A New Gibbs-Sampling Based Algorithm for Bayesian Model Updating of Linear Dynamic Systems with Incomplete Complex Modal Data

Cheung Sai Hung and Sahil Bansal

Abstract—**Model updating using measured system dynamic response has a wide range of applications in structural health monitoring, control and response prediction. In this paper, we are interested in model updating of a linear structural dynamic system with non-classical damping based on incomplete modal data including modal frequencies, damping ratios, and partial complex mode shapes of some of the dominant modes. To quantify the uncertainties and plausibility of the model parameters, a Bayesian approach is adopted. A new Gibbs-sampling based algorithm is proposed that allows for an efficient update of the probability distribution of the model parameters. The effectiveness and efficiency of the proposed method are illustrated by a numerical example involving a linear structural dynamic system with complex modes.**

Index Terms—**Bayesian model updating, linear, structural dynamic system, complex modes, Gibbs sampling**

I. Introduction

The need of structural model updating is usually motivated by the desire to improve the accuracy of prediction of the system response and control, and structural health monitoring. In general, the system is assumed to be linear and classically damped, i.e., its equation of motion can be transformed into a set of decoupled modal equations using the real-valued eigenvectors and eigenvalues. Most vibration data of structures are obtained under low amplitude excitation, thus the assumption that the structures behave approximately linearly during such vibration is valid. However, in many real situations a linear system can be non-classically damped. Such is the case when a system is made up of materials with different damping characteristics in different parts of the system. For example, a soil-structure system, a system with supplemental viscous dampers, or a coupled structure-equipment (primary-secondary) system is non-classically damped. Assuming a non-classically damped system to be classically damped might bring in errors in the updating process because of non-orthogonality of damping.

The usual approach to update a linear structural dynamic system is to first identify its modal properties and then use those to update the modeling parameters. There are several ambient or forced vibration based modal identification techniques available [1-7] that provide optimal estimates of the modal parameters. These techniques can be grouped into two types: probabilistic and deterministic. Probabilistic techniques, particularly the Bayesian approach, provide estimates of the optimal parameters along with their probability density function (PDF) that can be used to describe the complete picture of the uncertainty while the outcome of deterministic techniques is usually a unique set of parameters. Several researchers [8-10] have presented work on updating of the Finite element models, based on the experimental modal data. However, there are relatively few papers in structural model updating literature in which probabilistic model updating is considered [11-14]. Ching *et al*. [13] proposed a new Gibbs sampler based simulation approach for model updating of linear dynamic systems with classical damping. In this paper, a stochastic simulation algorithm based on Gibbs sampler is presented for Bayesian model updating of linear structural dynamic system based on incomplete complex modal data, corresponding to modal frequencies, damping ratios, and partial complex mode shapes of some of the dominant modes of a dynamical system with non-classical damping. The proposed method is robust to the dimension of the problem. Finally, to demonstrate the effectiveness and accuracy of the proposed method, a numerical example with complex modes is shown.

II. Bayesian Model updating

Bayesian model updating approach provides a robust and rigorous framework to characterize modeling uncertainties. Given the data D and prior PDF $p(\boldsymbol{\theta})$ of the uncertain system parameters, by applying Bayes' theorem the posterior PDF can be written as

$$p(\boldsymbol{\theta} \mid D) = \frac{p(D \mid \boldsymbol{\theta})\, p(\boldsymbol{\theta})}{\int p(D \mid \boldsymbol{\theta})\, p(\boldsymbol{\theta}) d\boldsymbol{\theta}}. \quad (1)$$

$p(\boldsymbol{\theta} \mid D)$ is often not known explicitly and only known up to a normalizing constant. However, if samples distributed according to PDF $p(\boldsymbol{\theta} \mid D)$ are available, statistical estimates such as mean, variance, or PDF of $\boldsymbol{\theta}$ can be estimated. Assuming that $\boldsymbol{\theta}$ is divided into G groups of uncertain parameter vectors, i.e., $\boldsymbol{\theta} = [\boldsymbol{\theta}_i : i = 1,..,G]$. The Gibbs Sampler [15] is one type of Markov chain Monte

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Cheung Sai Hung is an Assistant Professor at the School of Civil and Environmental Engineering, Nanyang Technological University, 50 Nanyang Avenue, Singapore 639798 (shcheung@ntu.edu.sg).

