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# Calculating the Degree of Similarity between Interval-Valued Fuzzy Numbers based on Map Distance

Shi-Jay Chen, Zhi-Yong Wang, Wei-Rou Li

***Abstract*—This study suggested a new similarity measured method that based on the map distance operator to solve before similarity measurement between interval-valued fuzzy numbers. In addition, some properties of the proposed similarity measure have been demonstrated, and 19 sets of interval-valued fuzzy numbers are adopted to compare the proposed method with existing similarity measures. The results of the comparison indicate that the proposed similarity measure outperforms existing methods.**

***Index Terms*—Interval-Valued Fuzzy Numbers, Similarity Measure, Map distance**

## I. Introduction

Interval-valued fuzzy numbers are very important in various domains. Guijun [10] et al. described interval-valued fuzzy numbers and their extended operations. Wang and Li [12] present the correlation coefficient of interval-valued fuzzy numbers and some of their properties. Lin [11] used interval-valued fuzzy numbers to represent vague processing time in job-shop scheduling problems. Yao and Lin [13] used interval-valued fuzzy numbers to represent unknown job processing time for constructing a fuzzy flow-shop sequencing model.

Some methods have been proposed for measuring the degree of similarity between interval-valued fuzzy numbers [4], [5], [6], [7], [8], [13]. However, existing similarity measures have some limitations. For example, they cannot correctly yield the degree of similarity between two interval-valued fuzzy numbers in some cases. Therefore, this study presents a new measure of similarity between interval-valued fuzzy numbers that is based on the standard deviation operator to overcome similarity measurement problems. Some properties of the proposed similarity measure are discussed. The proposed similarity measure is compared with five existing methods presented elsewhere by using 19 sets of interval-valued fuzzy numbers. The results of the comparison show that the proposed similarity measure overcomes the limitations of the existing methods.

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## II. Preliminaries

Chen [1], [2] definitions a generalized trapezoidal fuzzy number by $\widetilde{A}$=(a1, a2, a3, a4; $w_{\widetilde{A}}$), where 0 < $w_{\widetilde{A}}$ ≤ 1, 0 ≤ a1 ≤ a2 ≤ a3 ≤ a4 ≤1, and the value $w_{\widetilde{A}}$ denotes the degree of confidence of the linguistic opinion. Chen and Chen [3] presented the Simple Center of Gravity Method (SCGM) to calculate the COG points of generalized trapezoidal fuzzy numbers as follows:

$$y^*_{\widetilde{A}} = \begin{cases} \dfrac{w_{\widetilde{A}} \times (\dfrac{a_3 - a_2}{a_4 - a_1} + 2)}{6}, & \text{if } a_1 \neq a_4 \text{ and } 0 < w_{\widetilde{A}} \leq 1, \\ \dfrac{w_{\widetilde{A}}}{2}, & \text{if } a_1 = a_4 \text{ and } 0 < w_{\widetilde{A}} \leq 1, \end{cases} \quad (1)$$

$$x^*_{\widetilde{A}} = \frac{y^*_{\widetilde{A}}(a_3 + a_2) + (a_4 + a_1)(w_{\widetilde{A}} - y^*_{\widetilde{A}})}{2w_{\widetilde{A}}} \quad (2)$$

Yao and Lin [13] pointed out the interval-valued trapezoidal fuzzy number $\widetilde{\widetilde{A}}$ = [ $\widetilde{\widetilde{A}}^L$ , $\widetilde{\widetilde{A}}^U$ ] = [( $a_1^L$ , $a_2^L$ , $a_3^L$ , $a_4^L$ ; $w_{\widetilde{\widetilde{A}}^L}$ ), ( $a_1^U$ , $a_2^U$ , $a_3^U$ , $a_4^U$ ; $w_{\widetilde{\widetilde{A}}^U}$ )] as shown in Fig. 1, where $a_1^L \leq a_2^L \leq a_3^L \leq a_4^L$ , $a_1^U \leq a_2^U \leq a_3^U \leq a_4^U$ , $0 \leq w_{\widetilde{\widetilde{A}}^L} \leq 1$, $0 < w_{\widetilde{\widetilde{A}}^U} \leq 1$ , and $\widetilde{\widetilde{A}}^L \subset \widetilde{\widetilde{A}}^U$ .

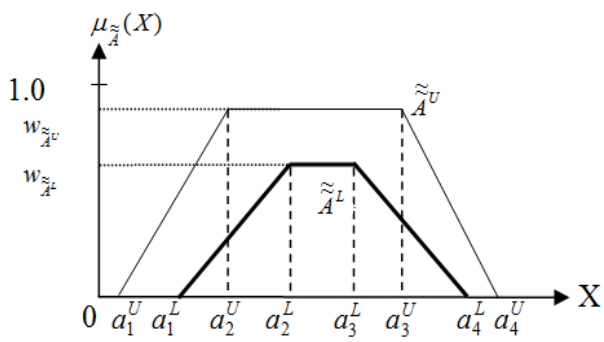


Fig. 1. Interval-valued trapezoidal fuzzy number $\widetilde{\widetilde{A}}$

Chen and Chen [6] presented a similarity measure between interval-valued trapezoidal fuzzy numbers as follows:

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$$S(\tilde{\tilde{A}},\tilde{\tilde{B}})=\sqrt{S(\tilde{\tilde{A}}^{L},\tilde{\tilde{B}}^{L})\times S(\tilde{\tilde{A}}^{U},\tilde{\tilde{B}}^{U})}\,, \qquad (3)$$

where

$$S(\tilde{\tilde{A}}^{L},\tilde{\tilde{B}}^{L})=\left[(1-\sum_{i=1}^{4}\left|a_i^L-b_i^L\right|\Big/4)\times(1-\left|x^*_{\tilde{\tilde{A}}^L}-x^*_{\tilde{\tilde{B}}^L}\right|)\right]^{1/2}\times\frac{\min(y^*_{\tilde{\tilde{A}}^L},y^*_{\tilde{\tilde{B}}^L})}{\max(y^*_{\tilde{\tilde{A}}^L},y^*_{\tilde{\tilde{B}}^L})}, \qquad (4)$$

$$S(\tilde{\tilde{A}}^{U},\tilde{\tilde{B}}^{U})=\left[(1-\sum_{i=1}^{4}\left|a_i^U-b_i^U\right|\Big/4)\times(1-\left|x^*_{\tilde{\tilde{A}}^U}-x^*_{\tilde{\tilde{B}}^U}\right|)\right]^{1/2}\times\frac{\min(y^*_{\tilde{\tilde{A}}^U},y^*_{\tilde{\tilde{B}}^U})}{\max(y^*_{\tilde{\tilde{A}}^U},y^*_{\tilde{\tilde{B}}^U})}, \qquad (5)$$

