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Hydropower Producer Day-ahead Market Strategic Offering Using Stochastic Bi-level Optimization

Yelena Vardanyan, Anthony Papavasiliou and Mohammad R. Hesamzadeh

***Abstract*—This paper proposes a bi-level stochastic optimization problem (a Stackelberg game) to generate optimal bids for a profit maximizing hydropower producer and presents a mathematical approach to solve it. The first level represents the strategically acting hydropower producer also called a Stackelberg leader, while the second level represents the transmission system operator (TSO) also called a Stackelberg follower. To solve the bi-level stochastic optimization problem, the second level is replaced by its KKT (Karush-Kuhn-Tucker) optimality conditions, which results in a stochastic MPEC (mathematical program with equilibrium constraints). Finally, the stochastic MPEC is reformulated as a stochastic MILP (mixed integer linear program) using linearization and SOS_1 variables (special ordered sets of type 1). Results are reported studying a small case, which point out the impact on the market outcomes when a hydropower producer behaves strategically.**

***Index Terms*—Optimal hydropower bidding, bi-level optimization, strategic behaviour, stochastic MPEC.**

I. Nomenclature

A. Sets

i	index for possible bid prices i = 1, ...,I;
j	index for hydro power plants j = 1, ...,J;
k	index for generation type k = 1, ...,K;
n	index for discharging segments n=1, ...,N;
t	index for planning periods t = 1, ...,T;
s	index for market price scenarios s = 1, ...,S_k;
R_j	set for power plants downstream of hydropower plant j;
$\perp$	Orthogonality symbol;

B. Parameters

ω_s	probabilities associated with the price scenarios;
$I_{j,t}$	inflow level to each power plant and time (HE);
$\bar{H}_j$	maximum power production at plant j (MW);
$\bar{G}_k$	maximum power production for generation type k (MW);
$D_{s,t}$	realized hourly demand scenarios (€/MWh);
$\bar{m}_j$	maximum reservoir content (HE);
m_j^o	initial reservoir content (HE);
$\eta_{j,n}$	marginal production equivalent at plant j segment n (MWh/HE);
γ_j	expected future production equivalent for plant j (MWh/HE);
$\bar{Q}_{j,n}$	maximum discharge level in plant j at segment n (HE);
p_f	expected future electricity price (€/MWh);
ρ_i	possible fixed bid prices for day-ahead market (€/MWh);
τ_j	delay time for the water between power plants (h);
c_k, α_k	intercept and slope of linear cost function ;
M_1, M_2	positive large numbers;

C. Variables

$p_{s,t}$	realized day-ahead market price scenarios (€/MWh);
$\mu_{s,k,t}$	dual variables
$H_{s,j,t}$	generation level at each hydro power plant, hour and scenario (MWh);
$G_{s,k,t}$	generation level at each generation type, hour and scenario (MWh);
$m_{s,j,t}$	content of reservoir j at the end of hour t, for each scenarios (HE);
$m_{s,j,T}$	reservoir content at the end of the planning period for each scenario (HE);
$Q_{s,j,t,n}$	discharged volume for hourly bids, for each power plant, segment, hour and scenarios (HE);
$S_{s,j,t}$	spillage from reservoir j during hour t, for each scenario (HE);
$x_{i,t}$	hourly bid volumes to day-ahead market corresponding to the possible bid prices (MWh);
$q_{s,t}$	dispatch level for each hour and scenario according to day-ahead market bids (MWh);
$d_{s,t,i}$	binary variable which is equal to 1 when realized price belongs to ith interval;
z	objective function value (€).

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II. Introduction

SHORT-TERM hydropower planning under uncertainty is a challenging task. Two main sources causing complexity in short term hydropower planning are on the one hand the deregulation of the electricity market and on the other hand, the continuous increase of wind power production share in the electricity market. In the competitive electricity market, electricity is now a commodity which can be sold and purchased at the market with the price defined by demand and supply for each hour [1]. However, due to intermittent wind power production in the power system, the hourly defined electric prices are getting increasingly volatile and difficult to predict. This creates a challenge for a hydropower producer , whose profits from short-term production are driven by prices Therefore, developing an optimal bidding strategy is very relevant and significantly important to hydropower producers.