Sahil Bansal is a PhD candidate at the School of Civil and Environmental Engineering, Nanyang Technological University, 50 Nanyang Avenue, Singapore 639798 (sahi0004@e.ntu.edu.sg)

Carlo (MCMC) algorithms that allow sampling from an arbitrary multivariate PDF if sample simulation according to the PDF of each group of uncertain parameters conditioned on all the others groups is possible. In Gibbs sampling algorithm, the full conditional PDFs $p_i(\boldsymbol{\theta}_i^* \mid \boldsymbol{\theta}_{1:i-1}^*, \boldsymbol{\theta}_{i+1:G}, D)$ are required for $i = 1,..,G$. Usually some initial portion of Markov chain samples are discarded before the stationary stage is reached. After the burn-in period the Markov chain samples obtained are distributed as the target PDF $p(\boldsymbol{\theta} \mid D)$. Statistical estimates such as the mean, variance, or PDFs can be estimated using the remaining samples.

A set of N_s experimental modal data identified from the structure under consideration, is considered for the Bayesian model updating problem. The modal data are denoted by

$$\hat{D} \equiv \left\{ \hat{\omega}_{m,s}, \hat{\zeta}_{m,s}, \hat{\phi}_{m,s} : m = 1 \ldots N_m, s = 1 \ldots N_s \right\}. \tag{2}$$

Where $\hat{\omega}_{m,s} \in \mathbb{R}$, $\hat{\zeta}_{m,s} \in \mathbb{R}$, and $\hat{\psi}_{m,s} \in \mathbb{C}^{N_o}$ are the observed modal frequency, damping ratio, and complex mode shape vector of the m-th mode in the s-th modal data set. Here, N_m is the number of modes identified and N_o is the number of observed DOF (degree of freedom).

III. Linear System updating model for complex modal data

In state-space, the equation of motion of a general N_d-DOF time invariant system can be expressed by a first order differential equation as follows

$$\dot{\mathbf{X}}(t) = \mathbf{AX}(t) + \mathbf{BU}(t). \tag{3}$$

$$\mathbf{X}(0) = \mathbf{X}_0. \tag{4}$$

$$\mathbf{A} = \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -\mathbf{M}^{-1}\mathbf{K} & -\mathbf{M}^{-1}\mathbf{C} \end{bmatrix}. \tag{5}$$

where $\mathbf{X}(t)$ and $\mathbf{U}(t)$ denote the state and excitation vectors at time t, respectively, and $\mathbf{X}_0$ denotes the initial conditions. The system matrix $\mathbf{A}$ is a function of mass, damping, and stiffness matrices $\mathbf{M}$, $\mathbf{C}$, and $\mathbf{K}$. Complex eigenvalues λ_m and eigenvector ψ_m, for $m = 1, 2, ..., N_m$, can be obtained from the solution of the eigenvalues problem corresponding to the system matrix $\mathbf{A}$ as

$$\mathbf{A}\psi_m = \lambda_m \psi_m. \tag{6}$$

$$\Psi_m = \begin{Bmatrix} \psi_m \\ \lambda_m \psi_m \end{Bmatrix}. \tag{7}$$

$$\lambda_m = -\zeta_m \omega_m + \mathrm{i}\,\omega_m \sqrt{1 - \zeta_m^2}. \tag{8}$$

The eigenvalues and eigenvectors occur in complex conjugate pairs. Using (5)-(6) and rearranging the terms gives the relationship between modal data and dynamic model parameters

$$\lambda_m^2 \mathbf{M}\psi_m + \lambda_m \mathbf{C}\psi_m + \mathbf{K}\psi_m = 0. \tag{9}$$

