Stability of a New Modified Iterative Algorithm

Ritu Sahni and Manoj Sahni

Abstract— **The stability of different fixed point iterative procedures plays an important role in the fixed point theory. Most of the real life problems can be easily rearranged as fixed point equation, which can be solved by some iterative procedure that converges towards the solution or the fixed point. During numerical computation it becomes important to study the stability of iterative procedure. In this paper, we have introduced a new iterative scheme and study the stability of that iterative scheme by using a general contractive condition.**

Index Terms— **Stability, Mann iteration, Ishikawa iteration, Noor iteration.**

I. Introduction and Preliminaries

MOST of the equations $y = f(x)$ arising in physical phenomenon can be easily transformed into a fixed point equation $Tx = x$ and then using an appropriate fixed point theorem we get information on the existence or existence and uniqueness of fixed point, that is, of the solution of the original equation. For the solution of equation of the type $Tx = x$, we approximate a sequence $\{x_n\}$ in X by some iterative procedures and because of rounding off or discretization in the function T, an approximated sequence $\{y_n\}$ is obtained in place of the actual sequence $\{x_n\}$. If $f(T, x_n\}$, that is the iterative scheme is stable and $\{x_n\}$ converges to a fixed point u of T, then $\{y_n\}$ converges to the same fixed point of T. Many results on the stability is established by several authors using different contractive mappings. The first result on the stability of the Picard iteration was proved by Ostrowski [16]. This study regarding the stability of iterative procedures enjoys a celebrated place in applicable mathematics due to chaotic behaviour of functions in discrete dynamics and other numerical computations (see, for instance, [2], [7]-[10], [15]-[16], [18] and references thereof).

Definition 1.1 [9]. Let (X, d) be a complete metric space and $T: X \to X$ be a self mapping. Suppose that $F(T) = \{p \in X, Tp = p\}$ is the set of all fixed points of T. Let $\{x_n\}_{n\in N} \subset X$ be the sequence generated by an iteration scheme involving T which is defined by

$$x_{n+1} = f(T, x_n),\ n = 0, 1, \dots \quad (1)$$

where $x_0 \in X$ is the initial point and f is a proper function.

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Suppose that sequence $\{x_n\}_{n\in N}$ converges to a fixed point p of T. Let $\{y_n\}_{n\in N} \subset X$ and set

$$\varepsilon_n = d\big(y_{n+1}, f(T, y_n)\big),\ n = 0, 1, \dots \quad (2)$$

Then the iteration process (1) is said to be T-stable or stable with respect to T if and only if $\lim_{n\to\infty} \varepsilon_n = 0$ implies $\lim_{n\to\infty} y_n = p$.

In a complete metric space, the Picard iteration [17] $\{x_n\}_{n=0}^{\infty}$ is defined as:

$$x_{n+1} = Tx_n, n = 0, 1, \dots \quad (3)$$

which was first used in Banach fixed point theorem to approximate the fixed points of mappings satisfying the inequality

$$d(Tx, Ty) \le a d(x, y) \quad (4)$$

for all $x,\ y \in X$ and $a \in [0, 1)$.

Condition (4) is called the Banach contraction condition.

The mapping T is called Kannan contraction condition [12] if there exists $b \in (0, \frac{1}{2})$ such that

$$d(Tx, Ty) \le b[d(x, Tx) + d(y, Ty)] \quad (5)$$

for all $x, y \in X$.

The mapping T becomes Chatterjea contraction [5] if there exists $c \in (0, \frac{1}{2})$ such that

$$d(Tx, Ty) \le c[d(x, Ty) + d(y, Tx)] \quad (6)$$

for all $x, y \in X$.

In 1972, Zamfirescu [19] obtained the following interesting fixed point theorem.

Theorem 1.1. Let (X, d) be a complete metric space and $T: X \to X$ a mapping for which there exists the real number $a,\ b$ and c satisfying $a \in (0, 1)$, $b, c \in (0, \frac{1}{2})$ such that for any pair $x, y \in X$, at least one of the following conditions holds:

(i) $d(Tx, Ty) \le a\ d(x, y)$

(ii) $d(Tx, Ty) \le b[d(x, Tx) + d(y, Ty)]$

(iii) $d(Tx, Ty) \le c[d(x, Ty) + d(y, Tx)]$

Then T has a unique fixed point p and the Picard iteration converges to p for any arbitrary but fixed point $x_0 \in X$. The conditions (i)-(iii) can be written in the following equivalent form:

$$d(Tx, Ty) \le h\, max\left\{d(x, y), \frac{d(x,Tx)+d(y,Ty)}{2}, \frac{d(x,Ty)+d(y,Tx)}{2}\right\} \quad (7)$$

for all $x, y \in X$ and $0 < h < 1$, which has been obtained by Ciric [6] in 1974.