The larger the value of S( $\tilde{\tilde{A}},\tilde{\tilde{B}}$ ), the greater the similarity between interval-valued fuzzy numbers $\tilde{\tilde{A}}$ and $\tilde{\tilde{B}}$ .

Chen [4] presented a similarity measure between interval-valued trapezoidal fuzzy numbers as follows:

$$S(\tilde{\tilde{A}},\tilde{\tilde{B}})=\sqrt{S(\tilde{\tilde{A}}^{L},\tilde{\tilde{B}}^{L})\times S(\tilde{\tilde{A}}^{U},\tilde{\tilde{B}}^{U})}\,, \qquad (6)$$

where

$$S(\tilde{\tilde{A}}^{L},\tilde{\tilde{B}}^{L})=\left[\sqrt[4]{\prod_{i=1}^{4}(2-\left|a_i^L-b_i^L\right|)}-1\right]\times\frac{\min(y^*_{\tilde{\tilde{A}}^L},y^*_{\tilde{\tilde{B}}^L})}{\max(y^*_{\tilde{\tilde{A}}^L},y^*_{\tilde{\tilde{B}}^L})}, \qquad (7)$$

$$S(\tilde{\tilde{A}}^{U},\tilde{\tilde{B}}^{U})=\left[\sqrt[4]{\prod_{i=1}^{4}(2-\left|a_i^U-b_i^U\right|)}-1\right]\times\frac{\min(y^*_{\tilde{\tilde{A}}^U},y^*_{\tilde{\tilde{B}}^U})}{\max(y^*_{\tilde{\tilde{A}}^U},y^*_{\tilde{\tilde{B}}^U})}, \qquad (8)$$

The larger the value of S( $\tilde{\tilde{A}},\tilde{\tilde{B}}$ ), the greater the similarity between interval-valued fuzzy numbers $\tilde{\tilde{A}}$ and $\tilde{\tilde{B}}$ .

Wei and Chen [13] presented a similarity measure between interval-valued trapezoidal fuzzy numbers as follows:

$$S(\tilde{\tilde{A}},\tilde{\tilde{B}})=\left[\frac{S(\tilde{\tilde{A}}^{L},\tilde{\tilde{B}}^{L})+S(\tilde{\tilde{A}}^{U},\tilde{\tilde{B}}^{U})}{2}\times(1-\Delta x)\times(1-\Delta y)\right]^{\left(\frac{1}{1+2t}\right)}\times\left(1-\left|w_{\tilde{\tilde{A}}^U}-w_{\tilde{\tilde{B}}^U}-w_{\tilde{\tilde{A}}^L}+w_{\tilde{\tilde{B}}^L}\right|\right)^{\frac{u}{2}}, \qquad (9)$$

The larger the value of S( $\tilde{\tilde{A}},\tilde{\tilde{B}}$ ), the greater the similarity between interval-valued fuzzy numbers $\tilde{\tilde{A}}$ and $\tilde{\tilde{B}}$ .

Chen and Chen [5] presented a similarity measure between interval-valued trapezoidal fuzzy numbers. The degree of similarity S( $\tilde{\tilde{A}},\tilde{\tilde{B}}$ )between the interval-valued trapezoidal fuzzy numbers $\tilde{\tilde{A}}$ and $\tilde{\tilde{B}}$ can be calculated as follows:

$$S(\tilde{\tilde{A}},\tilde{\tilde{B}})=\frac{S_X^U(\tilde{\tilde{A}}^U,\tilde{\tilde{B}}^U)\times\left(1-\left|w_{\tilde{\tilde{A}}^U}-w_{\tilde{\tilde{B}}^U}\right|\right)}{1+STD^U(\tilde{\tilde{A}}^U,\tilde{\tilde{B}}^U)}\times S_X(\tilde{\tilde{A}},\tilde{\tilde{B}})\times S_Y(\tilde{\tilde{A}},\tilde{\tilde{B}}) \qquad (10)$$

The larger the value of S( $\tilde{\tilde{A}},\tilde{\tilde{B}}$ ), the greater the similarity between interval-valued fuzzy numbers $\tilde{\tilde{A}}$ and $\tilde{\tilde{B}}$ .

Chen [7] presented a similarity measure between interval-valued trapezoidal fuzzy numbers based on geometric-mean operator. The degree of similarity S( $\tilde{\tilde{A}},\tilde{\tilde{B}}$ ) can be calculated as follows:

$$S(\tilde{\tilde{A}},\tilde{\tilde{B}})=\frac{S(\tilde{\tilde{A}}^{U},\tilde{\tilde{B}}^{U})\times\left[1+S(\tilde{\tilde{A}}^{\Delta},\tilde{\tilde{B}}^{\Delta})\right]}{2}, \qquad (11)$$

where S( $\tilde{\tilde{A}},\tilde{\tilde{B}}$ ) $\in$ [0, 1], and

$$S(\tilde{\tilde{A}}^{\Delta},\tilde{\tilde{B}}^{\Delta})=\left[\sqrt[4]{\prod_{i=1}^{4}(2-\left|\Delta a_i-\Delta b_i\right|)}-1\right]\times\left(1-\left|w_{\tilde{\tilde{A}}^L}-w_{\tilde{\tilde{B}}^L}\right|\right), \qquad (12)$$

