T \quad (15)$$

$$1 - a_{n,t+1} \leq \left(g_{n,t} - h_{n,t}\right)K + Kr_{n,t},\ n = 1,\dots,|N|,\ t = 1,..,T \quad (16)$$

$$y_{n,t} \leq Ya_{n,t},\ n = 1,\dots,|N|, t = 2,...,T+1 \quad (17)$$

$$g_{n,t} \leq G\left(1-a_{n,t}\right),\ n = 1,\dots,|N|,\ t = 2,...,T+1 \quad (18)$$

$$x_{n,t} \leq X_{max} a_{n,t},\ n = 1,\dots,|N|, t = 1,...,T \quad (19)$$

$$y_{n,t} \geq Y_{min} a_{n,t}, n = 1,\dots,|N|,\ t = 2,...,T+1 \quad (20)$$

$$g_{n,t} \geq G_{min}\left(1-a_{n,t}\right),\ n = 1,\dots,|N|,\ t = 2,...,T+1 \quad (21)$$

$$x_{n,t} \leq y_{n,t},\ n = 1,\dots,|N|,\ t = 1,...,T \quad (22)$$

$$h_{n,t} \leq g_{n,t},\ n = 1,\dots,|N|,\ t = 1,...,T \quad (23)$$

$$a_{n,1} = A1_n,\ n = 1,\dots,|N| \quad (24)$$

$$y_{n,1} = Y1_n,\ n = 1,\dots,|N| \quad (25)$$

$$g_{n,1} = G1_n,\ n = 1,\dots,|N| \quad (26)$$

$$x_{n,t}, h_{n,t} \geq 0;\ n = 1,\dots,|N|,\ t = 1,..,T \quad (27)$$

$$y_{n,t}, g_{n,t} \geq 0;\ n = 1,\dots,|N|,\ t = 2,..,T+1 \quad (28)$$

$$p_{n,t}, r_{n,t}, q_t \text{ binary},\ n = 1,\dots,|N|,\ t = 1,..,T \quad (29)$$

$$a_{n,t}, d_{n,t}, f_{n,t} \text{ binary},\ n = 1,\dots,|N|,\ t = 2,..,T+1 \quad (30)$$

The objective function (1) maximizes the cumulative residual flight time availability of the unit over the entire planning horizon, which consists of the sum of the individual fleet availabilities of all time periods. Constraint set (2) updates the residual flight time of each aircraft at the beginning of the next period, based on its residual flight time at the beginning of the previous period and the time that it flew during that period. Similarly, constraint set (5) updates the residual maintenance time of each aircraft at the beginning of the next period, based on its residual maintenance time at the beginning of the previous period and the time that it received maintenance during that period. Constraint sets (3), (4), (6) and (7) ensure that variables $d_{n,t}$ and $f_{n,t}$ take appropriate values, based on the values of variables $a_{n,t}$. Constraint set (8) ensures that the flight load in each time period is satisfied. Constraint sets (9) and (10) ensure that the time and space capacity constraints of the maintenance station are not violated in any time period. Constraint sets (11) and (12) ensure that the maintenance station does not remain idle whenever there is at least one aircraft waiting for service. Constraint sets (13) and (14) ensure that an aircraft's availability ceases as soon as its residual flight time drops to 0. Similarly, constraint sets (15) and (16) ensure that an aircraft becomes available as soon as its residual maintenance time drops to 0. Constraint set (17) states that the residual flight time of an aircraft cannot exceed Y, and ensures that it is equal to 0 whenever this aircraft is not available. Similarly, constraint set (18) states that the residual maintenance time of an aircraft cannot exceed G, and ensures that it is equal to 0 whenever this aircraft is available. Constraint set (19) imposes an upper bound on the maximum time that an aircraft can fly during a single time period. Such a restriction is usually present due to technical reasons. Constraint set (20) imposes a lower bound on the residual flight time of each available aircraft, and constraint (21) imposes a lower bound on the residual maintenance time of each non-available aircraft. These constraints are introduced to prevent an aircraft from ending up with negligible but positive residual flight or maintenance time. Constraint set (22) ensures that the total time that an aircraft flies during a single period does not exceed its residual flight time at the beginning of the same period. Similarly, constraint set (23) ensures that the total time that the maintenance crew works on an aircraft during a single period does not exceed the residual maintenance time of this aircraft at the beginning of the same period. Constraint sets (24), (25) and (26) are used to initialize the state of the system at the first period of the planning horizon. Finally, constraints (27), (28) and (29), (30) are the non-negativity and the integrality constraints, respectively.