A mapping satisfying equation (7) is called Ciric quasi-

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contraction. It is obvious that each of the conditions (i)−(iii) implies (7). An operator T satisfying the contractive conditions (i)-(iii) in the Theorem 1.1 is called Zamfirescu or Z-operator.

In some situations Picard iteration fails to converge. To overcome this difficulty, in 1953, Mann defined a new iterative scheme called Mann iteration [13] defined as:

$$x_{n+1} = (1 - \alpha_n)x_n + \alpha_n T x_n, \quad (8)$$

where $\{\alpha_n\}$ is a sequence of positive numbers in [0,1].

Again to deal with those situations in which Mann iteration does not converge or slow to converge, Ishikawa iteration or two step iteration is defined by Ishikawa in 1974 [11] given as follows:

$$x_{n+1} = (1 - \alpha_n)x_n + \alpha_n T y_n$$
$$y_n = (1 - \beta_n)x_n + \beta_n T x_n \quad (9)$$

where $\{\alpha_n\}$ and $\{\beta_n\}$ are sequences of positive numbers in [0,1].

In 2004, Berinde [3] proved the strong convergence theorem using Ishikawa iteration to approximate fixed points of Zamfirescu operator in an arbitrary Banach space . While proving the theorem, he uses following condition:

$$\|Tx - Ty\| \leq \delta\|x - y\| + 2\delta\|x - Tx\|, \quad (10)$$

which holds for any $x, y \in X$ and $0 \leq \delta < 1$.

In this paper, we employ a condition introduced in [4], which is more general than condition (10) and establish stability theorem in normed linear space. The condition is defined as follows:

Let C be a nonempty, closed, convex subset of a normed space E and $T: C \to C$ a self map. Then there exists a constant $L \geq 0$ such that for all $x, y \in C$, we have

$$\|Tx - Ty\| \leq e^{L\|x-Tx\|}(\delta\|x - y\| + 2\delta\|x - Tx\|) \quad (11)$$

where, $0 \leq \delta < 1$ and e^xdenotes the exponential function of $x \in C$.

Remark1.1. If $L = 0$, in the above condition, we obtain $\|Tx - Ty\| \leq \delta\|x - y\| + 2\delta\|x - Tx\|$, where $\delta = \max\{a, \frac{b}{1-b}, \frac{c}{1-c}\}$, $0 \leq \delta < 1$, and constants a, b and c are as defined in Theorem 1.1.

Example 1.1. Let X be the real line with the usual norm ‖.‖ and suppose K = [0,1]. Define $T: K \to K$ by $Tx = 1 - x$ for all x, $y \in K$. Obviously T is a self-mapping with a unique fixed point 1/2. Now we check that condition (11) is true. If x, $y \in [0,1]$, then $\|Tx - Ty\| = \|x - y\|$ and

$$e^{L\|x-Tx\|}(\delta\|x - y\| + 2\delta\|x - Tx\|)$$
$$= e^{L\|2x-1\|}(\delta\|x - y\| + 2\delta\|2x - 1\|)$$

Clearly, if we chose $x = 0$ and $y = 1$,then contractive condition (1.11) is satisfied since $\|Tx - Ty\| = \|x - y\| =$ 1and for L≥0, we choose $L \geq 0$,then

$$e^{L\|x-Tx\|}(\delta\|x - y\| + 2\delta\|x - Tx\|)$$
$$= e^{L\|2x-1\|}(\delta\|x - y\| + \delta\|x - 2\|)$$
$$= e^0(3\delta) = 3\delta, \text{ where } 0 < \delta < 1.$$

That is, $1 \leq 3\delta$, which implies $\delta \geq 1/3$. Now if we take $0 < \delta < 1$, then condition (1.11) is satisfied and ½ is the unique fixed point of T.

In 2000, Noor defined the three step Noor iteration scheme [14] as follows:

$$x_{n+1} = (1 - \alpha_n)x_n + \alpha_n T y_n$$
$$y_n = (1 - \beta_n)x_n + \beta_n T z_n$$
$$z_n = (1 - \gamma_n)x_n + \gamma_n T x_n, \quad (12)$$

where $\{\alpha_n\}, \{\beta_n\}$ and $\{\gamma_n\}$ are sequences of positive numbers in [0,1].