where i =1, 2, 3, 4 and S( $\tilde{\tilde{A}}^{\Delta}$ , $\tilde{\tilde{B}}^{\Delta}$ ) $\in$ [0, 1]. The value S( $\tilde{\tilde{A}}^U$ , $\tilde{\tilde{B}}^U$ ) denotes the degree of similarity between the upper trapezoidal fuzzy numbers $\tilde{\tilde{A}}^U$ and $\tilde{\tilde{B}}^U$ as follows:

$$S(\tilde{\tilde{A}}^{U},\tilde{\tilde{B}}^{U})=\left[\sqrt[4]{\prod_{i=1}^{4}(2-\left|a_i^U-b_i^U\right|)}-1\right]\times\frac{\min(w_{\tilde{\tilde{A}}^U},w_{\tilde{\tilde{B}}^U})}{\max(w_{\tilde{\tilde{A}}^U},w_{\tilde{\tilde{B}}^U})}, \qquad (13)$$

where S( $\tilde{\tilde{A}}^U$ , $\tilde{\tilde{B}}^U$ ) $\in$ [0, 1] and i =1, 2, 3, 4. The larger the value of S( $\tilde{\tilde{A}},\tilde{\tilde{B}}$ ), the greater the similarity between interval-valued trapezoidal fuzzy numbers $\tilde{\tilde{A}}$ and $\tilde{\tilde{B}}$ .

Chen and Kao [8] presented a similarity measure based on standard deviation for calculating the degree of similarity between interval-valued fuzzy numbers The degree of similarity S( $\tilde{\tilde{A}},\tilde{\tilde{B}}$ ) between the interval-valued trapezoidal fuzzy numbers $\tilde{\tilde{A}}$ and $\tilde{\tilde{B}}$ can be calculated as follows:

$$\mathrm{S}(\tilde{\tilde{\mathrm{A}}}^{\Delta},\tilde{\tilde{\mathrm{B}}}^{\Delta})=\left[1-\frac{\sqrt{\sum_{i=1}^{4}(\Delta a_i-\Delta b_i)^2}}{2}\right]\times\left[1-\sqrt{\frac{\left|\Delta S_a-\Delta S_b\right|}{2}}\right]\times\left[1-\frac{\left|\mathrm{w}_{\tilde{\tilde{\mathrm{A}}}^{\mathrm{L}}}-\mathrm{w}_{\tilde{\tilde{\mathrm{B}}}^{\mathrm{L}}}\right|}{\left|\mathrm{w}_{\tilde{\tilde{\mathrm{A}}}^{\mathrm{U}}}-\mathrm{w}_{\tilde{\tilde{\mathrm{B}}}^{\mathrm{U}}}\right|}\right] \qquad (14)$$

The larger the value of S( $\tilde{\tilde{A}},\tilde{\tilde{B}}$ ), the greater the similarity between interval-valued fuzzy numbers $\tilde{\tilde{A}}$ and $\tilde{\tilde{B}}$ .

## III. NEW METHOD FOR CALCULATING THE DEGREE OF SIMILARITY BETWEEN INTERVAL-VALUED FUZZY NUMBERS

This paper proposes a new similarity measure to calculate the degree of similarity between interval-valued trapezoidal fuzzy numbers, and shows some properties of this method. Let U be the universe of discourse, U= [0, 1]. Consider two interval-valued trapezoidal fuzzy numbers $\tilde{\tilde{A}}$ and $\tilde{\tilde{B}}$ , where $\tilde{\tilde{A}}$ = [ $\tilde{\tilde{A}}^L$ , $\tilde{\tilde{A}}^U$ ] = [( $a_1^L$ , $a_2^L$ , $a_3^L$ , $a_4^L$ ; $w_{\tilde{\tilde{A}}^L}$ ), ( $a_1^U$ , $a_2^U$ , $a_3^U$ , $a_4^U$ ; $w_{\tilde{\tilde{A}}^U}$ )] and $\tilde{\tilde{B}}$ = [ $\tilde{\tilde{B}}^L$ , $\tilde{\tilde{B}}^U$ ] = [( $b_1^L$ , $b_2^L$ , $b_3^L$ , $b_4^L$ ; $w_{\tilde{\tilde{B}}^L}$ ), ( $b_1^U$ , $b_2^U$ , $b_3^U$ , $b_4^U$ ; $w_{\tilde{\tilde{B}}^U}$ )], where $0 \le a_1^L \le a_2^L \le a_3^L \le a_4^L \le 1$, $0 \le a_1^U \le a_2^U \le a_3^U \le a_4^U \le 1$, $0 \le w_{\tilde{\tilde{A}}^L} \le w_{\tilde{\tilde{A}}^U} \le 1$ and $\tilde{\tilde{A}}^L \subset \tilde{\tilde{A}}^U$ ; $0 \le b_1^L \le b_2^L \le b_3^L \le b_4^L \le 1$, $\le b_1^U \le b_2^U \le b_3^U \le b_4^U \le 1$, $0 \le w_{\tilde{\tilde{B}}^L} \le w_{\tilde{\tilde{B}}^U} \le 1$ and

$\tilde{\tilde{B}}^L \subset \tilde{\tilde{B}}^U$. The degree of similarity between interval-value fuzzy numbers can be calculated as following steps.

Step 1: Calculate the distance values $\Delta a_i$ on the X-axis between the lower and upper trapezoidal fuzzy numbers $\tilde{\tilde{A}}^L$ and $\tilde{\tilde{A}}^U$ of the interval-valued trapezoidal fuzzy number $\tilde{\tilde{A}}$ shown as follows:

$$\Delta a_i = |a_i^U - a_i^L|, \quad (15)$$

where i =1, 2, 3, 4. In the same way, the distance values $\Delta b_i$ on the X-axis between the lower and upper trapezoidal fuzzy numbers $\tilde{\tilde{B}}^L$ and $\tilde{\tilde{B}}^U$ of the interval-valued trapezoidal fuzzy number $\tilde{\tilde{B}}$ can be calculated as formula (15).

