Proceedings of the International MultiConference of Engineers and Computer Scientists 2014 Vol II,
IMECS 2014, March 12 - 14, 2014, Hong Kong

Sightseeing Route Planning Responding Various Conditions with Fuzzy Random Satisfactions Dependent on Tourist's Tiredness

Takashi Hasuike, *Member, IAENG*, Hideki Katagiri, *Member, IAENG*, Hiroe Tsubaki, Hiroshi Tsuda

***Abstract*— This paper proposes a route planning problem for sightseeing with fuzzy random variables for traveling times and satisfaction of sightseeing activities under general sightseeing constraints and various conditions. In general, traveling times among sightseeing places and satisfactions of activities depend on weather and climate conditions. Furthermore, the satisfactions also depend on the tourist's tiredness. Therefore, not only fuzzy random variables for traveling times and satisfactions but also the tiredness-dependency is introduced. In addition, the tourist will like to do a route planning without drastically changing from the optimal route to each condition. A route planning problem is proposed to obtain the favorable common route supplying target satisfactions under various conditions. As a basic case of fuzzy numbers, trapezoidal fuzzy numbers and the order relation are introduced. From the order relation, the proposed model is transformed into an extended model of network optimization problems.**

***Index Terms*—Route planning for sightseeing; Fuzzy random and tiredness-dependent satisfactions; Network optimization; Mathematical modeling**

I. Introduction

THE development and popularization of information and communication technologies (ICT) including internet technologies enables tourists to collect a lot of information and plan personal sightseeing routes by themselves on the Web. Therefore, it is important to develop a sightseeing route planning system considering personal satisfactions under various sightseeing conditions. Furthermore, traveling and activity times are also important factors for sightseeing, and hence, it is necessary to do a suitable management of these times for the effective utilization of sightseeing activities. Thus, the tour planning should be prepared in advance, considering the above-mentioned factors, transportation networks, personal context, properties of activities, etc.

Previous researches of tour planning problems are broadly divided into several groups. Some studies devote solving the mathematical programming problems. Other studies devote dynamically planning an optimal itinerary which is related to designing intelligent tour planning systems based on the personalized tour recommender, and hence, there are various tour planning problems such as tourist trip design problem (Souffriau et al. [13]), tour planning problem in a multimodal and time-scheduled urban public transport network (Zografos and Androutsopoulos [18]), and time-dependent tour planning methodology to design a time-limited tour based on the maximum total priority value (Abbaspour and Samadzadegan [1]).

However, existing mathematical models for sightseeing do not include several important factors such as uncertainty of required traveling times and satisfaction values to sightseeing places. For instance, from historical traffic data, tourists may estimate a traveling time between two sightseeing places. However, since the actual traveling times are often different from the predictions in some degree, it is difficult to set traveling times as constant values. Therefore, it is important to consider uncertain traveling times in the given traffic network initially. Furthermore, weather and climate conditions are also important factors to change traveling times and satisfactions to sightseeing places. For instance, a zoo is a weather-sensitive sightseeing place due to outdoors. On the other hand, an aqua museum is a climate or weather-insensitive sightseeing place due to indoors. Therefore, a satisfaction of zoo more drastically changes than that of aqua museum dependent on weather conditions, and satisfactions of tourists to various sightseeing places may be different from each weather condition under uncertainty. As the other important factor, we need to consider the tourist's tiredness during sightseeing. If the tourist does activities to stand and walk all the way with the much tired, the tourist may not enjoy very well even if the landscape is much beautiful. In order to overcome the weakness in existing researches and to construct the more general framework of tour planning, we develop a mathematical model of tour planning with uncertain traveling times and tiredness-dependent satisfactions of activities. In this paper, we propose a tour planning problem with uncertain traveling times and fuzzy random and tiredness-dependent satisfactions from a mathematical point of view.

In addition, tourists often want not to change their sightseeing routes even if weather conditions are changed, i.e., tourists may obtain the only optimal route of fine, cloudy, and rainy days. If these routes are utterly different each weather condition, tourists are confused how they select

Takashi Hasuike is with Graduate School of Information Science and Technology, Osaka University, Japan (corresponding author to provide phone: +81-6-6879-7872; fax: +81-6-6879-7872; e-mail: thasuike@ist.osaka-u.ac.jp).

Hideki Katagiri is with Graduate School of Engineering, Hiroshima University, Japan. (e-mail: katagiri-h@hiroshima-u.ac.jp).