In general, the producer's interactions with the rest of the market players while preparing optimal offerings can be categorized as follows: 1) the producer is a price taker

[2], [3] , 2) the producer acts strategically [4], [5], [6] and 3) the producer acts strategically considering the strategic behavior of other market participants [7], [8]. A hydropower producer as a price taker is addressed in the following papers [9], [10], [11], [12], [13] and [14]. In all those papers the electricity market prices are modelled as exogenous variables and enter to the model as input data. The literature lacks models addressing the application of the other two bidding approaches for a hydropower producer.

In this paper, we present a stochastic optimal strategic day-ahead bidding model for a hydropower producer aiming to show that the hydropower producer can maximize its profit by acting strategically. The strategic bids for a hydropower producer is modelled as a Stackelberg game using bi-level optimization, where the hydropower producer acts as a 'leader' and other participants behave as 'followers'. Day-ahead market prices are calculated endogenously as a dual variable of the load balance constraint presented in the inner optimization problem. To solve the bi-level optimization problem the inner optimization problem is relaxed using KKT conditions. This relaxation results in a stochastic MPEC, where equilibrium constraints and the profit term in the objective function result in non-linearity. SOS_1 based approach is used to relax equilibrium constrains [15]. Finally, the linearization of the non-linear term in the objective function results in a stochastic MILP.

The paper is structured as follows. Section II provides the background of the topic and reviews the literature. Section III states the mathematical formulation. The results from a case are discussed in section IV. Section V concludes the paper.

III. Mathematical formulation

The mathematical formulation of the stochastic bi-level optimization problem is presented below:

$$Maximize \sum_{s=1}^{S}\omega_s[\sum_{t=1}^{T}(p_{s,t}q_{s,t} + p_f\sum_{j=1}^{J}\sum_{r\in R_j}\gamma_r m_{s,j,T}] \tag{1}$$

$subject\ to:$

$$\rho_i - M_1(1-d_{s,t,i}) \le p_{s,t} \le \rho_{i+1} + M_1(1-d_{s,t,i}) \tag{2}$$

$$\sum_{l=0}^{i} x_{i-l,t} - M_1(1-d_{s,t,i}) \le q_{s,t} \le \sum_{l=0}^{i} x_{i-l,t} + M_1(1-d_{s,t,i}) \tag{3}$$

$$\sum_{i=1}^{I} d_{s,t,i} = 1 \tag{4}$$

$$m_{s,j,t} = m_{s,j,t-1} - \sum_{n=1}^{N} Q_{s,j,t,n} - S_{s,j,t} + \sum_{n=1}^{N} Q_{s,j-1,t-\tau_j,n} + S_{s,j-1,t-\tau_j} + I_{j,t} \tag{5}$$

$$H_{s,j,t} \le \sum_{n=1}^{N}\eta_{j,n}Q_{s,j,t,n} \tag{6}$$

$$q_{s,t} = \sum_{j} H_{s,j,t} \tag{7}$$

$$Q_{s,j,t,n} \le \bar{Q}_{j,n} \tag{8}$$

$$m_{s,j,t} \le \bar{m}_j \tag{9}$$

$$Minimize \sum_{s=1}^{S}\omega_s[\sum_{t=1}^{T}\sum_{k=1}^{K}(c_k G_{s,k,t} + \frac{\alpha_k}{2}G_{s,k,t}^2)] \tag{10}$$

$subject\ to:$

$$\sum_{k=1}^{K} G_{s,k,t} + q_{s,t} = D_{s,t};\quad [p_{s,t}] \tag{11}$$

$$G_{s,k,t} \le \bar{G}_k;\quad [\mu_{s,k,t}] \tag{12}$$

The first level is a maximization problem for a profit maximizing hydropower producer. The second level is a TSO economic dispatch problem based on cost minimization. (1) is the profit of the hydropower producer, who acts strategically. The first term is the expected profit from day-ahead market trading and the second term is the expected profit from the stored water. In order to model the bidding process, the possible bidding prices are fixed: equidistance price points are selected and the corresponding bid volumes are considered as variables. Let i be the index for the possible bid prices and let ρ_i represent these prices. Then, a fundamental rule is applied to couple bid volumes $x_{i,t}$ and dispatched volumes q_t: for each hour if $\rho_i \le p_t \le \rho_{i+1}$ then $q_t = \sum_{l=0}^{i} x_{i-l,t}$, where l =1,2...i. Day-ahead market bids for each hour and dispatched quantities for each hour and scenario are coupled in constraints (2)-(4).