Step 2: Calculate the degree of similarity S( $\tilde{\tilde{A}}^\Delta, \tilde{\tilde{B}}^\Delta$ ) between the distance values $\Delta a_i$ and $\Delta b_i$ of the two interval-valued fuzzy numbers $\tilde{\tilde{A}}$ and $\tilde{\tilde{B}}$ as follows,

$$S(\tilde{\tilde{A}}^\Delta, \tilde{\tilde{B}}^\Delta) = \left[1 - \frac{\sqrt{\sum_{i=1}^{4}(\Delta a_i - \Delta b_i)^2}}{2}\right] \times \left[1 - \sqrt{\frac{|\Delta S_a - \Delta S_b|}{2}}\right] \quad (16)$$

$$\times \left[1 - \frac{\left|w_{\tilde{\tilde{A}}^L} - w_{\tilde{\tilde{B}}^L}\right|}{\left|w_{\tilde{\tilde{A}}^U} - w_{\tilde{\tilde{B}}^U}\right|}\right] \times T^\Delta,$$

where i=1, 2, 3, 4, S( $\tilde{\tilde{A}}^\Delta, \tilde{\tilde{B}}^\Delta$ ) $\in$ [0,1], $\Delta S_a = \left|S_{\tilde{\tilde{A}}^U} - S_{\tilde{\tilde{A}}^L}\right|$, and $\Delta S_b = \left|S_{\tilde{\tilde{B}}^U} - S_{\tilde{\tilde{B}}^L}\right|$. $T^\Delta$ denotes map distance between the lower and upper trapezoidal fuzzy number $\tilde{\tilde{A}}^L$ and $\tilde{\tilde{A}}^U$ of interval-valued trapezoidal fuzzy number $\tilde{\tilde{A}}$. $S_{\tilde{\tilde{A}}^U}$, $S_{\tilde{\tilde{A}}^L}$ and $T^\Delta$ can be calculated as follows:

$$S_{\tilde{\tilde{A}}^U} = \sqrt{\frac{\sum_{i=1}^{4}(a_i^U - \bar{a}^U)^2}{n-1}}, \quad (17)$$

$$S_{\tilde{\tilde{A}}^L} = \sqrt{\frac{\sum_{i=1}^{4}(a_i^L - \bar{a}^L)^2}{n-1}}, \quad (18)$$

$$T^\Delta = \frac{\left[\left(2 - \frac{1 + \max\{|\Delta a_2 - \Delta a_1|, |\Delta b_2 - \Delta b_1|\}}{1 + \min\{|\Delta a_2 - \Delta a_1|, |\Delta b_2 - \Delta b_1|\}}\right) + \left(2 - \frac{1 + \max\{|\Delta a_4 - \Delta a_3|, |\Delta b_4 - \Delta b_3|\}}{1 + \min\{|\Delta a_4 - \Delta a_3|, |\Delta b_4 - \Delta b_3|\}}\right)\right]}{2}, \quad (19)$$

where $\bar{a}^U$ denotes average of the four values $a_1^U$, $a_2^U$, $a_3^U$, and $a_4^U$ at the upper trapezoidal fuzzy number $\tilde{\tilde{A}}^U$, and $\bar{a}^L$ denotes average of the four values $a_1^L$, $a_2^L$, $a_3^L$, and $a_4^L$ at the lower trapezoidal fuzzy number $\tilde{\tilde{A}}^L$. In the same way, the values $S_{\tilde{\tilde{B}}^U}$ and $S_{\tilde{\tilde{B}}^L}$ can be calculated as formulae (17) and (18).

Step 3: Calculate the degree of similarity S( $\tilde{\tilde{A}}^U, \tilde{\tilde{B}}^U$ ) between the fuzzy numbers $\tilde{\tilde{A}}^U$ and $\tilde{\tilde{B}}^U$ as follows,

$$S(\tilde{\tilde{A}}^U, \tilde{\tilde{B}}^U) = \left[1 - \frac{\sqrt{\sum_{i=1}^{4}(a_i^U - b_i^U)^2}}{2}\right] \times \left[1 - \sqrt{\frac{\left|S_{\tilde{\tilde{A}}^U} - S_{\tilde{\tilde{B}}^U}\right|}{2}}\right] \quad (20)$$

$$\times \frac{\min(w_{\tilde{\tilde{A}}^U}, w_{\tilde{\tilde{B}}^U})}{\max(w_{\tilde{\tilde{A}}^U}, w_{\tilde{\tilde{B}}^U})} \times T^U,$$

$$T^U = \frac{\left[\left(2 - \frac{1 + \max\{|a_2^U - a_1^U|, |b_2^U - b_1^U|\}}{1 + \min\{|a_2^U - a_1^U|, |b_2^U - b_1^U|\}}\right) + \left(2 - \frac{1 + \max\{|a_4^U - a_3^U|, |b_4^U - b_3^U|\}}{1 + \min\{|a_4^U - a_3^U|, |b_4^U - b_3^U|\}}\right)\right]}{2}, \quad (21)$$

Step 4: Calculate the degree of similarity S( $\tilde{\tilde{A}}, \tilde{\tilde{B}}$ ) between the interval-valued trapezoidal fuzzy numbers $\tilde{\tilde{A}}$ and $\tilde{\tilde{B}}$ as follows,

$$S(\tilde{\tilde{A}}, \tilde{\tilde{B}}) = \frac{S(\tilde{\tilde{A}}^U, \tilde{\tilde{B}}^U) \times (1 + S(\tilde{\tilde{A}}^\Delta, \tilde{\tilde{B}}^\Delta))}{2}, \quad (22)$$

where S( $\tilde{\tilde{A}}, \tilde{\tilde{B}}$ ) $\in$ [0,1]. The larger the value of S( $\tilde{\tilde{A}}, \tilde{\tilde{B}}$ ), the greater the similarity between interval-valued fuzzy numbers $\tilde{\tilde{A}}$ and $\tilde{\tilde{B}}$ .