Hiroe Tsubaki is with Department of Data Science, The Institute of Statistical Mathematics, Japan. (e-mail: tsubaki@ism.ac.jp)

Hiroshi Tsuda is with Faculty of Science and Engineering, Doshisha University, Japan. (e-mail: htsuda@mail.doshisha.ac.jp)

an appropriate plan in these routes, and this approach will not become a better decision support system for sightseeing. Therefore, it is important for the tourist to have the same route plan to change adaptively according to current weather, climate and traffic conditions. Recently, we [7] have proposed a route planning system from mathematical point of view. It provides not only the optimal tour route of usual condition but also flexible routes under the other conditions which are the same or similar route of usual condition. On the other hand, it is also important to arrive at the final destination as soon as possible satisfying the target satisfactions, because the tourist can have leeway to stay favorable place longer or to visit other sightseeing places. Therefore, we construct a route planning problems to obtain a favorable common route plan under several conditions.

This paper is organized as follows. In Section 2, we introduce fuzzy random variables for uncertain traveling times and satisfactions. As a specific case of fuzzy numbers, we introduce a trapezoidal fuzzy number and define the order relation between two trapezoidal fuzzy numbers, called Yager's ranking method. In Section 3, we explain some assumptions of our proposed model in this paper, and formulate a proposed route planning problem for sightseeing in mathematical programming. Since this formulated problem includes fuzzy inequalities, we transform the previous problem into an extended model of network optimization problems using the order relation in Section 2. Finally, in Section 4, we conclude this paper.

II. Fuzzy Random Variable

We first introduce a fuzzy set theory before defining a fuzzy random variable. The fuzzy set theory was proposed by Zadeh [17] as a means of representing and manipulating data that was not precise, but rather fuzzy, and it was specifically designed to mathematically represent uncertainty and vagueness. Therefore, it allows decision making with estimated values under incomplete or uncertain information (Carlsson and Fuller [2]). The mathematical definition of fuzzy set is given as follows.

Definition 1

Let $\hat{X}$ be a nonempty set. A fuzzy set Φ in $\hat{X}$ is characterized by its membership function $\mu_\Phi : \hat{X} \to [0,1]$ and μ_Φ is interpreted as the degree of membership of element x in fuzzy set Φ for each $x \in \hat{X}$.

Consider the degree to which the statement "x is in Φ" is true. This definition means that the value 0 is used to represent complete non-membership, the value 1 is used to represent complete membership, and values in between are used to represent intermediate degrees of membership.

Using the definition of fuzzy sets as well as the probability theory, a fuzzy random variable was first defined by Kwakernaak [8], and Puri-Ralescu [11] established the mathematical basis. In this paper, consider the case where the realization of random variable is a fuzzy set. Accordingly, fuzzy random variables are defined as follows:

Definition 2

Let (Ω, B, P) be a probability space, $F(\Re)$ be the set of fuzzy numbers with compact supports, and $\hat{X}$ be a measurable mapping $\Omega \to F(\Re)$. Then, $\hat{X}$ is a fuzzy random variable if and only if given $\omega \in \Omega$, $\hat{X}_h(\omega)$ is a random interval for any $h \in [0,1]$ where $\hat{X}_h(\omega)$ is a h-level set of the fuzzy set $\hat{X}(\omega)$.

This definition corresponds to a special case of fuzzy random variables given by Kwakernaak and Puri-Ralesu. Though it is a simple definition, it would be useful for various applications.

As an assumption to simplify the following discussion, we consider that all fuzzy numbers are represented as trapezoidal fuzzy numbers $\tilde{a} = (a_L, a_U, \alpha, \beta)$ whose membership functions are defined as follows:

$$\mu_{\tilde{a}}(\omega) = \begin{cases} \dfrac{\omega - (a_L - \alpha)}{\alpha} & (a_L - \alpha \le \omega \le a_L) \\ 1 & (a_L \le \omega \le a_U) \\ \dfrac{(a_U + \beta) - \omega}{\beta} & (a_U \le \omega \le a_U + \beta) \\ 0 & \text{otherwise} \end{cases}$$

where a_L, a_U, α, β are constant values set by the decision maker. The trapezoidal fuzzy number is viewed as a special fuzzy number. A convenient approach for solving fuzzy mathematical programming problems is to use the ranking between two fuzzy numbers, and hence, several rankling functions have been proposed to compare two fuzzy numbers until now. In this paper, we introduce Yager's ranking method [16]. This is one of linear ranking functions, and the calculation process is simple. Therefore, many studies of fuzzy mathematical programming problems adopt the Yager's ranking method. Yager's ranking function for trapezoidal fuzzy numbers is defined as follows:

