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Transient Thermal Analysis of Power Cable-Considering Skin Effect

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***Abstract*—In order to determine the thermal rises of power cables under severe and short shock and size the proper type accordingly, this paper proposes analytical solutions and presents case study examples taking skin effect into consideration. Analyses show temperature arise upon exciting currents at high frequency exhibits quite different profile with that of low frequency currents whose skin depth outnumber the overall radius of power cable many times.**

***Index Terms*—power cable, skin effect, transient thermal analysis**

I. Introduction

POWER cables are apparatus widely used in electrical engineering. Engineers can easily size the appropriate type according to the datasheet provided by manufactures based on the magnitude of steady flowing current. Apart from steady carrying, cables may undergo shocks for very short time counting on seconds yet with magnitude more than tenfold of normal current. It's not complicated to calculate roughly the thermal rise assuming that current is distributed evenly on cross section and during the shock time the copper part is thermally isolated, i.e. no thermal energy can diffuse to parts other than copper. However, there are cases where appearance of overcurrent waveform look odd in a way quite different with sinusoid or even pulsing pattern with frequency in kilo Hertz. Such cases include sudden shorting of electrical machines, current through DC-link braking branch, expected transient overload and etc. Based on Fourier decomposition, harmonics arises which further leads to more current swarming close to the copper surface than areas far away from surface, a phenomenon usually termed as skin effect in electromagnetics. Since thermal arise is proportional to current density temporarily ignoring energy dissipated into protective sleeves, it will definitely lead to higher temperature rises on surface. Quantitative analyses cannot be obtained by elementary math with moderate effort and therefore prohibit engineers exploring the exact thermal stress and determine whether proper cables are selected accordingly. To overcome this obstacle, this paper gives analyses and solutions based on several reasonable assumptions that simplify the problem considerably.

In the following, firstly several assumptions are presented and elaborated. Next, the author proposes mathematical model based on well-established heat conduction theory and analytical solutions using eigenfunction expansion. Moreover, to make the result more vividly, a specific case study is analyzed which demonstrate that currents swarming near surface lead to much higher temperature than that of other areas and overlook of it may lay a bomb in system electrical design.

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II. Assumptions

A. Compressed Wires as One Solid Rod

A close look on cross section of power cable shows that the copper strings are tightened closely and can be treated as a single copper rod with constant thermal and electro-magnetic properties.

Fig. 1 Cross section of power cable

B. Thermally Isolated Copper During Transient Shock

The thermal boundary condition between copper and rubber sleeve can be modeled as (1) stated

$$\lambda_c \frac{\partial u_c}{\partial r}\bigg|_{r=l} = \lambda_r \frac{\partial u_r}{\partial r}\bigg|_{r=l} . \qquad (1)$$

Considering the thermal conductivity of rubber λ_c is hundred times larger than that of copper λ_r and during a short time there is no enough thermal energy to heat up the rubber thus the thermal gradient $\frac{\partial u_r}{\partial r}$ within rubber is small, we have a good reason to ignore the energy flowing from the heat source, namely copper, to the surrounding rubber. Hereby the boundary condition for copper part can be reduces as (2)

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$$\lambda_c \frac{\partial T_c}{\partial x}\bigg|_{x=l} = 0 \ . \quad (2)$$

C. Three Dimension Problem Reduced to One Counterpart

For simplicity, we assume cable under investigation has uniform thermal and electro-magnetic properties and therefore areas with same distance from the center of circular copper rod share equivalent temperature. Furthermore, suppose cable is long enough that areas with equivalent radius on each cross section along the cable also share the same temperature. Hereto we arrive at one dimensional temperature problem which diversifies with radius and time.

III. Mathematical Models

This part falls into two sections: one for thermal source model governed by Maxwell equations and another for heat conduction model derived from Fourier conduction theorem.