## IV. Comparing Existing Methods with the Proposed Similarity Measure

This section compares the proposed similarity measure with six existing similarity measures [5][4][5][6][7][8][13] using 19 sets of Interval-valued fuzzy numbers shown in Fig.2. Table 1 and Fig. 2 point out that six existing similarity measures have some drawbacks described as follows:

(1) In Set 4 of Fig. 2, the two interval-valued fuzzy numbers $\tilde{\tilde{A}}$ and $\tilde{\tilde{C}}$ are more similar than the two interval-valued fuzzy numbers $\tilde{\tilde{A}}$ and $\tilde{\tilde{B}}$. However, Table 1 shows that the methods of Chen and Chen[5], Chen[4], and Wei and Chen[13] yield an incorrect result that the two interval-valued fuzzy numbers $\tilde{\tilde{A}}$ and $\tilde{\tilde{B}}$ are more similar than the two fuzzy umbers $\tilde{\tilde{A}}$ and $\tilde{\tilde{C}}$.

(2) In Set 5 of Fig. 2, the two interval-valued fuzzy numbers $\tilde{\tilde{A}}$ and $\tilde{\tilde{B}}$ are more similar than the two interval-valued fuzzy numbers $\tilde{\tilde{A}}$ and $\tilde{\tilde{C}}$. However,

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Table 1 indicates that applying Chen's[4] method yields the same degree of similarity (i.e., S( $\tilde{\tilde{A}}$ , $\tilde{\tilde{B}}$ ) = S( $\tilde{\tilde{A}}$ , $\tilde{\tilde{C}}$ )) for the two sets ( $\tilde{\tilde{A}}$ , $\tilde{\tilde{B}}$ ) and ( $\tilde{\tilde{A}}$ , $\tilde{\tilde{C}}$ ) of fuzzy numbers $\tilde{\tilde{A}}$ , $\tilde{\tilde{B}}$ and $\tilde{\tilde{C}}$ .

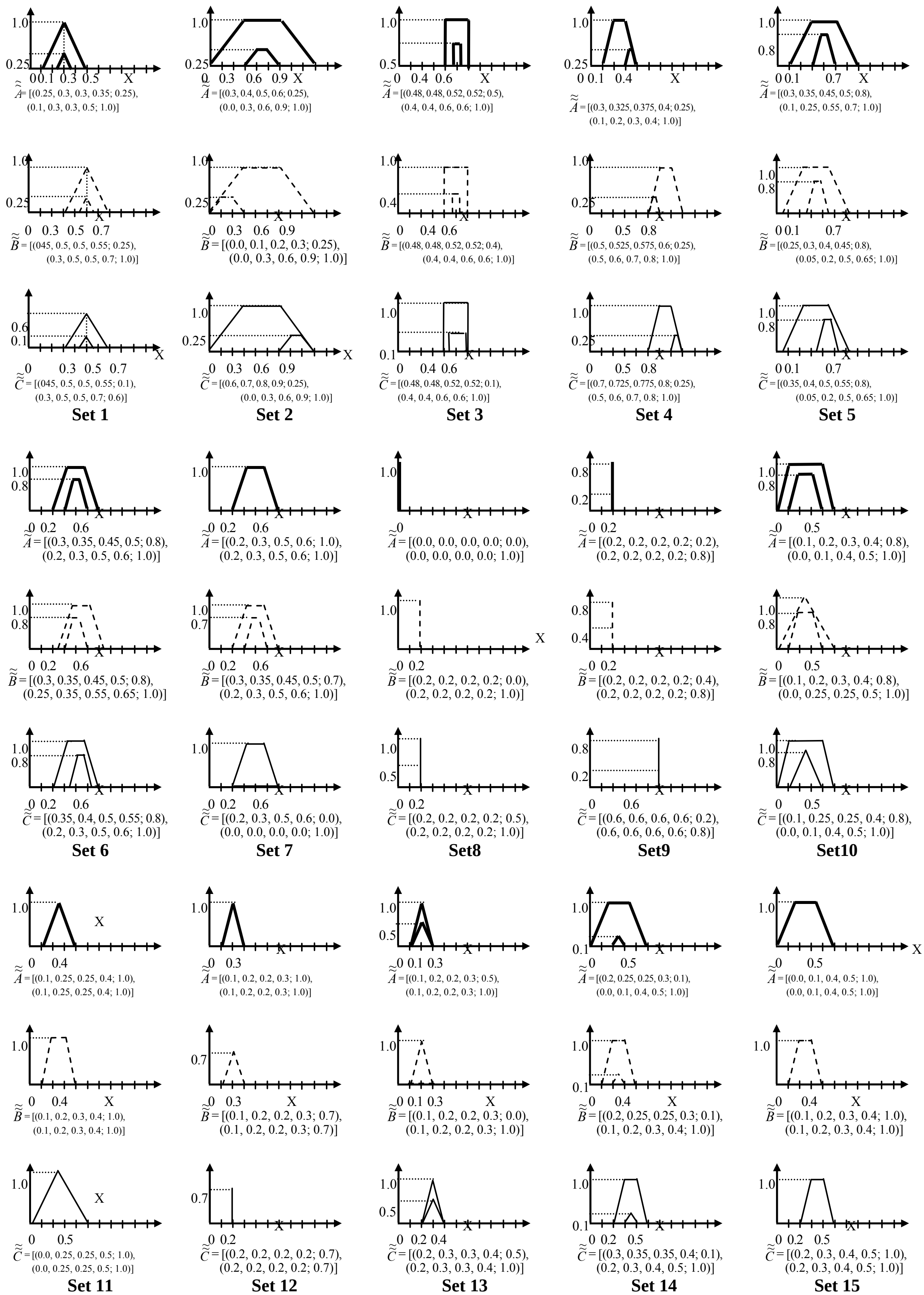


Fig. 2(a). The 19 sets of interval-valued fuzzy numbers.