The constraint (5) sets balance in the reservoirs: the new content of the reservoir is equal to the old content of the reservoir plus water inflow minus water outflow. The generation and discharge relation in each power plant is stated in constraint (6). The constraint (7) sets the bound on the dispatched quantity in day-ahead market. Maximum discharge capacity and maximum reservoir content are imposed by the constraints (8) and (9).

The constraint (10) is the objective function of the second level problem, which aims to minimize the total production cost. Load balance is set by (11), according to which the sum of the total production coming from different generation types plus the hydropower production have to satisfy the demand. Available production from different generation type is bounded by (12).

The bi-level stochastic optimization problem (1)-(12) can be solved by replacing the second level optimization problem by its KKT conditions [16]. The KKT conditions for the second level problem can be expressed as follows:

$$\sum_{k=1}^{K} G_{s,k,t} + q_{s,t} = D_{s,t} \tag{13}$$

$$0 \le \mu_{s,k,t} \perp (\bar{G}_k - G_{s,k,t}) \ge 0 \tag{14}$$

$$0 \le G_{s,k,t} \perp ((c_k + \alpha_k G_{s,k,t}) - p_{s,t} + \mu_{s,k,t}) \ge 0 \tag{15}$$

The math program (10)-(12) is replaced by its KKT conditions (13)-(15). The resulting stochastic non-linear optimization problem is presented below;

$$Maximize \sum_{s=1}^{S}\omega_s[\sum_{t=1}^{T}(p_{s,t}q_{s,t} + p_f\sum_{j=1}^{J}\sum_{r\in R_j}\gamma_r m_{s,j,T}] \tag{16}$$

$subject\ to:$

$$\rho_i - M_1(1-d_{s,t,i}) \le p_{s,t} \le \rho_{i+1} + M_1(1-d_{s,t,i}) \tag{17}$$

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$$\sum_{l=0}^{i} x_{i-l,t} - M_1(1-d_{s,t,i}) \le q_{s,t} \le \sum_{l=0}^{i} x_{i-l,t} + M_1(1-d_{s,t,i}) \quad (18)$$

$$\sum_{i=1}^{I} d_{s,t,i} = 1 \quad (19)$$

$$m_{s,j,t} = m_{s,j,t-1} - \sum_{n=1}^{N} Q_{s,j,t,n} - S_{s,j,t} + \sum_{n=1}^{N} Q_{s,j-1,t-\tau_j,n} + S_{s,j-1,t-\tau_j} + I_{j,t} \quad (20)$$

$$H_{s,j,t} \le \sum_{n=1}^{N} \eta_{j,n} Q_{s,j,t,n} \quad (21)$$

$$q_{s,t} = \sum_{j} H_{s,j,t} \quad (22)$$

$$Q_{s,j,t,n} \le \bar{Q}_{j,n} \quad (23)$$

$$m_{s,j,t} \le \bar{m}_j \quad (24)$$

$$\sum_{k=1}^{K} G_{s,k,t} + q_{s,t} = D_{s,t} \quad (25)$$

$$0 \le \mu_{s,k,t} \perp (\bar{G}_k - G_{s,k,t}) \ge 0 \quad (26)$$

$$0 \le G_{s,k,t} \perp ((c_k + \alpha_k G_{s,k,t}) - p_{s,t} + \mu_{s,k,t}) \ge 0 \quad (27)$$

The constraints causing non-linearity are (26)-(27). Non-linearity complicates the search for a globally optimal solution significantly. In order to facilitate the search for a globally optimal solution, we have to linearize constraints (26)-(27). First we use Schur's decomposition and then introduce special ordered sets of type 1 (SOS_1) variables [15].