$$\begin{aligned} Y(\tilde{a}) &= \frac{1}{2}\int_0^1 \left(\inf[\tilde{a}]_h + \sup[\tilde{a}]_h\right) dh \\ &= \frac{1}{2}\left(a_L + a_U + \frac{\beta - \alpha}{2}\right) \end{aligned} \tag{1}$$

where $[\tilde{a}]_h = \{\omega \mid \mu_{\tilde{a}}(\omega) \ge h\}$ means the h-cut of fuzzy number. Using the Yager's ranking method, we define orders between two trapezoidal fuzzy numbers $\tilde{a}$ and $\tilde{b}$ as follows:

$$\begin{aligned} &\tilde{a} \ge \tilde{b} \text{ if and only if } Y(\tilde{a}) \ge Y(\tilde{b}) \\ &\tilde{a} = \tilde{b} \text{ if and only if } Y(\tilde{a}) = Y(\tilde{b}) \\ &\tilde{a} \le \tilde{b} \text{ if and only if } Y(\tilde{a}) \le Y(\tilde{b}) \end{aligned} \tag{2}$$

III. Formulation of Our Proposed Route Planning Problem for Sightseeing

Let $G = (V, E)$ be a connected graph to represent a sightseeing area including m sightseeing places, one

ISBN: 978-988-19253-3-6
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

departure place 0, and one arrival place M, i.e., node set of places $V=(0,1,\ldots,m,M)$. c_{ij} denotes a positive deterministic number associated with arc (i,j) corresponding to costs necessary to traverse (i,j) from i to j and activity of sightseeing place j. In this paper, we assume the total of weather and climate conditions is K. As a fuzzy random variable, $\tilde{\bar{a}}_{ij}$ denotes the sum of satisfaction values to landscapes from sightseeing places i to j and the activity sightseeing place j. Fuzzy random variable $\tilde{\bar{a}}_{ij}$ is defined as the following trapezoidal fuzzy number under each condition:

$$\tilde{\bar{a}}_{ij}=\begin{cases}\tilde{a}_{ij}^{1}=\left(\underline{a}_{ij}^{1},\bar{a}_{ij}^{1},\alpha_{ij}^{1},\beta_{ij}^{1}\right)\\ \quad\vdots\\ \tilde{a}_{ij}^{k}=\left(\underline{a}_{ij}^{k},\bar{a}_{ij}^{k},\alpha_{ij}^{k},\beta_{ij}^{k}\right),(k=1,2,\ldots,K)\\ \quad\vdots\\ \tilde{a}_{ij}^{K}=\left(\underline{a}_{ij}^{K},\bar{a}_{ij}^{K},\alpha_{ij}^{K},\beta_{ij}^{K}\right)\end{cases} \tag{3}$$

$$\Pr\left\{\tilde{\bar{a}}_{ij}=\tilde{a}_{ij}^{k}\right\}=\gamma_k,\ \sum_{k=1}^{K}\gamma_k=1$$

where γ_k denotes a positive occurrence probability corresponding to kth condition.

In this paper, we assume that each satisfaction value of sightseeing place is dependent on tourist's tiredness. If the tourist visits the most favorable sightseeing place with the much tired, she or he may not enjoy this place very well. The sightseeing tiredness depends on the current total sightseeing time and the contents of sightseeing place such as stand and walk activity. Thus, it is important to consider the tourist's tiredness factor in the tour route planning problem. However, it is difficult to introduce this tiredness factor in a static network model based on the actual traffic network straightforwardly, because the static network cannot deal with time-dependent parameters directly. In order to overcome this weakness, we introduce the following network based on a tree structure as shown in Fig 1.

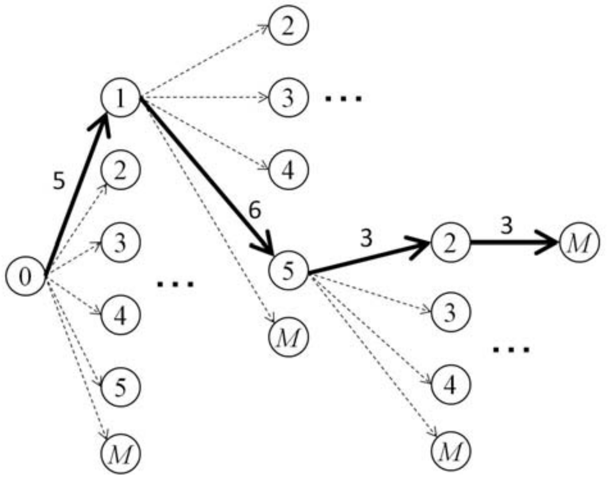

Fig 1. Partial given traffic network with 5 sightseeing places and tourist's tiredness under route 0-1-5-2-M