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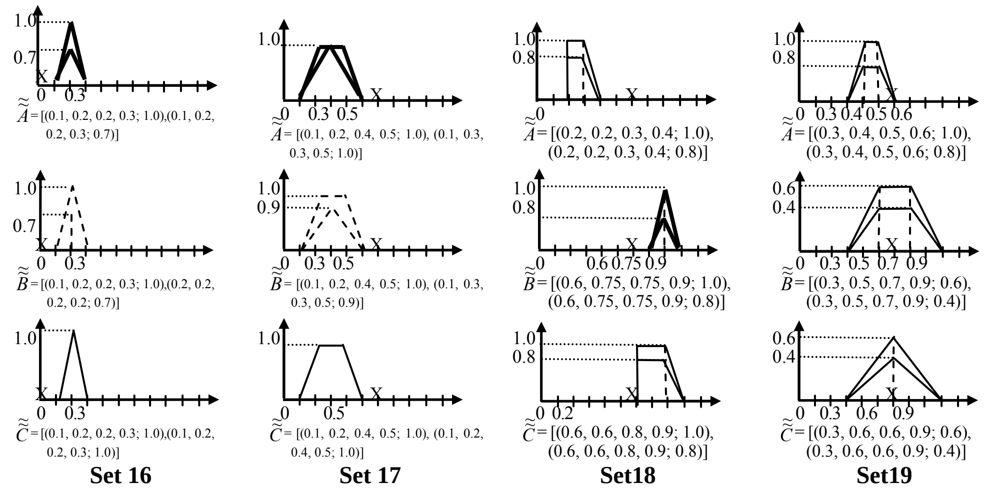


Fig. 2(b). The 19 sets of interval-valued fuzzy numbers.

TABLE 1

COMPARISON OF THE CALCULATION RESULTS OF THE PROPOSED SIMILARITY MEASURE AND THE EXISTING METHODS

| | Chen and Chen's Method[5] | | Chen's Method [4] | | Chen-and-Chen 's Method[6] | | Wei-and-Chen' s Method[13] | | Chen's Method [7] | | Chen-and-Kao's Method [8] | | The Proposed Method | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $\tilde{\tilde{A}},\tilde{\tilde{B}}$ | $\tilde{\tilde{A}},\tilde{\tilde{C}}$ | $\tilde{\tilde{A}},\tilde{\tilde{B}}$ | $\tilde{\tilde{A}},\tilde{\tilde{C}}$ | $\tilde{\tilde{A}},\tilde{\tilde{B}}$ | $\tilde{\tilde{A}},\tilde{\tilde{C}}$ | $\tilde{\tilde{A}},\tilde{\tilde{B}}$ | $\tilde{\tilde{A}},\tilde{\tilde{C}}$ | $\tilde{\tilde{A}},\tilde{\tilde{B}}$ | $\tilde{\tilde{A}},\tilde{\tilde{C}}$ | $\tilde{\tilde{A}},\tilde{\tilde{B}}$ | $\tilde{\tilde{A}},\tilde{\tilde{C}}$ | $\tilde{\tilde{A}},\tilde{\tilde{B}}$ | $\tilde{\tilde{A}},\tilde{\tilde{C}}$ |
| Set 1 | 0.8 | 0.3647 | 0.8 | 0.3919 | 0.8 | 0.4115 | 0.8 | 0.444 | 0.8 | 0.444 | 0.798 | 0.4575 | 0.8 | 0.4575 |
| Set 2 | 0.8367 | 0.8367 | 0.8367 | 0.8367 | 0.7 | 0.7 | 0.874 | 0.874 | 0.874 | 0.874 | 0.8677 | 0.8677 | 0.7253 | 0.7253 |
| Set 3 | 0.8944 | 0.4472 | 0.8944 | 0.4472 | 0.9983 | 0.9814 | 0.9668 | 0.8475 | 0.95 | 0.8 | 0.975 | 0.9 | 0.975 | 0.9 |
| Set 4 | 0.6928 | 0.4559 | 0.6928 | 0.6 | 0.48 | 0.6 | 0.7402 | 0.7114 | 0.5621 | 0.6 | 0.5563 | 0.598 | 0.5563 | 0.6 |
| Set 5 | 0.95 | 0.9372 | 0.95 | 0.95 | 0.95 | 0.855 | 0.9664 | 0.9539 | 0.95 | 0.9025 | 0.95 | 0.9025 | 0.95 | 0.8069 |
| Set 6 | 0.9747 | 0.9616 | 0.9747 | 0.9747 | 0.9025 | 0.95 | 0.9632 | 0.9805 | 0.9263 | 0.975 | 0.9261 | 0.975 | 0.8549 | 0.8809 |
| Set 7 | 0.8205 | * | 0.8046 | 0 | 0.4477 | 0.4167 | 0.8079 | 0.5 | 0.8237 | 0.5 | 0.8078 | 0.75 | 0.8718 | 0.75 |
| Set 8 | * | * | * | 0 | 0.8 | 0.8 | 0.4 | 0.2828 | 0.8 | 0.6 | 0.8 | 0.7 | 0.8 | 0.7 |
| Set 9 | 0.7071 | 0.6 | 0.7071 | 0.6 | 1 | 0.6 | 0.6708 | 0.6 | 0.9 | 0.6 | 0.9375 | 0.6 | 0.9375 | 0.6 |
| Set 10 | 0.8601 | 0.9018 | 0.8429 | 0.9141 | 0.7101 | 0.9652 | 0.9283 | 0.9875 | 0.9119 | 0.9874 | 0.7149 | 0.9546 | 0.6434 | 0.9582 |
| Set 11 | 0.8464 | 0.9682 | 0.8356 | 0.9494 | 0.9686 | 0.8783 | 0.95 | 0.8862 | 0.9748 | 0.9494 | 0.9091 | 0.7415 | 0.8679 | 0.6771 |
| Set 12 | 0.7 | 0.9283 | 0.7 | 0.9042 | 0.49 | 0.4304 | 0.7209 | 0.6215 | 0.595 | 0.5649 | 0.6382 | 0.4733 | 0.6382 | 0.426 |
| Set 13 | * | 0.9 | 0 | 0.9 | 0.8333 | 0.9 | * | 0.9322 | 0.75 | 0.9 | 0.875 | 0.9 | 0.875 | 0.9 |
| Set 14 | 0.9227 | 0.8279 | 0.8987 | 0.8513 | 0.7009 | 0.7009 | 0.9533 | 0.9031 | 0.855 | 0.8524 | 0.583 | 0.5562 | 0.6555 | 0.6254 |
| Set 15 | 0.8514 | 0.7843 | 0.8077 | 0.8053 | 0.8116 | 0.8116 | 0.8055 | 0.8055 | 0.9 | 0.8974 | 0.69 | 0.6582 | 0.69 | 0.6583 |
| Set 16 | 0.8061 | 0.8367 | 0.7956 | 0.8367 | 0.7283 | 0.5667 | 0.8937 | 0.8605 | 0.9747 | 0.85 | 0.8708 | 0.925 | 0.9182 | 0.925 |
| Set 17 | 0.9487 | 0.9413 | 0.9487 | 0.929 | 0.9667 | 0.4117 | 0.9744 | 0.9764 | 0.95 | 0.9747 | 0.975 | 0.919 | 0.975 | 0.9182 |
| Set 18 | 0.42 | 0.51 | 0.42 | 0.51 | 0.41 | 0.51 | 0.37 | 0.17 | 0.52 | 0.55 | 0.4613 | 0.457 | 0.4152 | 0.457 |
| Set 19 | 0.47 | 0.4 | 0/.47 | 0.39 | 0.3 | 0.29 | 0.32 | 0.32 | 0.34 | 0.34 | 0.3183 | 0.3241 | 0.2897 | 2657 |