Replacing system eigenvalues λ_m with observed eigenvalues $\hat{\lambda}_{m,s}$ gives

$$\hat{\lambda}_{m,s}^2 \mathbf{M}\psi_m + \hat{\lambda}_{m,s} \mathbf{C}\psi_m + \mathbf{K}\psi_m = \boldsymbol{\varepsilon}_{m,s}. \tag{10}$$

where the system mode shape ψ_m is related to the observed mode shape $\hat{\psi}_{m,s}$ through a selection matrix Γ that picks the observed DOF from the system mode shape

$$\hat{\psi}_{m,s} = \Gamma \psi_m + \mathbf{e}_{m,s}. \tag{11}$$

In the above equation $\boldsymbol{\varepsilon}_{m,s}$ and $\mathbf{e}_{m,s}$ are the complex random vectors representing the model prediction errors, i.e., the errors between the response of the structure under consideration and that of the assumed model. The mass, damping and stiffness matrices in (10) are represented as a linear sum of contribution of the corresponding mass, damping, and stiffness matrices from the individual prescribed substructures

$$\mathbf{M}(\alpha) = \mathbf{M}_0 + \sum_{i=1}^{N_\alpha} \alpha_i \mathbf{M}_i. \tag{12}$$

$$\mathbf{C}(\beta) = \mathbf{C}_0 + \sum_{i=1}^{N_\beta} \beta_i \mathbf{C}_i. \tag{13}$$

$$\mathbf{K}(\eta) = \mathbf{K}_0 + \sum_{i=1}^{N_\eta} \eta_i \mathbf{K}_i. \tag{14}$$

where $[\boldsymbol{\alpha}, \boldsymbol{\beta}, \boldsymbol{\eta}]$ are the contribution parameters to be updated ($[\boldsymbol{\alpha}, \boldsymbol{\beta}, \boldsymbol{\eta}] = [1,..,1]$ gives the nominal matrices). Damping matrix can also be represented in terms of mass and stiffness matrix (as in the case of classical damping), and contribution from other damping sources (as in the case of viscous damping). Other parameters which are unknown in (10)-(11) and need to be updated are the system mode shapes ψ_m and the parameters defining the probabilistic models of the model prediction errors. Separating the real and imaginary parts, (10)-(11) are transformed to

$$\mathrm{Re}(\hat{\lambda}_{m,s}^2 \mathbf{M}(\alpha)\psi_m + \hat{\lambda}_{m,s} \mathbf{C}(\beta)\psi_m + \mathbf{K}(\eta)\psi_m) = \mathrm{Re}(\boldsymbol{\varepsilon}_{m,s}). \tag{15}$$

$$\mathrm{Im}(\hat{\lambda}_{m,s}^2 \mathbf{M}(\alpha)\psi_m + \hat{\lambda}_{m,s} \mathbf{C}(\beta)\psi_m + \mathbf{K}(\eta)\psi_m) = \mathrm{Im}(\boldsymbol{\varepsilon}_{m,s}). \tag{16}$$

$$\mathrm{Re}(\hat{\psi}_{m,s} - \Gamma \psi_m) = \mathrm{Re}(\mathbf{e}_{m,s}). \tag{17}$$

$$\mathrm{Im}(\hat{\psi}_{m,s} - \Gamma \psi_m) = \mathrm{Im}(\mathbf{e}_{m,s}). \tag{18}$$