Let's assume we have a non-linear constraint $y^T g(x,y) = 0;\ \ y \ge 0,\ \ g(x,y) \ge 0$. To apply Schur's decomposition we introduce variables u, v depending on x, y in the following way: $u = \frac{y+g(x,y)}{2}$, $v = \frac{y-g(x,y)}{2}$ and $u^T u - v^T v = 0$. The first two equations do not contain any non-linear terms. The last one has to be approximated with a piecewise-linear function. Considering the assumptions that $y \ge 0,\ \ g(x,y) \ge 0$, it is easy to notice that only positive u is feasible for $\sqrt{u^2}$. Thus the last equation can be written as $u - \mid v \mid = 0$. To simplify things the transformation of the absolute value is needed. For that purpose $\mid v \mid$ is expressed as the sum of the positive and negative parts, where at most one variable out of the variable pair is non-zero. Hence, the linearization of the non-linear equation $y^T g(x,y) = 0$ is supported by the following three linear equations. $u = \frac{y+g(x,y)}{2}$, $(v^+ - v^-) = \frac{y-g(x,y)}{2}$ and $u - (v^+ + v^-) = 0$, where $v^+ \ \ and \ \ v^-$ are SOS1 type variables.

Applying these transformations on non-linear equations (26)-(27) we obtain the following optimization problem.

$$Maximize \ \sum_{s=1}^{S} \omega_s [\sum_{t=1}^{T} (p_{s,t} q_{s,t} + p_f \sum_{j=1}^{J} \sum_{r \in R_j} \gamma_r m_{s,j,T}] \quad (28)$$

$subject\ to:$

$$\rho_i - M_1(1-d_{s,t,i}) \le p_{s,t} \le \rho_{i+1} + M_1(1-d_{s,t,i}) \quad (29)$$

$$\sum_{l=0}^{i} x_{i-l,t} - M_1(1-d_{s,t,i}) \le q_{s,t} \le \sum_{l=0}^{i} x_{i-l,t} + M_1(1-d_{s,t,i}) \quad (30)$$

$$\sum_{i=1}^{I} d_{s,t,i} = 1 \quad (31)$$

$$m_{s,j,t} = m_{s,j,t-1} - \sum_{n=1}^{N} Q_{s,j,t,n} - S_{s,j,t} + \sum_{n=1}^{N} Q_{s,j-1,t-\tau_j,n} + S_{s,j-1,t-\tau_j} + I_{j,t} \quad (32)$$

$$H_{s,j,t} \le \sum_{n=1}^{N} \eta_{j,n} Q_{s,j,t,n} \quad (33)$$

$$q_{s,t} = \sum_{j} H_{s,j,t} \quad (34)$$

$$Q_{s,j,t,n} \le \bar{Q}_{j,n} \quad (35)$$

$$m_{s,j,t} \le \bar{m}_j \quad (36)$$

$$\sum_{k=1}^{K} G_{s,k,t} + q_{s,t} = D_{s,t} \quad (37)$$

$$\bar{G}_k - G_{s,k,t} \ge 0, \ \ \mu_{s,k,t} \ge 0 \quad (38)$$

$$(c_k + \alpha_k G_{s,k,t}) - p_{s,t} + \mu_{s,k,t} \ge 0, \ \ G_{s,k,t} \ge 0 \quad (39)$$

$$u1_{s,k,t} = \frac{\mu_{s,k,t} + (\bar{G}_k - G_{s,k,t})}{2} \quad (40)$$

$$v1^+_{s,k,t} - v1^-_{s,k,t} = \frac{\mu_{s,k,t} - (\bar{G}_k - G_{s,k,t})}{2} \quad (41)$$

$$u1_{s,k,t} - (v1^+_{s,k,t} + v1^-_{s,k,t}) = 0 \quad (42)$$

$$u2_{s,k,t} = \frac{G_{s,k,t} + ((c_k + \alpha_k G_{s,k,t}) - p_{s,t} + \mu_{s,k,t})}{2} \quad (43)$$

$$v2^+_{s,k,t} - v2^-_{s,k,t} = \frac{G_{s,k,t} - ((c_k + \alpha_k G_{s,k,t}) - p_{s,t} + \mu_{s,k,t})}{2} \quad (44)$$

$$u2_{s,k,t} - (v2^+_{s,k,t} + v2^-_{s,k,t}) = 0 \quad (45)$$

$$u \ge 0; \ \ v^+, \ \ v^- \ \ are \ \ SOS1 \ \ type \ \ of \ \ variables. \quad (46)$$

We now focus on the non-linear term in the objective function, $p_{s,t} q_{s,t}$. The non-linear term in the objective function is relaxed by discretizing the generation level.