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IV. Solution Methodology

We propose a solution algorithm which utilizes the fact that the cumulative fleet availability of the unit depends solely on the combination of aircraft that enter and exit the maintenance station and that the number of such combinations is finite. The algorithm solves a relaxation of the original problem first, in order to identify a valid upper bound on the optimal objective function value. Starting from this bound, the algorithm reduces gradually the value of the objective, until a feasible flight and maintenance plan that attains it is identified.

In general, there exist several aircraft combinations that can result in the same cumulative fleet availability. Each time one such combination is identified, the algorithm performs two separate checks for feasibility. The first check involves a typical *flow balance* calculation on the number of aircraft that enter and exit the maintenance station. The second check investigates whether it is possible for this particular aircraft combination to be realized by a specific flight and maintenance plan. If both these checks are successful, then the associated solution is optimal and the algorithm terminates. If not, a cut is added to the model, excluding this combination from further consideration. To check a particular aircraft combination for feasibility, we utilize the original formulation, after adjusting accordingly the model constraints to "force" the realization of this aircraft combination. The following subsections portray in more detail each step of the proposed solution algorithm.

A. *Bounding the Optimal Objective Function Value*

As determined by constraint (8), the fleet time availability of the unit drops in each time period by the corresponding flight load, independently of the particular flight and maintenance time allocation to the aircraft of the unit. Therefore, the cumulative fleet availability of the unit is maximized when the number of aircraft that enter the station for service over the entire planning horizon is the maximum possible, and each grounded aircraft completes its service as early as possible.

To see why this is the case, note that it is suboptimal for the maintenance station to interrupt the service of a grounded aircraft once this has begun, since this could potentially delay the addition of this aircraft's phase interval to the fleet availability of the unit by one or more time periods. This would clearly result in lower cumulative fleet availability, since the availability of any individual time period is more heavily weighted in the objective function than that of any succeeding one. Thus, as far as the maintenance decisions in each time period are concerned, it is always optimal for the station to begin its service on the grounded aircraft with the lowest residual maintenance time and to work continuously on this aircraft until its service is completed. Of course, this service may be spread out over more than one time periods if the time capacity of the station is not sufficient. In extend, a first-in-first-out policy is optimal for each subsequent aircraft that will enter the station for service.

Given this maintenance priority policy, an upper bound on the optimal objective function value can be obtained by computing the maximum possible number of aircraft that can enter the station for service over the entire planning horizon. This number can be found by grounding each available aircraft as early as possible. For time savings, the value of this bound is computed independently of whether the associated aircraft combination satisfies all the constraints of the original formulation. This implies that the corresponding solution this bound will be associated with will not necessarily be feasible. Note that, for each individual time period, the number of aircraft that cumulatively enter or exit the station according to this aircraft combination is the maximum possible in any feasible solution.

This aircraft combination is the first one that is checked for feasibility. If this check is successful, then the associated solution is optimal and the algorithm terminates. If not, the algorithm adds a cut that excludes this particular combination from further consideration, and searches for the next best combination.

B. *Checking a Particular Aircraft Combination for Feasibility*

Checking a particular aircraft combination for feasibility is quite trivial. It can be easily shown that there exists an optimal solution to the problem defined by (1)-(30), which preserves a steady rotation of the aircraft in and out of the maintenance station, in non-decreasing order of their residual flight/maintenance times. In practice, no such restriction is present. Aircraft are allowed to enter and exit the maintenance in any feasible order, and their indices are updated accordingly to represent their relative order in terms of their residual flight/maintenance times. With this in mind, the index of each aircraft at the beginning of the next period should be a decision variable allowed to take any feasible value. Since adding this degree of freedom complicates things unnecessarily, in what follows we impose a steady rotation of the aircraft in and out of the maintenance station. The proof that this has no effect whatsoever on the optimal solution is identical to the proof of Proposition 1 in [7]; for space consideration, we do not repeat this proof here.

With this in mind, the check of whether a particular aircraft combination is feasible reduces to a check of whether there exists a feasible flight and maintenance plan that realizes this combination. This is equivalent to checking the original formulation for feasibility with known the values of decision variables $a_{n,t}$, $d_{n,t}$ and $f_{n,t}$, for $n = 1,\ldots,|N|$ and $t = 2,\ldots,T+1$. In conjunction with the preservation of the order of aircraft, this simplifies things considerably. As a result, this check can be performed straightforwardly in negligible computational time, as is also demonstrated by the computational results that we present in Section V.

C. *Generating a Cut for the Exclusion of a Particular Aircraft Combination*

For the sake of illustration of how a cut excluding a particular aircraft combination is generated, we introduce the following additional notation:

$in_{t,k}$: number of aircraft that enter the station for service at the end of time period t in aircraft combination k;

$out_{t,k}$: number of aircraft that exit the station at the end of time period t in aircraft combination k

$m_{t,k:}$: number of grounded aircraft at the beginning of time period t in aircraft combination k

Suppose now that a particular aircraft combination with index $k = 1$ is proven infeasible. In this case, we need to add a valid inequality excluding this combination, in order to check if the current cumulative fleet availability can be attained by a different combination with index $k = 2$. A suitable cut that achieves this is the following: $\sum_{t=1}^{T}| in_{t,1} - in_{t,2} | + \sum_{t=1}^{T}| out_{t,1} - out_{t,2} | \geq 1$.