Based on the Principle of Maximum Entropy [16], the PDFs for vectors $\mathrm{Re}(\boldsymbol{\varepsilon}_{m,s}), \mathrm{Im}(\boldsymbol{\varepsilon}_{m,s}), \mathrm{Re}(\mathbf{e}_{m,s}), \mathrm{Im}(\mathbf{e}_{m,s})$ are zero-mean Gaussian PDF and their covariance matrices are assumed to be equal to scaled versions of the identity matrix $\mathbf{I}$ of appropriate order

$$\mathrm{Re}(\boldsymbol{\varepsilon}_{m,s}) \sim N(0, \sigma_{\mathrm{Re},m}^2 \mathbf{I}). \tag{19}$$

$$\mathrm{Im}(\boldsymbol{\varepsilon}_{m,s}) \sim N(0, \sigma_{\mathrm{Im},m}^2 \mathbf{I}). \tag{20}$$

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$$\mathrm{Re}(\mathbf{e}_{m,s}) \sim N(0, \delta_{\mathrm{Re},m}^2 \mathbf{I}). \tag{21}$$

$$\mathrm{Im}(\mathbf{e}_{m,s}) \sim N(0, \delta_{\mathrm{Im},m}^2 \mathbf{I}). \tag{22}$$

The variance parameters $\delta_{\mathrm{Re},m}^2$ and $\delta_{\mathrm{Im},m}^2$ are assumed to be known or are directly estimated from the sample variance of the experimental modal data

$$\hat{\delta}_{\mathrm{Re},m}^2 = \frac{1}{N_s N_o} \sum_{j=1}^{N_s} \left| \mathrm{Re}(\hat{\psi}_{m,s} - \bar{\psi}_m) \right|^2. \tag{23}$$

$$\hat{\delta}_{\mathrm{Im},m}^2 = \frac{1}{N_s N_o} \sum_{j=1}^{N_s} \left| \mathrm{Im}(\hat{\psi}_{m,s} - \bar{\psi}_m) \right|^2. \tag{24}$$

where $\bar{\psi}_m$ is the averaged mode shape for m-th mode. The variance parameters $\sigma_{\mathrm{Re},m}^2, \sigma_{\mathrm{Im},m}^2$ are left for updating. In total, the parameters to be updated are the contribution parameters $[\boldsymbol{\alpha}, \boldsymbol{\beta}, \boldsymbol{\eta}]$, mode shapes $[\mathrm{Re}(\psi_1), \mathrm{Im}(\psi_1), ..., \mathrm{Re}(\psi_{Nm}), \mathrm{Im}(\psi_{Nm})]$, and prediction error variance $[\sigma_{\mathrm{Re},1}^2, \sigma_{\mathrm{Im},1}^2, .., \sigma_{\mathrm{Re},Nm}^2, \sigma_{\mathrm{Im},Nm}^2]$.

IV. Prior PDFs

To define the Gibbs Sampler algorithm, three groups of parameters are considered

$$\begin{aligned}
\boldsymbol{\theta}_1 &= [\boldsymbol{\alpha}, \boldsymbol{\beta}, \boldsymbol{\eta}], \\
\boldsymbol{\theta}_2 &= [\mathrm{Re}(\psi_1), \mathrm{Im}(\psi_1), ..., \mathrm{Re}(\psi_{Nm}), \mathrm{Im}(\psi_{Nm})], \\
\boldsymbol{\theta}_3 &= [\sigma_{\mathrm{Re},1}^2, \sigma_{\mathrm{Im},1}^2, .., \sigma_{\mathrm{Re},Nm}^2, \sigma_{\mathrm{Im},Nm}^2]
\end{aligned}$$