This figure shows a partial given network with 5 sightseeing places, particularly focused on route 0-1-5-2-M. For instance, the tourist can go to sightseeing places and arrival place M from the departure place 0, and hence, we set directed arcs from departure place 0 to each sightseeing place. Then, in the case that the tourist goes to sightseeing place 1, the next sightseeing places are selected from sightseeing places 2 to 5 except for sightseeing place 1, and hence, directed arcs from sightseeing place 1 to the other places. In addition, value 5 on directed arc from departure place 0 to sightseeing place 1 means the value of tiredness the tourist goes this route. By counting values of f tiredness until the present location on the sightseeing route, tiredness-dependent satisfactions of sightseeing places are straightforwardly obtained. In this paper, fuzzy random and tiredness-dependent satisfactions are defined as $\tilde{a}_{ij}^{k}(p)=\left(\underline{a}_{ij}^{k}(p),\bar{a}_{ij}^{k}(p),\alpha_{ij}^{k}(p),\beta_{ij}^{k}(p)\right)$ where p is a feasible path in the given tree structure-based network such as Fig 1.

As another fuzzy random variable, $\tilde{\bar{t}}_{ij}$ denotes the sum of necessary times to travel from sightseeing places i to j and to do the activity for sightseeing place j, i.e., fuzzy random variable $\tilde{\bar{t}}_{ij}$ is formulated as follows:

$$\tilde{\bar{t}}_{ij}=\begin{cases}\tilde{t}_{ij}^{1}=\left(\underline{t}_{ij}^{1},\bar{t}_{ij}^{1},\xi_{ij}^{1},\eta_{ij}^{1}\right)\\ \quad\vdots\\ \tilde{t}_{ij}^{k}=\left(\underline{t}_{ij}^{k},\bar{t}_{ij}^{k},\xi_{ij}^{k},\eta_{ij}^{k}\right),(k=1,2,\ldots,K)\\ \quad\vdots\\ \tilde{t}_{ij}^{K}=\left(\underline{t}_{ij}^{K},\bar{t}_{ij}^{K},\xi_{ij}^{K},\eta_{ij}^{K}\right)\end{cases} \tag{4}$$

$$\Pr\left\{\tilde{\bar{t}}_{ij}=\tilde{t}_{ij}^{k}\right\}=\gamma_k,\ \sum_{k=1}^{K}\gamma_k=1$$

The other parameters denote as follows:

- b : minimum visiting points initially decided by the tourist
- C : total budget available for the tour
- $\tilde{a}_G^k$: target satisfaction value under the kth condition given as interval $[\underline{a}_G^k,\bar{a}_G^k]$
- $\tilde{t}_G^k$: target traveling time under the kth condition given as interval $[\underline{t}_G^k,\bar{t}_G^k]$
- x_{ij} : 0-1 decision variables to favorable common route satisfying the following condition;

$$x_{ij}=\begin{cases}1,\ \text{if the tourist travels from places } i \text{ to } j\\ 0,\ \text{otherwise}\end{cases}$$

The main object of our propose model is to minimize the maximum total traveling time satisfying the following situations:

- The total satisfaction value under kth condition is more than the target value $\tilde{a}_G^k=\left(\underline{a}_G^k,\bar{a}_G^k,0,0\right)$ in terms of ordering relation based on the Yager's ranking method in Section 2.

Based on these situations and considering occurrence probabilities γ_k corresponding to kth condition, we formulate a tour planning problem for sightseeing minimizing the maximum weighted under several conditions as follows:

$$\text{Minimize} \max_k \left\{ w(\gamma_k) \sum_{(i,j)\in E} \tilde{t}_{ij}^{\,k} x_{ij} \right\}$$