The corresponding formulation that we utilize in order to search if such a combination exists is the following:

$$Max \sum_{t=1}^{T}(Y(T-t+1)out_{t,2}) \quad (31)$$

$$\text{s.t.} \sum_{t=1}^{\tau} out_{t,2} \leq \sum_{t=1}^{\tau} out_{t,1},\ \tau = 1,..,T \quad (32)$$

$$\sum_{t=1}^{\tau} in_{t,2} \leq \sum_{t=1}^{\tau} in_{t,1},\ \tau = 1,..,T \quad (33)$$

$$m_{t,2} = m_{t-1,2} + in_{t-1,2} - out_{t-1,2},\ t = 2,..,T+1 \quad (34)$$

$$m_{t,2} \leq C,\ t = 2,..,T+1 \quad (35)$$

$$out_{t,2} \leq m_{t,2},\ t = 1,..,T \quad (36)$$

$$\sum_{t=1}^{T}| in_{t,1} - in_{t,2} | + \sum_{t=1}^{T}| out_{t,1} - out_{t,2} | \geq 1, \quad (37)$$

$$in_{t,2}, out_{t,2} \text{ integer} \geq 0,\ t = 1,..,T; \quad (38)$$

$$m_{t,2} \text{ integer} \geq 0,\ t = 2,..,T+1 \quad (39)$$

The objective function of the above formulation is equivalent to the objective function of the original problem. Constraint sets (32) and (33) impose the upper bounds on the cumulative number of aircraft that exit and enter the station, as determined by the initially identified aircraft combination. Constraint set (34) updates the number of grounded aircraft based on the number of aircraft that enter and exit the maintenance station. Constraint set (35) ensures that the space capacity of the maintenance station is not violated in any time period. Constraint set (36) states that the number of aircraft exiting the station at the beginning of each time period cannot exceed the number of grounded aircraft. Constraint (37) is the valid cut that excludes the previously identified infeasible combination. Of course, if more than one such combinations are found, one such cut needs to be added for each of them. Finally, constraint sets (38) and (39) impose the non-negativity and the integrality of the decision variables.

D. A Small Numerical Example

In this section, we illustrate the application of the proposed algorithm on a small numerical example. Consider a unit comprising of 6 aircraft, 5 of which are available and 1 of which is grounded at the beginning of the planning horizon.

TABLE I
RESIDUAL FLIGHT/MAINTENANCE TIMES (y_{ip}/g_{jp}) (HOURS)

$i = 1$	$i = 2$	$i = 3$	$i = 4$	$i = 5$	$j = 1$
5	38	186	213	257	**70**

Table I presents the residual flight times of the available aircraft and the residual maintenance times of the grounded aircraft. In this table, bold-style entries denote maintenance times of grounded aircraft and plain-style entries denote flight times of available aircraft.

Table II presents the flight load requirements and the time capacity of the maintenance station in each time period.

TABLE II
FLIGHT LOAD REQUIREMENTS
AND TIME CAPACITY OF THE MAINTENANCE STATION

t	1	2	3	4	5	6
S_t	97	115	99	121	121	113
B_t	129	148	154	144	126	135

Initially, we utilize the procedure for obtaining the upper bound on the optimal cumulative fleet availability. Table III shows the associated combination of aircraft that enter and exit the maintenance station in each time period.

TABLE III
INITIAL AIRCRAFT COMBINATION

t	1	2	3	4	5	6
in	2	0	0	1	1	1
out	1	0	0	1	0	1

First, we check if this combination satisfies the flow balance of the aircraft. Since it does, we next check if this combination leads to a feasible solution for the original formulation. This reduces to checking if there exists a feasible allocation of the flight load and the station's time capacity for which this combination is realized. This is not true here; therefore, we add a valid-cut excluding this combination, and we solve the sub-problem defined by (31)-(39) to see if there exists another combination that results in the same cumulative fleet availability. The optimal solution of this sub-problem identifies the aircraft combination shown in Table IV.