For this purpose valid generation levels are fixed $\hat{q}_{s,t,y}$ for $y = 1, \ldots, Y$ [4]. Then, the binary variables $d_{s,t,y}$ and the profit variables $\nu_{s,t,y}$ are defined in the following way:

$$d_{s,t,y} = \begin{cases} 1 & \text{if fixed generation level } \hat{q}_{s,t,y} \text{ is selected} \\ 0 & \text{otherwise} \end{cases} \quad (47)$$

$$\nu_{s,t,y} = \begin{cases} \hat{q}_{s,t,y}\ p_{s,t} & \text{if } d_{s,t,y} = 1 \\ 0 & \text{otherwise} \end{cases} \quad (48)$$

The corresponding constraints which are used for linearising the non-linear term are the following:

$$q_{s,t} \ge \sum_{y=1}^{Y} d_{s,t,y} \hat{q}_{s,t,y} \quad (49)$$

$$\sum_{y=1}^{Y} d_{s,t,y} = 1 \quad (50)$$

$$\hat{q}_{s,t,y}p_{s,t} - M_2(1-d_{s,t,y}) \leq \nu_{s,t,y} \leq \hat{q}_{s,t,y}p_{s,t} + M_2(1-d_{s,t,y}) \quad (51)$$

$$-M_2 d_{s,t,y} \leq \nu_{s,t,y} \leq M_2 d_{s,t,y} \quad (52)$$

where again M_2 is a sufficiently large positive number. Finally, considering the linearization constraints (49)-(52) the stochastic MILP will have the following form:

$$Maximize \sum_{s=1}^{S}\omega_s[\sum_{t=1}^{T}\sum_{y=1}^{Y}\nu_{s,t,y} + p_f\sum_{j=1}^{J}\sum_{r\in R_j}\gamma_r m_{s,j,T}] \quad (53)$$

$$subject\ to:$$

$$q_{s,t} \geq \sum_{y=1}^{Y} d_{s,t,y}\hat{q}_{s,t,y} \quad (54)$$

$$\sum_{y=1}^{Y} d_{s,t,y} = 1 \quad (55)$$

$$\hat{q}_{s,t,y}p_{s,t} - M_2(1-d_{s,t,y}) \leq \nu_{s,t,y} \leq \hat{q}_{s,t,y}p_{s,t} + M_2(1-d_{s,t,y}) \quad (56)$$

$$-M_2 d_{s,t,y} \leq \nu_{s,t,y} \leq M_2 d_{s,t,y} \quad (57)$$

$$\rho_i - M_1(1-d_{s,t,i}) \leq p_{s,t} \leq \rho_{i+1} + M_1(1-d_{s,t,i}) \quad (58)$$

$$\sum_{l=0}^{i} x_{i-l,t} - M_1(1-d_{s,t,i}) \leq q_{s,t} \leq \sum_{l=0}^{i} x_{i-l,t} + M_1(1-d_{s,t,i}) \quad (59)$$

$$\sum_{i=1}^{I} d_{s,t,i} = 1 \quad (60)$$

$$m_{s,j,t} = m_{s,j,t-1} - \sum_{n=1}^{N} Q_{s,j,t,n} - S_{s,j,t} + \sum_{n=1}^{N} Q_{s,j-1,t-\tau_j,n} + S_{s,j-1,t-\tau_j} + I_{j,t} \quad (61)$$

$$H_{s,j,t} \leq \sum_{n=1}^{N}\eta_{j,n}Q_{s,j,t,n} \quad (62)$$

$$q_{s,t} = \sum_{j} H_{s,j,t} \quad (63)$$

$$Q_{s,j,t,n} \leq \bar{Q}_{j,n} \quad (64)$$

$$m_{s,j,t} \leq \bar{m}_j \quad (65)$$

$$\sum_{k=1}^{K} G_{s,k,t} + q_{s,t} = D_{s,t} \quad (66)$$

$$\bar{G}_k - G_{s,k,t} \geq 0, \quad \mu_{s,k,t} \geq 0 \quad (67)$$

$$(c_k + \alpha_k G_{s,k,t}) - p_{s,t} + \mu_{s,k,t} \geq 0, \quad G_{s,k,t} \geq 0 \quad (68)$$

$$u1_{s,k,t} = \frac{\mu_{s,k,t} + (\bar{G}_k - G_{s,k,t})}{2} \quad (69)$$

$$v1^+_{s,k,t} - v1^-_{s,k,t} = \frac{\mu_{s,k,t} - (\bar{G}_k - G_{s,k,t})}{2} \quad (70)$$