In this paper, we introduce a new modified three step iteration process which is the generalization of Noor iteration process as follows:

$$x_0 = x \in C,$$
$$f(T, x_n) = (1 - \alpha_n)x_n + \beta_n T x_n + (\alpha_n - \beta_n)T y_n$$
$$y_n = (1 - \gamma_n)x_n + \xi_n T x_n + (\gamma_n - \xi_n)T z_n$$
$$z_n = (1 - \eta_n)x_n + \eta_n T x_n, \quad (13)$$

for $n \geq 0$, where, $\{\alpha_n\}$, $\{\beta_n\}$, $\{\gamma_n\}$, $\{\eta_n\}$ and $\{\xi_n\}$ satisfy the following conditions:

(i) $\alpha_n \geq \beta_n$, $\gamma_n \geq \xi_n$

$(ii)\{\alpha_n - \beta_n\}_{n=0}^{\infty}$, $\{\gamma_n - \xi_n\}_{n=0}^{\infty}$, $\{\alpha_n\}_{n=0}^{\infty}$, $\{\beta_n\}_{n=0}^{\infty}$, $\{\gamma_n\}_{n=0}^{\infty}$, $\{\xi_n\}_{n=0}^{\infty}$ and $\{\eta_n\}_{n=0}^{\infty} \in [0,1]$.

In the following remark, we show that the new iteration process is more general than the Noor, Ishikawa, Mann and Picard iteration.

Remark1.2.

(1) If $\xi_n = \beta_n = 0$, it reduces to Noor iteration.

(2) If $\xi_n = \beta_n = \eta_n = 0$, it reduces to Ishikawa iteration.

(3) If $\beta_n = \eta_n = \gamma_n = 0$, it reduces to Mann iteration.

(4) If $\eta_n = \gamma_n = 0, \alpha_n = 1$, it reduces to Picard iteration.

In the sequel, we shall use the following Lemma which is contained in Berinde [1].

Lemma1.1. Let δ be a real number satisfying $0 \leq \delta < 1$, and $\{\varepsilon_n\}$ a positive sequence satisfying $\lim_{n\to\infty} \varepsilon_n = 0$. Then, for any positive sequence $\{u_n\}$ satisfying

$$u_{n+1} \leq \delta u_n + \varepsilon_n$$

it follows that $\lim_{n\to\infty} u_n = 0$.

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II. MAIN RESULTS

Theorem2.1. Let $(E, ||.||)$ be Banach space and $T: E \to E$ a self mapping with fixed point p with respect to condition (11). Let $\{x_n\}_{n\in N}$ be modified three step iteration process converges to fixed point of T, where $\alpha_n \geq \beta_n, \gamma_n \geq \xi_n$, and $\{\alpha_n - \beta_n\}_{n=0}^{\infty}$, $\{\gamma_n - \xi_n\}_{n=0}^{\infty}$, $\{\alpha_n\}_{n=0}^{\infty}$, $\{\beta_n\}_{n=0}^{\infty}$, $\{\gamma_n\}_{n=0}^{\infty}$, $\{\xi_n\}_{n=0}^{\infty}$ and $\{\eta_n\}_{n=0}^{\infty} \in [0,1]$ such that

$0 < \alpha \leq \alpha_n$, $0 < \gamma \leq \gamma_n$, $0 < \beta \leq \beta_n$ and $0 < \eta \leq \eta_n$ for all n. (14)

Then modified three step iteration process is T-stable.

Proof. Suppose $\{x_n\}_{n=0}^{\infty}$ converges to p. Suppose also that $\{y_n\} \subset E$ is an arbitrary sequence in *E*. Define

$$\varepsilon_n = \|y_{n+1} - (1-\alpha_n)y_n + (\alpha_n - \beta_n)Tq_n + \beta_n Ty_n\|,$$
$n = 0,1,\dots$

where

$$q_n = (1-\gamma_n)y_n + (\gamma_n - \xi_n)Tr_n + \xi_n Ty_n \text{ and}$$
$$r_n = (1-\eta_n)y_n + \eta_n Ty_n.$$