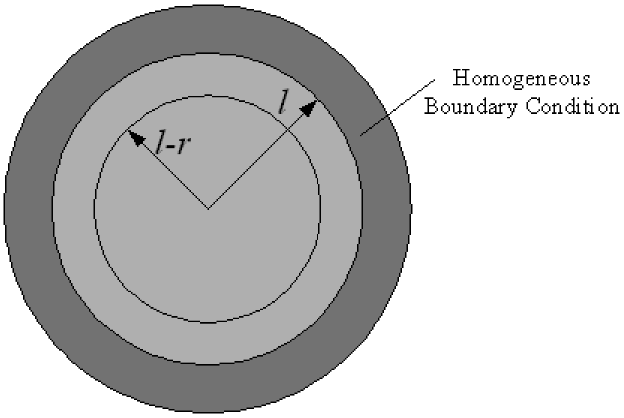

Fig. 2 Idealized cross section

A. Thermal Source Model

For simplicity again, consider Transverse Electromagnetic Waves (TEM) propagate along copper rod and both electric field and magnetic field are time harmonic vectors [1]. The current density within cross section of copper is governed by (3)

$$J_r = J_0 e^{\frac{r-l}{\delta}} \ . \quad (3)$$

Where

- J_r Current density of concentric circle at distance r away from copper surface
- J_0 Current density of copper surface
- l Radius of copper
- δ Skin depth of copper, variant with current frequency

Due to the overall current I flowing through cross section is normally given, surface current density J_0 can be derived if we take a double integration as

$$I = \iint_s J_r ds = \int_0^{2\pi}\left(\int_0^l J_0 e^{\frac{r-l}{\delta}} r dr\right) d\theta \ . \quad (4)$$

Simple math manipulation leads to expression of J_0 as

$$J_0 = \frac{I}{2\pi\left(\frac{\delta^2}{e^{1/\delta}} - \delta(\delta-1)\right)} \ . \quad (5)$$

As skin depth δ goes to infinity, J_0 approaches to $I/(\pi l^2)$ which means uniform current distribution.

Finally we have the heat source formula which represents energy per unit volume provided by copper itself as

$$p(r) = \frac{heat}{volume} = \frac{J_0{}^2 e^{\frac{2(r-l)}{\delta}}}{\sigma} \ . \quad (6)$$

B. Heat Conduction Model

Based from Fourier heat conduction theorem which depicts the proportional relationship between heat flow per unit area and temperature gradient, it’s not hard to the following (7) which we skip the derivation process found notable in every heat transfer textbook [2].

$$\eta \frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + f(r) \quad (7)$$

Where

- η Reciprocal of thermal diffusivity of copper
- $f(r)$ Heat source function $p(r)$ divided by thermal conductivity of copper

Together with boundary condition equation as

$$\frac{\partial u}{\partial r}\bigg|_{r=l} = 0 \ . \quad (8)$$

And initial condition as

$$u(r,0) = b \ . \quad (9)$$

C. Analytical Solution

For (7) (8) (9), the analytical solution is derived based on Partial Differential Equation (PDE) theory [3], [4] as follows.

First we suppose a tentative solution exist as

$$u(r,t) = \sum_{n=0}^{\infty} T_n(t) R_n(r) \ . \quad (10)$$

Substitute variable F with $u(r,t)$ in Helmholtz equation

$$\Delta(F) = -kF = -\lambda_n{}^2 F \ . \quad (11)$$

After simple manipulation

$$r^2 R_{rrn} + r R_{rn} + \lambda_n{}^2 r^2 R_n = 0 \ . \quad (12)$$

Where

- R_{rrn} Second order derivative of R_n on r
- R_{rn} First order derivative of R_n on r

Equation (12) takes the form of 0 order Bessel equation and the solution is Bessel function $J_0(\lambda r)$.

Solution (10) should comply with boundary condition (8) and thus another equation is obtained as

$$T_n(t) R_n(l) = 0 \quad (13)$$

Further manipulation leads to

$$J_0^{'}(\lambda_n l) = 0 \tag{14}$$

i.e.

$$J_1(\lambda_n l) = 0 . \tag{15}$$

In another word, (15) must hold if solution (10) comply with Helmholtz equation and boundary condition and the unknown parameter λ_n should be equal to roots of first order Bessel function which we denote here as $a_n\left(n = 0,1,\cdots\right)$, divided by copper radius l.