$$u1_{s,k,t} - (v1^+_{s,k,t} + v1^-_{s,k,t}) = 0 \quad (71)$$

$$u2_{s,k,t} = \frac{G_{s,k,t} + ((c_k + \alpha_k G_{s,k,t}) - p_{s,t} + \mu_{s,k,t})}{2} \quad (72)$$

$$v2^+_{s,k,t} - v2^-_{s,k,t} = \frac{G_{s,k,t} - ((c_k + \alpha_k G_{s,k,t}) - p_{s,t} + \mu_{s,k,t})}{2} \quad (73)$$

$$u2_{s,k,t} - (v2^+_{s,k,t} + v2^-_{s,k,t}) = 0 \quad (74)$$

$$u \geq 0;\ \ v^+,\ \ v^-\ \ are\ \ SOS1\ \ type\ \ of\ \ variables. \quad (75)$$

IV. Case Study

A. Input data

A cascaded three-reservoir hydro system is used to represent a player who acts strategically as shown in Fig. 1. Other electricity market players who do not exercise market power own aggregated wind, hydro and thermal units.

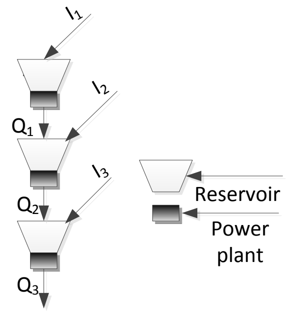

Fig. 1: Cascaded hydro system.

The data related to the physical characteristics of the reservoirs and power plants of the three-reservoir hydro system are summarized in Table I.

TABLE I: Data for the three-reservoir hydro system

Reservoir	$\bar{m}_j$ (HE)	$Q_j (m^3/s)$	H_j(MW)	τ_j (h)	m^o_j (MW)
1	4008	340	95	0	2400
2	1392	310	50	0.5	835
3	4008	330	90	2	2400

Moreover, data related to the maximum capacity and marginal cost of the aggregated hydro and thermal units are set in Table II.

TABLE II: Data for aggregated units

Generation type	G_k MW	c_k	α_k
Hydro	12000	0	0
Thermal	60000	10	0.0013

In addition, wind and demand scenarios used in the model are depicted in Fig.2. It is assumed that all scenarios are equally likely to occur.

Finally, the water opportunity cost is estimated based on the financial market and taken 40 $€/MWh$.

B. Results

The stochastic bi-level optimization model is implemented in GAMS and solved by CPLEX. Fig. 3 demonstrates the price scenarios for the planning period calculated as dual variables of the load balance constraints in the market clearing problem. Depending on the realized demand and wind power output for a specific hour, shown in Fig.2, the production volume needed from the most expensive generation type to meet the demand will vary, which result

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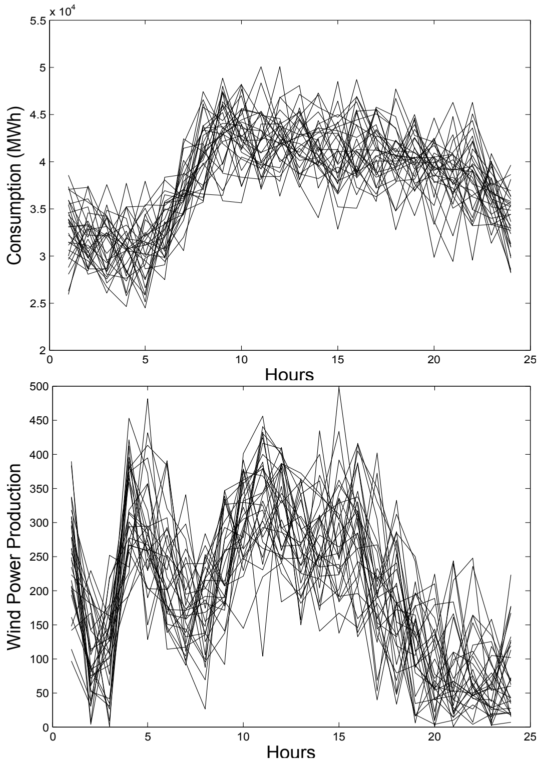

Fig. 2: Scenarios for demand (upper figure) and scenarios for wind power production (lower figure).

in the varying price scenarios shown in Fig. 3. Having these prices calculated internally, the player who acts strategically will decide to sell the power to the day-ahead market or store it and sell in the future with the water opportunity cost.