Assume that $\lim_{n\to\infty} \varepsilon_n = 0$, then using the contractive condition and the triangle inequality; we shall prove that $\lim_{n\to\infty} y_n = p$ as follows:

$$\|y_{n+1} - p\| \leq$$

$$\|y_{n+1} - (1-\alpha_n)y_n + (\alpha_n - \beta_n)Tq_n + \beta_n Ty_n\| + \|(1-\alpha_n)y_n + (\alpha_n - \beta_n)Tq_n + \beta_n Ty_n - p\|$$

$$\leq \varepsilon_n + \|(1-\alpha_n)y_n + (\alpha_n - \beta_n)Tq_n + \beta_n Ty_n - p\|$$

$$\leq \varepsilon_n + (1-\alpha_n)\|y_n - p\| + (\alpha_n - \beta_n)\|Tq_n - p\| + \beta_n\|Ty_n - p\|$$

$$= \varepsilon_n + (1-\alpha_n)\|y_n - p\| + (\alpha_n - \beta_n)\|Tp - Tq_n\| + \beta_n\|Tp - Ty_n\|$$

$$\leq \varepsilon_n + (1-\alpha_n)\|y_n - p\| + (\alpha_n - \beta_n)\left[e^{L\|p-Tp\|}(2\delta\|p - Tp\| + \delta\|p - q_n\|)\right] + \beta_n\left[e^{L\|p-Tp\|}(2\delta\|p - Tp\| + \delta\|p - y_n\|)\right]$$

$$= \varepsilon_n + (1-\alpha_n)\|y_n - p\| + (\alpha_n - \beta_n)\left[e^{L\|p-p\|}(2\delta\|p - p\| + \delta\|p - q_n\|)\right] + \beta_n\left[e^{L\|p-p\|}(2\delta\|p - p\| + \delta\|p - y_n\|)\right]$$

$$= \varepsilon_n + (1-\alpha_n)\|y_n - p\| + (\alpha_n - \beta_n)\left[e^{L(0)}(2\delta(0) + \delta\|p - q_n\|)\right] + \beta_n\left[e^{L(0)}(2\delta(0) + \delta\|p - y_n\|)\right]$$

$$= \varepsilon_n + (1-\alpha_n)\|y_n - p\| + (\alpha_n - \beta_n)\delta\|p - q_n\| + \beta_n\delta\|p - y_n\|$$

$$= \varepsilon_n + (1-\alpha_n + \delta\beta_n)\|y_n - p\| + (\alpha_n - \beta_n)\delta\|q_n - p\|. \tag{15}$$

For the estimate of $\|q_n - p\|$ in (15), we get

$$\|q_n - p\| = \|(1-\gamma_n)y_n + (\gamma_n - \xi_n)Tr_n + \xi_n Ty_n - p\|$$

$$= \left\|(1-\gamma_n)y_n + (\gamma_n - \xi_n)Tr_n + \xi_n Ty_n - \left((1-\gamma_n) + \gamma_n\right)p\right\|$$

$$\leq (1-\gamma_n)\|y_n - p\| + \gamma_n\|Tr_n - p\| + \xi_n\|Ty_n - Tr_n\|$$

$$\leq (1-\gamma_n)\|y_n - p\| + \gamma_n\|Tr_n - Tp\| + \xi_n\|Ty_n - Tp + Tp - Tr_n\|$$

$$\leq (1-\gamma_n)\|y_n - p\| + (\gamma_n + \xi_n)\|Tp - Tr_n\| + \xi_n\|Tp - Ty_n\|$$

$$\leq (1-\gamma_n)\|y_n - p\| + (\gamma_n + \xi_n)\left[e^{L\|p-Tp\|}(2\delta\|p - Tp\| + \delta\|p - r_n\|)\right] + \xi_n\left[e^{L\|p-Tp\|}(2\delta\|p - Tp\| + \delta\|p - y_n\|)\right]$$

$$= (1-\gamma_n)\|y_n - p\| + (\gamma_n + \xi_n)\left[e^{L\|p-p\|}(2\delta\|p - p\| + \delta\|p - r_n\|)\right] + \xi_n\left[e^{L\|p-p\|}(2\delta\|p - p\| + \delta\|p - y_n\|)\right]$$

$$= (1-\gamma_n)\|y_n - p\| + (\gamma_n + \xi_n)\left[e^{L(0)}(2\delta(0) + \delta\|p - r_n\|)\right] + \xi_n$$

$$= (1-\gamma_n)\|y_n - p\| + (\gamma_n + \xi_n)\delta\|p - r_n\| + \xi_n\delta\|p - y_n\|$$

$$=(1-\gamma_n + \xi_n\delta)\|y_n - p\| + (\gamma_n + \xi_n)\delta\|p - r_n\| \tag{16}$$

