Multiple LFM Signals Detection Method Based on Pseudo-Wigner-Ville Distribution and Binary Integration of Hough Transform

Kameliya I. Kaneva, Zhigang Li

Abstract—The Wigner-Ville Distribution (WVD) and Pseudo-Wigner-Ville Distribution (PWVD) are very popular and effective methods for detecting Linear Frequency Modulated (LFM) signals in noisy environments. Hough transform is also widely used method for detection and parameter estimation of chirp signals. However, when there are signals with very different strength from each other, it is difficult to simultaneously detect both of them. The paper proposes a method for solving this problem. The algorithm is based on PWVD and Binary Transform in Hough space. Simulations show that the strong and the weak signal can be detected simultaneously and their parameters can be estimated.

Index Terms—Binary integration, Hough transform, LFM signal, PWVD.

I. INTRODUCTION

Linear frequency modulated (LFM) signals, also known as chirp signals, are often encountered in many applications such as radar, sonar and communications [1]. They are appreciated for their excellent characteristics of long time interval and wide frequency band. The Wigner-Ville Distribution (WVD) can perfectly show the features of a LFM signal in the time-frequency plane [2], however, if there are two or more chirp signals, the cross terms produced by WVD will decrease the performance of the algorithm. One method for reducing the negative effect of the interference terms is by using Pseudo-Wigner-Ville Distribution (PWVD). It uses a time-domain window and thus smooths the cross terms in the frequency domain. The Hough transform is a popular image processing technique which is capable of finding straight lines. Since the chirp signal is transformed to a straight line in the Time-Frequency (TF) domain, Hough transform is an appropriate and widely used method for detection and parameter estimation of LFM signals. However, when dealing with signals with very different strengths from each other, it is very difficult to detect all LFM signals with the WVD-Hough or PWVD-Hough transforms at the same time because the platform of strong signals will cover weak signals [3]. One way of approaching the problem is by using binary detection in Hough transform parameter space [3], [4]. This paper introduces a signal detection method based on PWVD and binary integration in Hough domain.

II. PSEUDO-WIGNER-VILLE TRANSFORM

Wigner-Ville Distribution (WVD) is perfectly suitable and gives excellent results when we deal only with one chirp signal. WVD is given as [5]:

$$W(t,f) = \int_{-\infty}^{\infty} x\left(t + \frac{\tau}{2}\right) \cdot x^*\left(t - \frac{\tau}{2}\right) e^{-j2\pi f n} d\tau, \quad (1)$$

where x(t) is the analyzed signal, $x^*(t)$ is its complex conjugate. However, if we have to process two or more LFM signals, the WVD unavoidably gives cross terms due to its nonlinearity. Pseudo-Wigner-Ville Distribution (PWVD) is a method which uses a time domain window for smoothing in the frequency domain. PWVD is defined as [5]:

$$PW(k, f) = \sum_{n=-N+1}^{N-1} x(k+n) \cdot x^*(k-n) \cdot w(n) \cdot w(-n) \cdot e^{-j2\pi fn}, \quad (2)$$

where w(n) is the real window of 2N length. The result from the PWVD is that the cross terms are decreased. Another consequence is that the frequency resolution decreases. The longer the filter, the higher the frequency resolution. In this case, the cross terms rejection is also very good, however, the time resolution is low. Moreover, if the filter is longer than the signal, the cross terms rejection is not so effective. Choosing the appropriate length of the filter, as well as the two thresholds, which will be explained later, is critical for the success of the method.

III. HOUGH TRANSFORM

The Hough transform is an image processing technique which can find straight lines. It is also robust to extraneous noise because the integration carried out by the Hough transform increases the SNR [6]. As it can be seen from Fig. 1 [1], each point in the TF plane is mapped to a sinusoid in the Hough domain by [5, 6]:

$$\rho = t\cos\theta + f\sin\theta, \tag{3}$$

where ρ is the length of the shortest line segment between the line (target trajectory) and the origin, and θ is the angle between this line segment and the range axis of the data space [4]. If the points lie on one line in the TF domain, their corresponding sinusoids in the parameter space will intersect in one point. By the line integration, a peak appears in the Hough space and its coordinates are directly related to the parameters of the lines [6].

After performing PWVD, the chirp signal is transformed into a straight line in an image. Moreover, PWVD reduces

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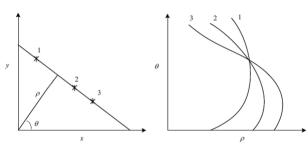


Figure 1. The illustration of coordinate plane and Hough plane.

the influence of the interference terms. After that, Hough transform is employed which also decreases the negative effect of the noise. As stated in [7], this means that the detection and parameter estimation of a chirp signal embedded in noise can be recast as the search for a peak in the domain (ρ , θ).