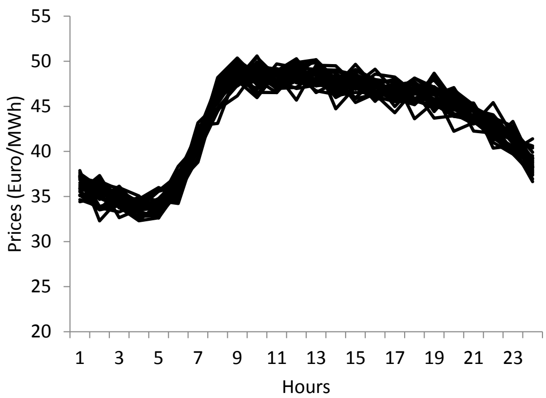

Fig. 3: Price scenarios calculated as dual variables of the load balance constraint

According to the calculated prices illustrated in Fig. 3, the price driven bid offerings to the day-ahead market for the whole planning period are summarised in Table III. These bid offerings are the result of the first level optimization problem. According to the Table III the model suggests selling at peak hours when the realised market prices are higher than the water opportunity cost and holding the energy back during off-peak hours when the prices are low.

As the suggested model ('original') simulates the results of strategic behaviour, a benchmark case ('benchmark') is defined in order to analyse the impact of the strategic behaviour on market settlement. The benchmark model is the case of perfect competition. The first comparison results

TABLE III: The optimal bid offering to the day-ahead market

$Hour$	Bid volume	Hour	Bid volume
1	0	13	235
2	0	14	235
3	0	15	235
4	0	16	235
5	0	17	235
6	0	18	235
7	159	19	235
8	235	20	235
9	235	21	235
10	235	22	210
11	235	23	130
12	235	24	70

based on the dispatch volumes for scenario 1 are illustrated in Fig. 4. The benchmark model dispatches in all hours. In contrast, the original model dispatches only under the high prices.

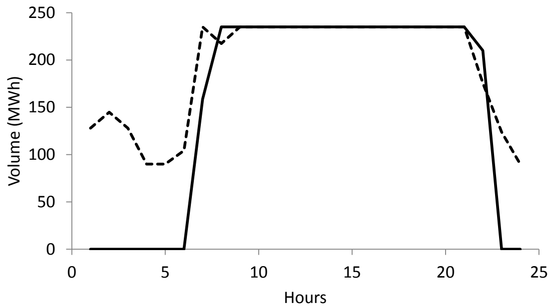

Fig. 4: Dispatch volume for original (solid line) and benchmark (dashed line) cases for scenario 1.

Second, the model statistics for both cases are provided in Table IV. Note that the benchmark model has a quadratic cost function, thus it is faster to solve using the MOSEK solver [17]. The CPLEX solver required 19.71 seconds in order to solve the problem.

TABLE IV: Model statistics for the three-reservoir hydropower system

	Original model	Benchmark model
Continuous variables	61,993	12,961
Discrete variables	20,160	0
Single equations	106,561	13,681
Computational time (second)	42.35	0.359

Finally, Table V shows the impact of the strategic behaviour on the generation cost. In the original case where the hydropower producer behaves strategically the generation cost increases by $16,630$€, which is an increase of 0.06%. For this specific example, when the hydropower producer owns only three-reservoir system, the cost increase is modest , but would be exacerbated id additional hydro production assets were owned by the same firm. Thus, the hydropower producer can behave strategically and increase its profit.

TABLE V: Generation cost minus profit from the stored water calculated for both models.

	Generation cost-profit from stored water
Original model	26,728,540
Benchmark model	26,711,910
Difference	16,630
Difference in %	0.06 %

V. Conclusion

In this work, a stochastic MPEC model is used for representing a Stackelberg game applicable for a strategic hydropower producer that maximizes profit. Using linearization and SOS_1 variables, the stochastic MPEC is transformed to a stochastic MILP, which guarantees better numerical behaviour. The results are analysed on a case study, where an electricity market player owning hydropower acts strategically. Results show that the strategic behaviour of a hydropower producer alters the market outcomes. The execution time of the model is less than a minute, thus the approach can be applied to larger system.

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