It will be more convenient to choose Bayesian conjugate priors which will allow exact sampling from the full conditional PDFs $p(\boldsymbol{\theta}_1 | \boldsymbol{\theta}_2, \boldsymbol{\theta}_3, \hat{D})$, $p(\boldsymbol{\theta}_2 | \boldsymbol{\theta}_1, \boldsymbol{\theta}_3, \hat{D})$, and $p(\boldsymbol{\theta}_3 | \boldsymbol{\theta}_1, \boldsymbol{\theta}_2, \hat{D})$. Thus, the initial PDF for the system parameters $\boldsymbol{\theta}_1$ is taken to be the product of independent Gaussian PDFs, $\boldsymbol{\theta}_1 \sim N(\boldsymbol{\theta}_1^{(0)}, P^{(0)})$ with mean $\boldsymbol{\theta}_1^{(0)}$ and diagonal covariance matrix $P^{(0)}$ to express the initial uncertainties. Similarly, the initial PDF for the system mode shapes $\boldsymbol{\theta}_2$ is taken to be the product of either independent Gaussian PDFs in case any prior information is available, or independent uniform PDFs in case of no prior information (as for the case with unknown components of the mode shapes). The initial PDF for prediction error variances $\boldsymbol{\theta}_3$ is taken to be the product of independent inverse gamma PDFs, $IG(\rho_0, \kappa_0)$.

V. Conditional PDFs

Equation (15)-(16) are linear with respect to $\boldsymbol{\theta}_1$, i.e., they can be written in the following form

$$\mathbf{Y} - \mathbf{A}\boldsymbol{\theta}_1 = \mathbf{E}. \tag{25}$$

Then, the full conditional PDF $p(\boldsymbol{\theta}_1 | \boldsymbol{\theta}_2, \boldsymbol{\theta}_3, \hat{D})$ is Gaussian whose first two moments are given by

$$\begin{aligned}
&\mathrm{E}(\boldsymbol{\theta}_1 | \boldsymbol{\theta}_2, \boldsymbol{\theta}_3, \hat{D}) \\
&\quad = \left(P^{(0)\,-1} + \mathbf{A}^T \Sigma_1^{-1} \mathbf{A} \right)^{-1} (P^{(0)\,-1} \boldsymbol{\theta}_1^{(0)} + \mathbf{A}^T \Sigma_1^{-1} \mathbf{Y}).
\end{aligned} \tag{26}$$

$$\begin{aligned}
&\mathrm{Cov}(\boldsymbol{\theta}_1 | \boldsymbol{\theta}_2, \boldsymbol{\theta}_3, \hat{D}) \\
&\quad = \left(P^{(0)\,-1} + \mathbf{A}^T \Sigma_1^{-1} \mathbf{A} \right)^{-1}.
\end{aligned} \tag{27}$$

Similarly, the PDF $p(\boldsymbol{\theta}_2 | \boldsymbol{\theta}_1, \boldsymbol{\theta}_3, \hat{D})$ is also a Gaussian and its first two moments can be obtained in a similar manner. $p(\boldsymbol{\theta}_3 | \boldsymbol{\theta}_1, \boldsymbol{\theta}_2, \hat{D})$ is an inverse gamma given by

$$p(\boldsymbol{\theta}_3 | \boldsymbol{\theta}_1, \boldsymbol{\theta}_2, \hat{D}) = IG\left(\rho_0 + \frac{N_d N_s}{2}, \kappa_0 + \frac{1}{2} \sum_{j=1}^{N_s} \mathbf{E}^T \mathbf{E} \right). \tag{28}$$

VI. Gibbs Sampling Algorithm

1) Initialize samples, drawn from prior or choose nominal values and let k=1.
2) Sample contribution parameters $\boldsymbol{\theta}_1^{(k)}$ from $p(\boldsymbol{\theta}_1^{(k)} | \boldsymbol{\theta}_2^{(k-1)}, \boldsymbol{\theta}_3^{(k-1)}, \hat{D})$.
3) Sample mode shape $\boldsymbol{\theta}_2^{(k)}$ from $p(\boldsymbol{\theta}_2^{(k)} | \boldsymbol{\theta}_1^{(k)}, \boldsymbol{\theta}_3^{(k-1)}, \hat{D})$.
4) Sample prediction error variance $\boldsymbol{\theta}_3^{(k)}$ from $p(\boldsymbol{\theta}_3^{(k)} | \boldsymbol{\theta}_1^{(k)}, \boldsymbol{\theta}_2^{(k)}, \hat{D})$.
5) Let $k=k+1$ and go to step 2, until N samples are obtained.