**Note**: " * " means that the similarity measure cannot calculate the degree of similarity between two interval-valued fuzzy numbers.

" " means incorrect results

(3) In Set 6 of Fig. 2, the degrees of similarity S($\tilde{\tilde{A}},\tilde{\tilde{B}}$) and S($\tilde{\tilde{A}},\tilde{\tilde{C}}$) of the two sets of interval-valued fuzzy numbers ($\tilde{\tilde{A}},\tilde{\tilde{B}}$) and ($\tilde{\tilde{A}},\tilde{\tilde{C}}$) are different. However, Table 1 shows that Chen's[4] method yields the same degrees of similarity for the two sets ($\tilde{\tilde{A}},\tilde{\tilde{B}}$) and ($\tilde{\tilde{A}},\tilde{\tilde{C}}$) of interval-valued fuzzy numbers $\tilde{\tilde{A}},\tilde{\tilde{B}}$ and $\tilde{\tilde{C}}$.

(4) In Set 7 of Table 1, the degree of similarity between the interval-valued fuzzy numbers $\tilde{\tilde{A}}$ and $\tilde{\tilde{C}}$ cannot be correctly calculated using Chen and Chen's[5] Method, because $x^*_{\tilde{\tilde{C}}^L} = \infty$. Furthermore, in Set 7 of Fig. 2, the degree of similarity S($\tilde{\tilde{A}},\tilde{\tilde{C}}$) is not zero. However, Table 1 indicates that Chen's[4] method yields S($\tilde{\tilde{A}},\tilde{\tilde{C}}$)= 0.

(5) In Set 8 of Table 1, the degrees of similarity S($\tilde{\tilde{A}},\tilde{\tilde{B}}$) and S($\tilde{\tilde{A}},\tilde{\tilde{C}}$) cannot be correctly calculated using Chen and Chen's[5] Method because $x^*_{\tilde{\tilde{A}}^L} = \infty$ and $x^*_{\tilde{\tilde{B}}^L} = \infty$. The degree of similarity S($\tilde{\tilde{A}},\tilde{\tilde{B}}$)cannot be correctly calculated using Chen's[4] method because S($\tilde{\tilde{A}}_L$, $\tilde{\tilde{B}}_L$) = ∞. Additionally, in Set 8 of Fig. 2, the degree of similarity S($\tilde{\tilde{A}},\tilde{\tilde{C}}$) is not zero. However, Table 1 indicates that the methods of Chen's[4] method yields an incorrect result S($\tilde{\tilde{A}},\tilde{\tilde{C}}$)=0. The degrees of similarity S($\tilde{\tilde{A}},\tilde{\tilde{B}}$) and S($\tilde{\tilde{A}},\tilde{\tilde{C}}$)of the two sets of interval-valued fuzzy numbers ($\tilde{\tilde{A}},\tilde{\tilde{B}}$) and ($\tilde{\tilde{A}},\tilde{\tilde{C}}$) are different. However, Table 1 demonstrates that Chen and Chen's[5] method yields the same degree of similarity for the two sets ($\tilde{\tilde{A}},\tilde{\tilde{B}}$) and ($\tilde{\tilde{A}},\tilde{\tilde{C}}$) of interval-valued fuzzy numbers $\tilde{\tilde{A}},\tilde{\tilde{B}}$ and $\tilde{\tilde{C}}$.

(6) In Set 9 of Fig. 2, the interval-valued fuzzy numbers $\tilde{\tilde{A}}$ and $\tilde{\tilde{B}}$ not the same. However, according to Table 1, Chen and Chen's[5] method yields S($\tilde{\tilde{A}},\tilde{\tilde{B}}$)= 1.

(7) In Set 11 of Fig. 2, the two interval-valued fuzzy numbers $\tilde{\tilde{A}}$ and $\tilde{\tilde{B}}$ have higher similarity than the two interval-valued fuzzy numbers $\tilde{\tilde{A}}$ and $\tilde{\tilde{C}}$. However, Table 1 indicates that the methods of Chen and Chen[5], and Chen[4] yield an incorrect result that the two interval-valued fuzzy numbers $\tilde{\tilde{A}}$ and $\tilde{\tilde{C}}$ are more similar than the two fuzzy numbers $\tilde{\tilde{A}}$ and $\tilde{\tilde{B}}$.