TABLE IV
SECOND AIRCRAFT COMBINATION

t	1	2	3	4	5	6
in	1	0	0	1	1	1
out	1	0	0	1	0	1

This combination is also infeasible; therefore, a new cut excluding it is added. The procedure continues similarly until the optimal solution of the problem is identified. This happens at the 6th combination which is shown in table V:

TABLE V
OPTIMAL AIRCRAFT COMBINATION

t	1	2	3	4	5	6
in	1	0	1	0	1	0
out	1	0	0	1	0	1

V. COMPUTATIONAL RESULTS

The proposed solution algorithm was implemented in C/C++ interfacing with LINGO 11.0 [12] through LINGO Dynamic Link Library (DLL) callback functions and its performance was compared against that of the MILP model of Section III, which was developed in LINGO 11.0. Our computational experiments were performed on an Intel Core 2 Duo E6750 @ 2.66GHz processor with 2 GB system memory.

We used 7 different values for $|N|$ and solved 20 random problem instances for each of them. We chose a considerably smaller problem size for the first 3 sets of problems, in order to enable their exact solution with the MILP model. In order to test the efficiency of the proposed solution algorithm on large scale problems too, we also tested it on problem instances with $|N|$ = 100, 200, 300, and 400. We were not able to apply the MILP model on these problems due to its excessive computational requirements. Typical combat wings of the HAF may consist of up to 100 aircraft; therefore, a high speed solution algorithm, such as the one that we propose is extremely important.

The value of T was always taken equal to 6, since the flight requirements are typically issued for a planning horizon of 6 monthly periods. The required flight time for each squadron and period combination was a random number distributed uniformly in the interval $[16|N|, 21|N|]$. The time capacity of the maintenance station in each time period was a random number distributed uniformly in the interval $[21|N|, 26|N|]$, and the space capacity was set equal to $0.1|N|$, rounded up to the nearest integer. These figures correspond to actual FMP configurations encountered in the HAF. We generated the number of grounded aircraft randomly, using a discrete probability function that considered integer values between 0 and C, inclusive. We set parameters Y and G equal to their actual values, i.e., 300 and 320 hours, respectively. The residual flight time of each available aircraft was a random number distributed uniformly in the interval $[Y_{min},Y]$, whereas the residual maintenance time of each grounded aircraft was a random number distributed uniformly in the interval $[G_{min},G]$. We used actual values drawn from the real application for the remaining problem parameters, i.e., $X_{max} = 50$, $Y_{\min} = 0.1$ and $G_{min} = 0.1$. We performed several checks to ensure that each randomly generated problem instance was feasible.

Table VI presents the average and maximum required computational times of the MILP LINGO model and of our proposed solution algorithm. The superiority of the latter becomes immediately clear, since its computational requirements are significantly lower than those of the MILP LINGO model. As the results of Table VI demonstrate, the computational savings increase considerably for large scale problem instances, for which the application of LINGO is impracticable.

TABLE VI
COMPUTATIONAL REQUIREMENTS (IN SECONDS)

	MILP LINGO		Proposed algorithm	
$\|N\|$	Avg	Max	Avg	Max
12	9.70	68.39	0.46	0.50
16	14500.24	33966.27	0.47	0.50
20	57979.32	277450.66	0.47	0.50
100			0.56	0.58
200			0.67	0.70
300			0.78	0.80
400			0.90	0.94

The number of iterations that the proposed solution algorithm performs is relatively small on the average. A short computational study reveals that the size of the problem alone is not indicative of the computational effort needed to reach an optimal solution. In order to make the task of finding the optimal solution more challenging for the algorithm, we opted for larger values of C equal to $0.35|N|$ and $0.4|N|$, rounded up to the nearest integer, although this is not in agreement with the actual value of C for realistic problems. We solved 20 random problem instances for each of the large scale problem values with $|N|$ = 100, 200, 300, and 400. The results are presented in Table VII.

TABLE VII
EFFECT OF VALUE OF C ON COMPUTATIONAL REQUIREMENTS

	$C = \lceil 0.35\|N\| \rceil$		$C = \lceil 0.4\|N\| \rceil$	
$\|N\|$	Avg	Max	Avg	Max
100	3.84	11.55	4.85	19.77
200	4.73	16.31	7.42	72.48
300	5.63	23.59	4.58	16.89
400	14.46	154.7	5.05	32.91

As the results of Table VII demonstrate, the computational burden of the proposed solution algorithm increases considerably when the value of C increases. However, the computational requirements are still very reasonable even for such large scale problem instances, for which the application of LINGO is impracticable.

VI. CONCLUSIONS & FUTURE RESEARCH

In this work, we developed an exact solution algorithm for the FMP problem, i.e., for the problem of issuing a joint flight and maintenance plan for a group of aircraft that comprise a unit, so as to maximize the unit's fleet availability. Our experimental results demonstrate that the proposed algorithm is capable of handling even large FMP instances quite effectively. An interesting extension for future research is to include a second objective in the model formulation that will minimize the variability of the fleet availability, so that this does not vary significantly in each time period of the planning horizon.

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