Normalize $R_n\left(r\right)$ as

$$R_n\left(r\right) = \frac{\sqrt{2}J_0(\lambda_n r)}{lJ_0(\lambda_n l)} . \tag{16}$$

Expand heat source function $f\left(r\right)$ and initial condition b with $R_n\left(r\right)$

$$f(r) = \sum_{n=0}^{\infty} f_n R_n(r) . \tag{17}$$

Where

$$f_n = \int_0^l rf\left(r\right)R_n\left(r\right)dr$$

And

$$b = \sum_{n=0}^{\infty} b_n R_n(r) . \tag{18}$$

Where

$$b_n = \int_0^l rbR_n\left(r\right)dr$$

Substitute equation (16) (17) (18) into equation (7), we conclude a equation on $T_n\left(t\right)$

$$\eta T_n^{'}\left(t\right) + \left(\frac{a_n}{l}\right)^2 T_n\left(t\right) = f_n . \tag{19}$$

Equation (19) is constant coefficient ordinary equation with general solution as

$$T_n\left(t\right) = C_n e^{\frac{(a_n/l)^2 t}{\eta}} + \left(\frac{a_n}{l}\right)^2 f_n . \tag{20}$$

Hereto the original tentative solution become

$$u(\mathrm{r},\mathrm{t}) = \sum_{n=0}^{\infty}\left(C_n e^{\frac{(a_n/l)^2 t}{\eta}} + \left(\frac{a_n}{l}\right)^2 f_n\right)\frac{\sqrt{2}J_0(\lambda_n \mathrm{r})}{lJ_0(\lambda_n l)} . \tag{21}$$

Use initial condition

$$\sum_{n=0}^{\infty}\left(C_n + \left(\frac{a_n}{l}\right)^2 f_n\right)R_n\left(r\right) = b = \sum_{n=0}^{\infty} b_n R_n(r) , \tag{22}$$

We find the last unknown coefficient C_n as

$$C_n = b_n - \left(\frac{a_n}{l}\right)^2 f_n . \tag{23}$$

Remarks about the solution above:

1) From the very beginning, the author assumes the exciting current behaves as time harmonic function and the current denotation I is actually RMS value of sinusoidal current with amplitude of $\sqrt{2}I$. For arbitrary periodic waveforms, we first decompose it into components of different frequencies, solve the PDEs as (7) (8) (9) and then add all the solutions. It will work due to the linearity of governing equations.
2) Engineers lost in the derivation above should feel relived with the help of numerous numerical PDE solvers, commercially available as Matlab, Maple and etc. or self-made algorithms based on Finite Difference Method, Finite Element Method and etc. No matter which one engineers select, it can solve specific problems on no condition of demanding knowledge of solving PDE analytically. However, acquaintance of derivation above helps to understand the most underlying principles which brew the original thoughts where the exploring comes from and what new may happen at another time.

IV. CASE STUDY

After lengthy mathematical derivation at which engineers frown, some real life cases are introduced here to demonstrate what the temperature profiles look like upon transient shocks and how the skin effect influences the whole process.

Consider a very long 20 mm^2 power cable endures 0.6 second shock by several types of waveforms.

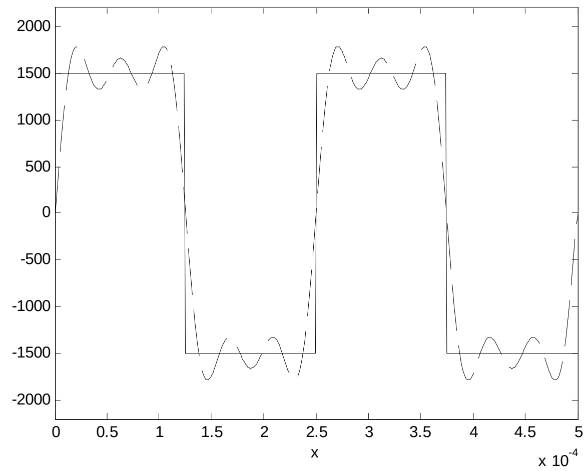

Fig. 3 Square wave (solid line) and Fourier expansions sum (dashed line) counting up to 5rd harmonic. The appropriate number of decomposition to be taken depends on the influence on temperature rise on hottest spot of high order harmonics.