(8) In Set 12 of Fig. 2, the two interval-valued fuzzy numbers $\tilde{\tilde{A}}$ and $\tilde{\tilde{B}}$ are more similar than the two fuzzy numbers $\tilde{\tilde{A}}$ and $\tilde{\tilde{C}}$. However, Table 1 indicates that the methods of Chen and Chen[5] , and Chen[4] yield an incorrect result that the two interval-valued fuzzy numbers $\tilde{\tilde{A}}$ and $\tilde{\tilde{C}}$ are more similar than the two fuzzy numbers $\tilde{\tilde{A}}$ and $\tilde{\tilde{B}}$ .

(9) In Set 13 of Table 1, the degree of similarity S( $\tilde{\tilde{A}}$ , $\tilde{\tilde{B}}$ ) cannot be correctly determined using Chen and Chen's[5] method and Wei-and-Chen's[13] method because $x^*_{\tilde{\tilde{A}}^L} = \infty$ and $x^*_{\tilde{\tilde{B}}^L} = \infty$. Furthermore, in Set 13 of Fig. 2, the degree of similarity S( $\tilde{\tilde{A}}$ , $\tilde{\tilde{B}}$ ) is not zero. However, Table 1 indicates that Chen's[4] method yields S( $\tilde{\tilde{A}}$ , $\tilde{\tilde{C}}$ ) = 0.

(10) In Set 14 of Fig. 2, the degrees of similarity S( $\tilde{\tilde{A}}$ , $\tilde{\tilde{B}}$ ) and S( $\tilde{\tilde{A}}$ , $\tilde{\tilde{C}}$ ) of the two sets of interval-valued fuzzy numbers ( $\tilde{\tilde{A}}$ , $\tilde{\tilde{B}}$ ) and ( $\tilde{\tilde{A}}$ , $\tilde{\tilde{C}}$ ) are different. However, Table 1 indicates that Chen and Chen's[5] method yields the same degrees of similarity for the two sets ( $\tilde{\tilde{A}}$ , $\tilde{\tilde{B}}$ ) and ( $\tilde{\tilde{A}}$ , $\tilde{\tilde{C}}$ ) of interval-valued fuzzy numbers $\tilde{\tilde{A}}$ , $\tilde{\tilde{B}}$ and $\tilde{\tilde{C}}$.

(11) In Set 15 of Fig. 2, the degrees of similarity S( $\tilde{\tilde{A}}$ , $\tilde{\tilde{B}}$ ) and S( $\tilde{\tilde{A}}$ , $\tilde{\tilde{C}}$ ) of the two sets of interval-valued fuzzy numbers ( $\tilde{\tilde{A}}$ , $\tilde{\tilde{B}}$ ) and ( $\tilde{\tilde{A}}$ , $\tilde{\tilde{C}}$ ) are different. However, Table 1 indicates that the methods of Chen and Chen[5], and Wei-and-Chen[13] yield the same degrees of similarity for the two sets ( $\tilde{\tilde{A}}$ , $\tilde{\tilde{B}}$ ) and ( $\tilde{\tilde{A}}$ , $\tilde{\tilde{C}}$ ).

(12) In Set 16 of Fig. 2, the two interval-valued fuzzy numbers $\tilde{\tilde{A}}$ and $\tilde{\tilde{C}}$ are more similar than the two interval-valued fuzzy numbers $\tilde{\tilde{A}}$ and $\tilde{\tilde{B}}$ . However, Table 1 shows that the methods of Chen and Chen[5], and Wei and Chen[13] and Chen[7] yield an incorrect result that the two interval-valued fuzzy numbers $\tilde{\tilde{A}}$ and $\tilde{\tilde{B}}$ are more similar than the two fuzzy numbers $\tilde{\tilde{A}}$ and $\tilde{\tilde{C}}$.

(13) In Set 17 of Fig. 2, the two interval-valued fuzzy numbers $\tilde{\tilde{A}}$ and $\tilde{\tilde{B}}$ are more similar than the two interval-valued fuzzy numbers $\tilde{\tilde{A}}$ and $\tilde{\tilde{C}}$ . However, Table 1 shows that the methods of Wei and Chen[13] and Chen[7] yield an incorrect result that the two interval-valued fuzzy numbers $\tilde{\tilde{A}}$ and $\tilde{\tilde{C}}$ are more similar than the two fuzzy numbers $\tilde{\tilde{A}}$ and $\tilde{\tilde{B}}$ .

(14) In Set 18 of Fig. 2, the two interval-valued fuzzy numbers $\tilde{\tilde{A}}$ and $\tilde{\tilde{C}}$ are more similar than the two interval-valued fuzzy numbers $\tilde{\tilde{A}}$ and $\tilde{\tilde{B}}$ . However, Table 1 shows that the method of Chen and Kao [8] yield an incorrect result that the two interval-valued fuzzy numbers $\tilde{\tilde{A}}$ and $\tilde{\tilde{B}}$ are more similar than the two fuzzy numbers $\tilde{\tilde{A}}$ and $\tilde{\tilde{C}}$.

(15) In Set 19 of Fig. 2, the two interval-valued fuzzy numbers $\tilde{\tilde{A}}$ and $\tilde{\tilde{B}}$ are more similar than the two interval-valued fuzzy numbers $\tilde{\tilde{A}}$ and $\tilde{\tilde{C}}$ . However, Table 1 shows that the method of Chen and Kao [8] yield an incorrect result that the two fuzzy numbers $\tilde{\tilde{A}}$ and $\tilde{\tilde{C}}$ are more similar than the two fuzzy numbers $\tilde{\tilde{A}}$ and $\tilde{\tilde{B}}$ .

Table 1 and Fig. 2 indicate that the proposed method overcomes the drawbacks of the existing methods.

## V. CONCLUSION

This study presents a new approach for calculating similarity measure between interval-valued fuzzy numbers. Some properties of the proposed similarity measure were demonstrated, and 19 sets of generalized fuzzy numbers were adopted to compare the proposed similarity measure with four existing similarity measures. Table 1 indicates that the proposed similarity measure overcomes the drawbacks of the existing similarity measures. The proposed similarity measure provides a useful way to calculate the degree of similarity between internal-valued fuzzy numbers.

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