It can be seen that the proposed approach is a generalization of what was proposed by Ching *et al.* [13] with steps 1 to 5 being the same and **A**, **E**, **Y**, $\boldsymbol{\theta}_1$, $\boldsymbol{\theta}_2$ and $\boldsymbol{\theta}_3$ being different. For a classically damped system, (9) reduces to $(\mathbf{K} - \omega_r^2 \mathbf{M})\psi_r = 0$ and the dimension of the problem reduces by half as all the imaginary components are equal to zero.

VII. Illustrative Example

The system selected for the illustrative example is a 4-DOF mechanical system considered in [6] as shown in Fig. 1, and has the following properties: m_1=m_2=m_3=m_4=1 kg, k_1=k_3=k_5=7000 N/m, k_2=k_4=8000 N/m, c_1=c_3=c_5=0.7 N s/m, c_2=c_4=0.8 N s/m. The modal data for the model updating problem using Gibbs sampler for this system consists of 10 sets of modal data (N_s=10) with the first two modal frequencies, modal damping ratios, and partial complex mode shapes (corresponding to DOFs 1, 2, and 4, i.e., N_o=3) for each data set (N_m=2). Variability in modal data is introduced by perturbing the masses by 5%, and stiffnesses and dampings by 10%.

For Bayesian identification purpose, a dynamic model structure based on same 4-DOF mechanical system is considered and the uncertain parameters to be updated for this model class are contribution parameters $[\alpha_1, \alpha_2, \alpha_3, \alpha_4]$, $[\beta_1, \beta_2, \beta_3, \beta_4, \beta_5]$, $[\eta_1, \eta_2, \eta_3, \eta_4, \eta_5]$, prediction error variances $[\sigma_{\mathrm{Re},1}^2, \sigma_{\mathrm{Im},1}^2, \sigma_{\mathrm{Re},2}^2, \sigma_{\mathrm{Im},2}^2]$, and complete complex mode shapes $[\psi_1, \psi_2]$. Masses are assumed to be known with sufficient accuracy and thus the initial PDFs for

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$[\alpha_1, \alpha_2, \alpha_3, \alpha_4]$ are chosen with mean values equal to 1 and coefficient of variation (c.o.v., i.e., the ratio of standard deviation to mean) for each equal to 1%; and prior mean values for $[\beta_1, \beta_2, \beta_3, \beta_4, \beta_5]$ and $[\eta_1, \eta_2, \eta_3, \eta_4, \eta_5]$ are assumed equal to 1, with prior c.o.v. for each equal to 20%. Flat independent priors are taken for $[\psi_1, \psi_2]$. Independent inverse gamma prior PDFs with $\rho_0 = 2, \kappa_0 = 0$ are taken for $[\sigma_{\mathrm{Re},1}^2, \sigma_{\mathrm{Im},1}^2, \sigma_{\mathrm{Re},2}^2, \sigma_{\mathrm{Im},2}^2]$ (Jeffreys' non-informative prior). The total number of parameters to be updated is 34 (14 for the contribution parameters, 16 for the two mode shapes, and 4 prediction error variances).