$$\text{subject to } \boldsymbol{x} \in X$$

$$X = \left\{ \boldsymbol{x} \left| \begin{array}{l} \sum_{(i,j)\in E} \tilde{a}_{ij}^{k}(p) x_{ij} \geq \tilde{a}_G^k, \\ \sum_{(i,j)\in E} c_{ij} x_{ij} \leq C, \\ \sum_{(i,j)\in E} x_{ij} \geq b, \\ \sum_{\{j|(i,j)\in E\}} x_{ij} - \sum_{\{j|(j,i)\in E\}} x_{ji} \\ = \begin{cases} 1 \text{ if } i = 0 \\ 0 \text{ if } i \neq 0, M \\ \qquad (i = 1,2,\ldots,M) \\ -1 \text{ if } i = M \end{cases} \\ x_{ij} \in \{0,1\}, \forall (i,j) \in E \\ (k = 1,2,\ldots,K) \end{array} \right. \right\} \tag{5}$$

where $w(\gamma_k)$ is the fixed weight value to probability γ_k. In feasible set X of our proposed model, the first constraint represents that the total satisfaction values of possible conditions from departure place to arrival place is more than the target values. The second constraint is the budget constraint, and the fourth constraint means that tourists visit more than the target visiting points b. The other constraints are derived from the general network flow for given network G. Tourists solve the proposed model, and obtain the favorable common route under several weather and climate conditions. The obtained route can be flexibly changed responding each condition, because the objective function is derived from the minimization of the total traveling time and the tourist can have leeway to stay favorable place longer or to visit other sightseeing places. Therefore, it will be easy for tourists to do the more favorable sightseeing planning than the other existing tour planning problems.

In proposed problem (5), the objective function is equivalently transformed into the following form by introducing decision variable θ:

$$\text{Minimize} \max_k \left\{ w(\gamma_k) \sum_{(i,j)\in E} \tilde{t}_{ij}^{\,k} x_{ij} \right\}$$

$$\Leftrightarrow \begin{cases} \text{Minimize } \theta \\ \text{subject to } w(\gamma_k) \sum_{(i,j)\in E} \tilde{t}_{ij}^{\,k} x_{ij} \leq \theta \end{cases} \tag{6}$$

Furthermore, constraints in X and $w(\gamma_k) \sum_{(i,j)\in E} \tilde{t}_{ij}^{\,k} x_{ij} \leq \theta$ are transformed into appropriate inequalities in terms of ordering relation (2) to trapezoidal fuzzy numbers, and problem (5) is also transformed into the following problem:

Minimize θ

subject to $\boldsymbol{x} \in X'$

$$X' = \left\{ \boldsymbol{x} \left| \begin{array}{l} \sum_{(i,j)\in E} \Big(\underline{a}_{ij}^k(p) + \overline{a}_{ij}^k(p) \\ \qquad + \dfrac{\beta_{ij}(p) - \alpha_{ij}(p)}{2} \Big) x_{ij} \geq \underline{a}_G^k + \overline{a}_G^k, \\ \sum_{(i,j)\in E} \Big(\underline{t}_{ij}^k(p) + \overline{t}_{ij}^k(p) \\ \qquad + \dfrac{\eta_{ij}(p) - \xi_{ij}(p)}{2} \Big) x_{ij} \leq \dfrac{1}{w(\gamma_k)} \theta, \\ \sum_{(i,j)\in E} c_{ij} x_{ij} \leq C, \\ \sum_{(i,j)\in E} x_{ij} \geq b, \\ \sum_{\{j|(i,j)\in E\}} x_{ij} - \sum_{\{j|(j,i)\in E\}} x_{ji} \\ = \begin{cases} 1 \text{ if } i = 0 \\ 0 \text{ if } i \neq 0, M \\ \qquad (i = 1,2,\ldots,M) \\ -1 \text{ if } i = M \end{cases} \\ x_{ij} \in \{0,1\}, \forall (i,j) \in E \\ (k = 1,2,\ldots,K) \end{array} \right. \right\} \tag{6}$$

Each feasible set X' consists of a constrained network flow, and this problem is one extended model of network optimization problems. Satisfaction values $\underline{a}_{ij}^k(p), \overline{a}_{ij}^k(p), \alpha_{ij}^k(p), \beta_{ij}^k(p)$ are also obtained as constant values using given tree structure-based network such as Fig 1 initially. Therefore, this problem is solved by strict network optimization method strictly or heuristic solution algorithms efficiently; branch-and-cut algorithm (Fischetti et al. [4]), 2-opt or n-opt algorithm (Chao et al. [3]), LP relaxation (Kennington and Nicholso [10]), Genetic algorithm (Abbaspour and Samadzadegan [1]), Neural network (Wang et al. [15]), Tabu-search (Gendreau et al. [6], Tang and Miller-Hooks [14]), Ant Colony Optimization (Ke et al. [9]), etc.