Substitute (16) into (15) gives,

$$\|y_{n+1} - p\| \leq \varepsilon_n + (1-\alpha_n + \delta\beta_n)\|y_n - p\| + (\alpha_n - \beta_n)\delta[(1-\gamma_n + \xi_n\delta)\|y_n - p\| + (\gamma_n + \xi_n)\delta\|r_n - p\|]$$

$$\|y_{n+1} - p\| \leq \varepsilon_n + [1 - \alpha_n(1 - \delta + \gamma_n\delta - \delta^2\xi_n) + \beta_n(\gamma_n\delta - \delta^2\xi_n)]\|y_n - p\| + (\alpha_n - \beta_n)(\gamma_n + \xi_n)\delta^2\|r_n - p\|. \tag{17}$$

For $\|r_n - p\|$ into (17) gives

$$\|r_n - p\|$$
$$= \|(1-\eta_n)y_n + \eta_n Ty_n - p\|$$

$$= \left\|(1-\eta_n)y_n + \eta_n Ty_n - \left((1-\eta_n) + \eta_n\right)p\right\|$$

$$\leq (1-\eta_n)\|y_n - p\| + \eta_n\|Ty_n - p\|$$

$$\leq (1-\eta_n)\|y_n - p\| + \gamma_n\|Tp - Ty_n\|$$

$$\leq (1-\eta_n)\|y_n - p\| + \eta_n\left[e^{L\|p-Tp\|}(2\delta\|p - Tp\| + \delta\|p - y_n\|)\right]$$

$$= (1-\eta_n)\|y_n - p\| + \eta_n\left[e^{L\|p-p\|}(2\delta\|p - p\| + \delta\|p - y_n\|)\right]$$

$$= (1-\eta_n)\|y_n - p\| + \eta_n\left[e^{L(0)}(2\delta(0) + \delta\|p - y_n\|)\right]$$

$$= (1-\eta_n)\|y_n - p\| + \eta_n\delta\|p - y_n\|$$

$$=(1-\eta_n + \eta_n\delta)\|y_n - p\| \tag{18}$$

Substituting (18) into (17) and using (14)

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$\|y_{n+1} - p\| \le \varepsilon_n + [1 - \alpha(1 - \delta + \gamma\delta - \gamma\delta^2 - 2\xi\delta^2 + \eta\gamma\delta^2 - \eta\xi\delta^2 - \gamma\eta\delta^3 - \xi\eta\delta^3) + \beta(\gamma\delta - \gamma\delta^2 - 2\xi\delta^2 + \eta\gamma\delta^2 - \eta\xi\delta^2 - \gamma\eta\delta^3 - \xi\eta\delta^3]\|y_n - p\|$

$$= \varepsilon_n + [1 - \alpha(1 - \delta) - (\alpha - \beta)(1 - \delta + \gamma\delta - \gamma\delta^2 - 2\xi\delta^2 + \eta\gamma\delta^2 - \eta\xi\delta^2 - \gamma\eta\delta^3 - \xi\eta\delta^3)]\|y_n - p\| \quad (19)$$

Observe that,

$$0 \le [1 - \alpha(1 - \delta) - (\alpha - \beta)(1 - \delta + \gamma\delta - \gamma\delta^2 - 2\xi\delta^2 + \eta\gamma\delta^2 - \eta\xi\delta^2 - \gamma\eta\delta^3 - \xi\eta\delta^3)] < 1.$$

Therefore, taking the limit as $n \to \infty$ of both sides of the inequality (19), and using Lemma 1.1, we get $\lim_{n\to\infty}\|y_n - p\| = 0$, that is, $\lim_{n\to\infty} y_n = p$. This completes the proof.
Theorem 2.1 yields following corollaries:

Corollary 2.1. Let $(E, ||.||)$ be Banach space and $T: E \to E$ a self mapping with fixed point p with respect to condition (10). Let $\{x_n\}_{n\in N}$ be three step iteration process or Noor iteration, converges to fixed point of T. Then Noor iteration is *T*-stable.

Proof. If $\xi_n = \beta_n = L = 0$, we arrive the result.

Corollary 2.2. Let $(E, ||.||)$ be Banach space and $T: E \to E$ a self mapping with fixed point p with respect to condition (10). Let $\{x_n\}_{n\in N}$ be two step iteration process or Ishikawa iteration, converges to fixed point of T. Then Ishikawa iteration is *T*-stable.

Proof. Putting, $\xi_n = \beta_n = \eta_n = L = 0$, we arrive the result.

Corollary 2.3. Let $(E, ||.||)$ be Banach space and $T: E \to E$ a self mapping with fixed point p with respect to condition (10). Let $\{x_n\}_{n\in N}$ be one step iteration process or Mann iteration, converges to fixed point of T. Then Mann iteration is *T*-stable.

Proof. Putting, $\beta_n = \eta_n = \gamma_n = L = 0$, we arrive the result.

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