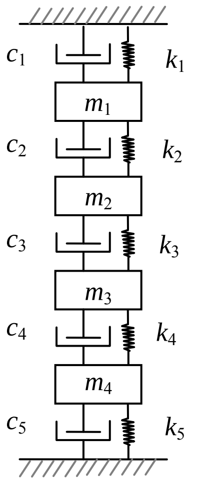
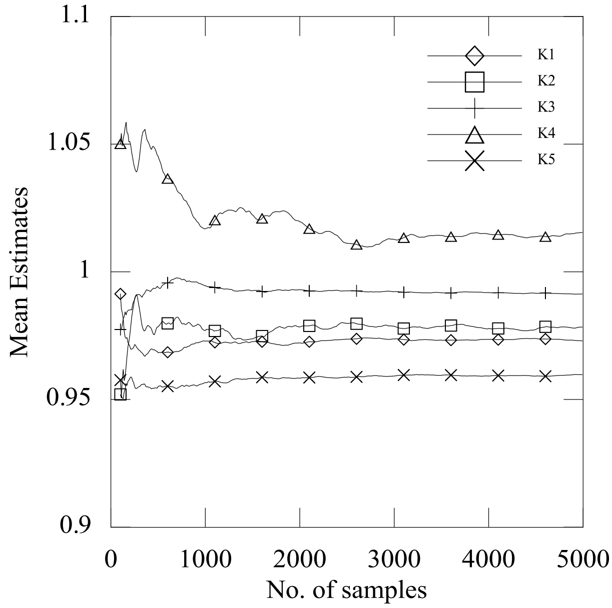


Fig. 1. The 4-DOF mechanical system

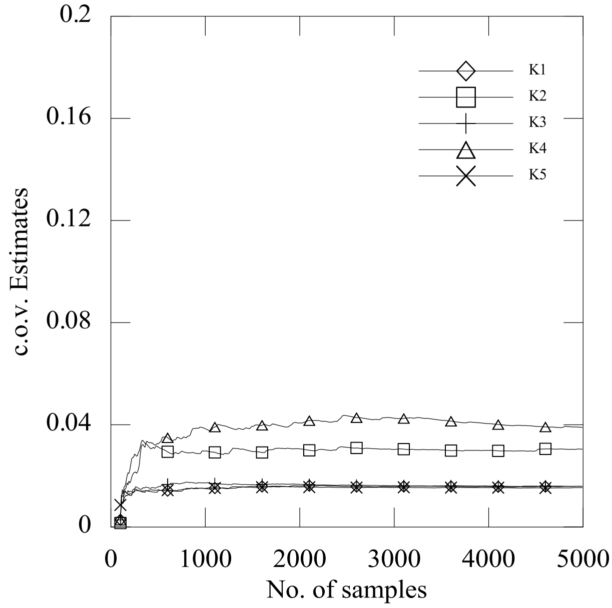
TABLE I
BAYESIAN IDENTIFICATION RESULTS

Parameter	Mean $\bar{\boldsymbol{\theta}}_1$	Standard Deviation	c.o.v. (%)	$\dfrac{\lvert \bar{\boldsymbol{\theta}}_1 - \tilde{\boldsymbol{\theta}}_1 \rvert}{\sigma}$
m_1	0.974	0.016	1.670	1.621
m_2	0.982	0.031	3.150	0.600
m_3	0.993	0.016	1.600	0.468
m_4	1.009	0.039	3.880	0.226
c_1	0.960	0.015	1.570	2.673
c_2	0.959	0.020	2.050	2.101
c_3	0.996	0.052	5.240	0.082
c_4	0.992	0.020	2.050	0.385
c_5	1.031	0.064	6.200	0.490
k_1	1.013	0.021	2.060	0.641
k_2	0.995	0.009	0.940	0.545
k_3	0.995	0.010	0.990	0.561
k_4	0.997	0.009	0.950	0.287
k_5	0.997	0.010	0.990	0.290

Using the proposed Gibbs sampling based algorithm, N=5,000 samples of contribution parameters, mode shapes and prediction errors variances are obtained. The burn-in period is less than 100 samples. Table I shows the statistical properties of the contribution parameters samples. It shows the estimated posterior mean values $\bar{\boldsymbol{\theta}}_1$ (column 2),



(a)