IV. CONCLUSION

In this paper, we have proposed a sightseeing route planning model with fuzzy random traveling times among sightseeing places and tiredness-dependent satisfactions. From general concepts that tourists like to do their favorable common route planning without changing the tour route even if the weather and climate condition changes and to have leeway, we have formulated a mathematical model whose objective function is minimizing the maximum weighted total traveling time under possible conditions. Furthermore, we have transformed the initial proposed problem into a constrained network optimization problem using the Yager's ranking method for trapezoidal fuzzy numbers in fuzzy random parameters.

ISBN: 978-988-19253-3-6
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

As a drawback of our proposed model in terms of computation algorithm, the proposed problem is strongly NP-hard (Fischetti et al. [5]) due to mixed integer linear programming problem. Furthermore, with respect to the tiredness degree, we need to construct a tree structure with all sightseeing places and conditions. Therefore, the proposed model with many sightseeing places and conditions is more complex, and obviously, it is computationally difficult to solve large-scale tour planning problems. Therefore, we are now developing efficient heuristic solution algorithms based on soft computing approaches.

REFERENCES

[1] R.A. Abbaspour and F. Samadzadegan, “Time-dependent personal tour planning and scheduling in metropolises”, Expert Systems with Applications, 38, pp. 12439-12452, 2011.

[2] C. Carlsson and R. Fuller, Fuzzy Reasoning in Decision Making and Optimization, Physica Verlag, 2002.

[3] I.M. Chao, B.L. Golden, and E.A. Wasil, “A fast and effective heuristic for the orienteering problem”, European Journal of Operational Research, 88(3), pp. 475-489, 1996.

[4] M. Fischetti, J.S. Gonzalez, and P. Toth, “Solving the orienteering problem through branch-and-cut”, INFORMS Journal on Computing, 10(2), pp. 133-148, 1998.

[5] M. Fischetti, J.J. Salazar-González, and P. Toth, “The generalized traveling salesman and orienteering problems”, In G. Gutin & A. P. Punnen (Eds.), The traveling salesman problem and its variations, Dordrecht: Kluwer Academic Publisher, pp. 609-662, 2002.

[6] M. Gendreau, G. Laporte, and F. Semet, “A tabu search heuristic for the undirected selective traveling salesman problem”, European Journal of Operational Researc_, 106(2–3), pp. 539-545, 1998.

[7] T. Hasuike, H. Katagiri, H. Tsubaki, and H. Tsuda, “Flexible route planning for sightseeing with fuzzy random and fatigue-dependent satisfactions”, submitted to an international journal.

[8] H. Kwakernaak, “Fuzzy random variable-I”, Information Sciences, 15, pp. 1-29, 1978.

[9] L. Ke, C. Archetti, and Z. Feng, “Ants can solve the team orienteering problem”, Computers & Industrial Engineering, 54(3), pp. 648-665, 2008.

[10] J.L. Kennington and C.D. Nicholson, “The uncapacitated time-space fixed-charge network flow problem; an empirical investigation of procedures for arc capacity assignment”, INFORMS Journal of Computing, 22, pp. 326-337, 2009.

[11] M.L. Puri and D.A. Ralescu, “Fuzzy random variables”, Journal of Mathematical Analysis and Applications, 114, pp. 409-422, (1986)

[12] M.R. Silver and O.L. de Weck, “Time-expanded decision networks: a framework for designing evolvable complex systems”, Systems Engineering, 10(2), pp. 167-186, 2007.

[13] W. Souffriau, P. Vansteenwegen, J. Vertommen, G.V. Berghe, and D.V. Oudheusden, “A personalized tourist trip design algorithm for mobile tourist guides”, Applied Artificial Intelligence, 22(10), pp. 964-985, 2008.

[14] H. Tang and E. Miller-Hooks, “A tabu search heuristic for the team orienteering problem”, Computers & Operations Research, 32, pp. 1379-1407, 2005

[15] Q. Wang, X. Sun, B.L. Golden, and J. Jia, “Using artificial neural networks to solve the orienteering problem”, Annals of Operations Research, 61, pp. 111-120, 1995.

[16] R.R. Yager, “A procedure for ordering fuzzy subsets of the unit interval”, Information Sciences, 24, pp. 143-161, 1981.

[17] L.A. Zadeh, “Fuzzy Sets”, Information and Control, 8, pp. 338-353, 1965.

[18] K.G. Zografos and K.N. Androutsopoulos, “Algorithms for itinerary planning in multimodal transportation networks”, IEEE Transactions on Intelligent Transportation System, 9(1), pp. 175-184, 2008.