A. Square Pulse Excitation

In case of square pulse excitation, as mentioned above, Fourier expansion can be used to decompose the original pulse with combination of numerous sinusoidal waves as Fig.3 shows. Here we assume the magnitude and frequency as 1500A and 4000 Hz. The number of harmonics should be considered relating to the decaying influencing pattern of high order components. We can safely substitute the original square pulse with combination of harmonic components up to the exact order that higher ones stop contribute the temperature rise notably. Comparison two plots in Fig. 4

shows that counting up to the first 5 components for analysis will not lead to much difference with the result of original square pulses.

B. Low Frequency Sinusoidal Excitation

As a comparison, consider 0.01Hz, a very low frequency sinusoidal excitation with the same amplitude as the fundament of square pulse in the last section. The result in Fig. 5 clearly demonstrates that current spreading evenly on cross section leads to uniform temperature profile. The temperature of hottest area, namely the boundary surface, equals to nearly one half that of square pulse excitation.

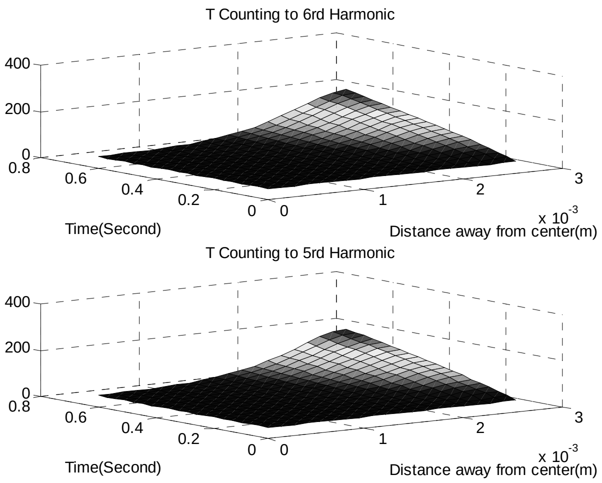

Fig. 4 Temperature rises caused by the first 6 and the first 5 components make no obvious difference.

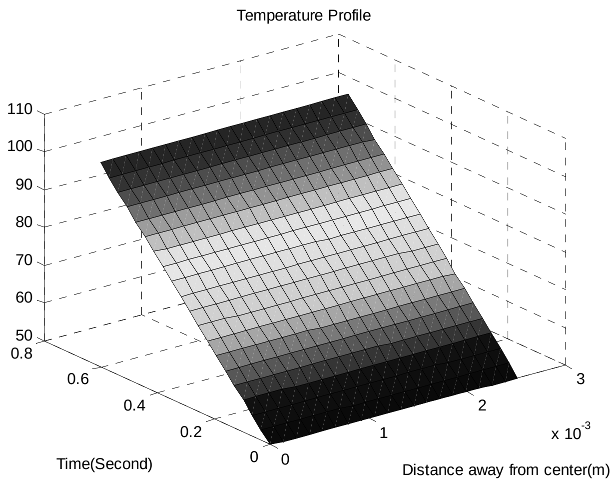

Fig. 5 Temperature rises caused by the very low frequency excitation. The current distributes evenly on cross section and finally lead to uniform temperature profile at each constant.

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REFERENCES

[1] D. K. Cheng, Field and Wave Electromagnetics, 2rd ed.. NJ : Pearson Addison Wesley, 1989.

[2] H. S. Carslaw and J.C. Jaeger, Conduction of Heat in Solid, 2rd ed.. Oxford : Clarendon Press, 1959.

[3] N. H. Asmar, Partial Differential Equations with Fourier Series and Boundary Value Problems, 2rd ed.. NJ : Pearson Pretice Hall, 2004.

[4] D. W. Trim, Applied Partial Differential Equations. London : PWS, 1990.

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