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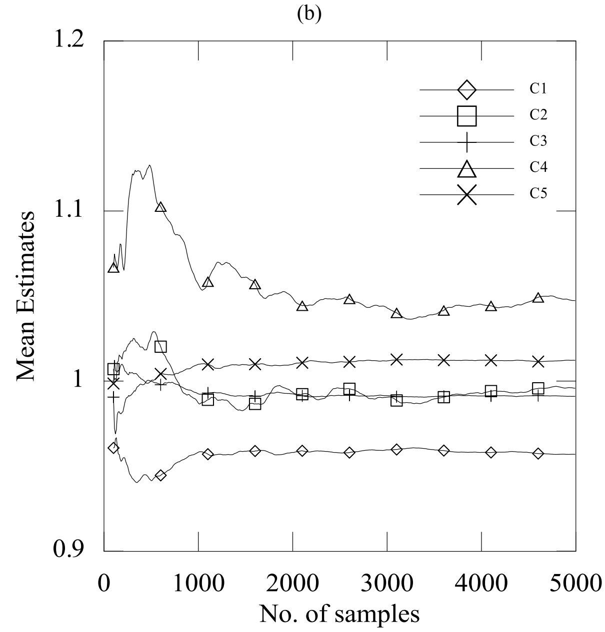

(c)

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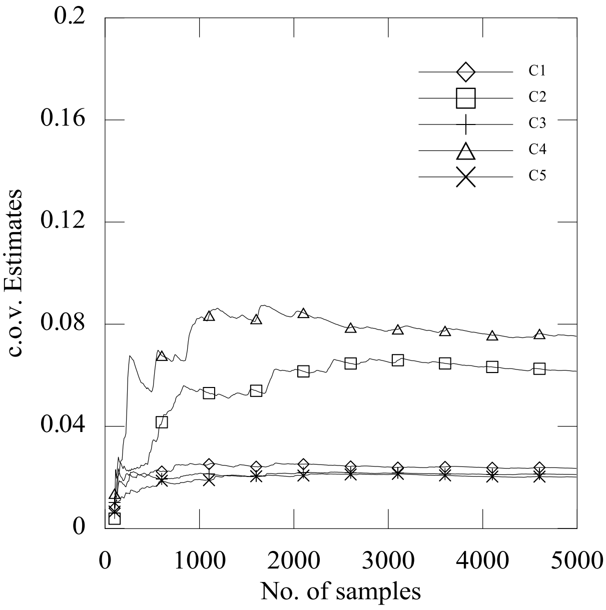

(d)

Fig. 2. Posterior mean and c.o.v. estimates for stiffness (a, b), and damping (c, d) contribution parameters as functions of the number of Gibbs samples, stabilizing after about 2000 samples.

posterior standard deviation (column 3), the posterior c.o.v. (column 4), and the normalized distance (column 5). The normalized distance represents the absolute value of difference between the estimated mean value and the true mean value $\tilde{\boldsymbol{\theta}}_1$ of structural parameters, normalized with respect to the estimated standard deviation. It can be seen that the estimated posterior mean parameters are close to the true mean values of the system and their c.o.v. is much smaller than that what was initially assumed. Table II shows the identified missing mode shape components (corresponding to DOF-3). Again the Bayesian identification results are quite close to the true mean values.

Table II
IDENTIFIED MODE SHAPE COMPONENT, DOF-3

Mode	True mean (abs, angle)		Posterior mean (abs, angle)	
1	1.529	0.004	1.528	0.003
2	0.654	3.134	0.635	3.136

The simulation results stabilized at around 2000 samples, as shown in Fig. 2 where the posterior mean and c.o.v. estimates of the stiffness and damping contribution parameters are reported as functions of the number of Gibbs samples.

VIII. Conclusion

A new Gibbs sampling based approach for model updating of a linear structural dynamic system with non-classical damping based on incomplete modal data including modal frequencies, damping ratios and partial complex mode shapes of some of the dominant modes is proposed. The results from the example demonstrate that all the updated parameters are reasonable when compared with the true mean values, indicating the effectiveness of the procedure. The proposed method also allows the uncertainty of the parameters to be updated efficiently even if there are a large number of uncertain parameters. Like most problems in identification, the quality of the updated results can only be as good as the quality of the data available.

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