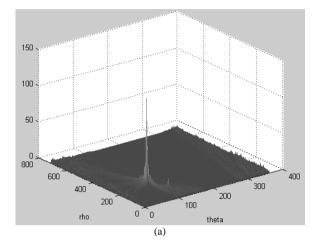
IV. BINARY INTEGRATION OF HOUGH TRANSFORM

One way of simultaneously detecting a strong signal (large target) and a weak signal (small target) is to use binary integration in the Hough domain. For this method an "analog" power threshold is set in the TF plane to establish a set of primary detections. Then, only these points higher than the first threshold are mapped to the parameter space and all of the resulting Hough curves are given unit weight. This step makes small targets and large targets look the same size in Hough space [4]. After that, if the peaks in the parameter domain are higher than a second threshold value, then the peaks are assumed to be targets.

The Hough transform subdivides the $\rho\theta$ parameter space into so-called accumulator cells [8]. Instead of summing the signal powers, associated with the sinusoidal curve segments in a given cell, the binary integration simply counts the number of segments in the cell [4]. That is why it shows distinguishable peaks for strong and weak signals. However, in noisy environments there will be interference terms producing as many segments as the weak signal. This means that both the noise and the small target will look the same in the Hough plane which will lead to false alarm detections.

V. SIMULATION

Two LFM signals are generated with the following parameters: amplitudes are 1 and 0.35, center frequency f0 equals 40 Hz and 100 Hz respectively and the chirp rate k of these signals is 100 Hz/s and 175 Hz/s respectively. The number of samples is 1000. In the simulation the length of the Gaussian filter is equal to the length of the signal. The two chirps are processed with PWVD-Hough Transform and PWVD and binary integration of Hough space. White Gaussian noise is added into the signals. The simulations are conducted for SNR equals to -3 dB, 0 dB and 3 dB. Fig. 2 (a) shows that when the SNR = -3 dB, the weak signal almost cannot be seen, only the strong signal can be detected. After the signal is processed by the new method (Fig. 2 b), the weak signal is still not noticeable enough. Since the SNR is too low, some of the interference terms produce almost the same number of curves in the Hough space, which makes them and the weak signal look the same size in the parameter domain. The results after performing PWVD and Hough transform when SNR is 0 dB and 3 dB (Fig. 3 (a) and Fig. 4 (a)) are that the weak signal does not increase significantly although the SNR is better. However, if the method proposed is used, the small target can easily be detected simultaneously with the big target.



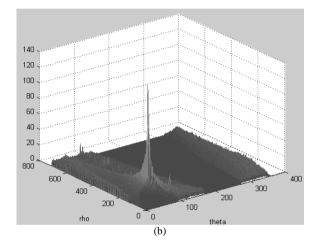
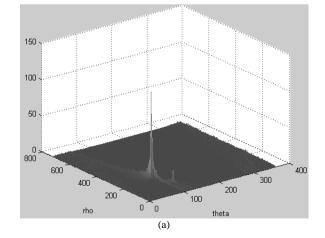


Figure 2. The (a) PWVD and Hough transform and the PWVD and binary integration of Hough space (b) (SNR = -3 dB).



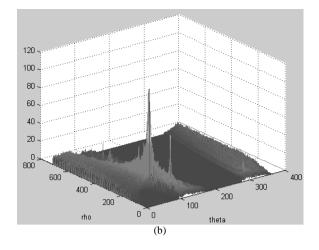
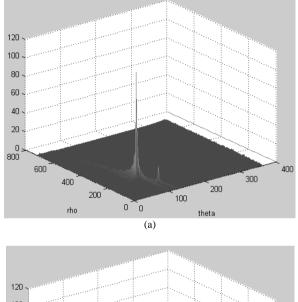


Figure 3. The (a) PWVD and Hough transform and the PWVD and binary integration of Hough space (b) (SNR = 0 dB).



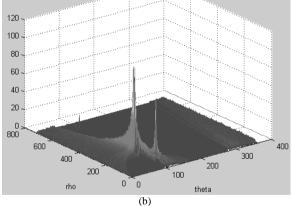


Fig. 4. The (a) PWVD and the Hough transform and the PWVD and binary integration of Hough space (b) (SNR = 3 dB)

VI. CONCLUSION

When the SNR is low, it is crucial that the length of the filter used for PWVD sufficiently suppresses the cross terms but simultaneously, does not filter out the weak signal. Simulations done prove that by using PWVD and binary integration in Hough space, the small target is much more noticeable in the parameter domain. This means that the weak and the strong signals can be detected simultaneously.

However, the success of the method proposed in the

paper relies on appropriately choosing the parameters of the filter for PWVD and the two thresholds.

The additional filtering is what gives the benefits of PWVD over WVD so in future works, the comparison between WVD and binary integration in parameter domain and the method, proposed here, should be conducted.

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