Proceedings of the International MultiConference of Engineers and Computer Scientists 2016 Vol II,
IMECS 2016, March 16 - 18, 2016, Hong Kong

Bragg Scattering Mechanism Effect on Potential Velocity of Water Wave

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Abstract--- A potential velocity is important properties on water wave's propagation which is analogous as the first derivative function of the surface displacement of the water. Here, we will study the effect of Bragg scattering mechanism to the potential velocity. Assume that the water is incompressible and irrotational, and also satisfying linear equation as a governing equation. We apply the multiple scale expansion to solve the boundary value problem analytically. Reflection coefficient as a solution shows that Bragg resonance gives the significant effect to the potential velocity. We found that a larger amplitude disturbance leads to larger reflected wave amplitude. This result explains that the sinusoidal beds can reduce the amplitude of incident wave. At the end, we show that the maximum reflection coefficient occurs when the wavelength of the monochromatic wave is twice the wavelength of sinusoidal beds, which is Bragg condition.

Index Terms— Potential velocity, Bragg resonance

I. INTRODUCTION

Many researcher studies about breakwater design to minimizing disadvantages of tidal waves or tsunami. Breakwater can be in the form of strong and powerful bar with certain size and distance as needed. The optimum bar size can reduce the incoming wave's amplitude optimally[1]. Bottom undulation on seabed can occur naturally and it forms can also split the wave into transmission and reflection. Based on this analysis the author is interested to learn about the effect of the bottom undulation on seabed in minimizing the amplitude of the wave that transmitted to the beach.

Bragg resonance gives a significant influence to reduced wave amplitude. It occurs when a wave propagates through the impermeable sinusoidal beds which wave number is twice the incoming wave numbers[2]. The influences of Bragg resonance and current on the wave propagation over permeable beds have also been studied[3,5]. In another side, LH Wiryanto[6] has also been reviewing propagation on unsteady wave through the permeable beds.

The presence of hard-wall beach on the right of sinusoidal patch will increase the incident wave amplitude that hit the shore much higher, and hence increase the hazard to the shore. The situations will less severe if the shore can absorb the wave partially[4,7].

These researches about the modeling of wave propagation through the bottom undulation on deviation of the water surface. The other factors that can affect on waves propagation is potential velocity of water. The potential velocity is the velocity of water particles; so that the velocity will be different on each position and times. Viska and Wikaria[8] analyze whether potential velocity give the significant effect for wave amplitude or not. The method used to obtain approximations solution is multiple scale asymptotic expansion method. Here, we analyze more parameter on simulation such as amplitude and length of sinusoidal bar.

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II. BOUNDARY VALUE PROBLEM

Consider the depth of water as follows

$$y(x) = h + \varepsilon c(x), \tag{1}$$

where c(x) is a function that representation of bottom topography, h is flat depth, ε is small undimensioneless parameter ($\varepsilon \ll 1$). The parameter ε is used to express that the amplitude of bottom topography is relatively small compared with the flat depth.

Assume that the water is *incompressible* (material density is constant) and *irrotational*, so that we use Laplace equation for governing equation as follows

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0, \qquad -\infty < x < \infty, 0 \leq y \leq h + \varepsilon c(x) \tag{2}$$

$$\frac{\partial \phi}{\partial y} + K\phi = 0, \qquad y = 0 \tag{3}$$

$$\frac{\partial \phi}{\partial n} = 0, \qquad y(x) = h + \varepsilon c(x) \tag{4}$$

where $\phi(x, y)$ is potential velocity of surface wave at position x and time t, $K = \frac{\sigma^2}{g}$ is angular frequency constant of incoming wave with time function $e^{-i\sigma t}$, g is gravitation coefficient , and $\frac{\partial \phi}{\partial n}$ directional derivative at point (x, y).

When a wave meets a different depth, it will scatter into a transmitted and reflected wave. Let us imagine an incident wave running above a flat bottom with a small undulation. During its evolution, there occur many scattering processes.

Assume that the beach on the right of the small undulation beds can absorb wave completely, then the transmitted wave is wave that running to right and reflected wave is the opposite. Therefore, the potential velocity can be written as

ISBN: 978-988-14047-6-3
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

$$\phi(x,y) = \begin{cases} (e^{ik_0x} + Re^{-ik_0x})\cosh\ k_0(h-y), & x \to -\infty \\ Te^{ik_0x}\cosh\ k_0(h-y), & x \to +\infty \end{cases} \quad (5)$$

where R and T are reflected wave and transmitted wave coefficient, respectively.

When the bottom satisfy the condition $\frac{\partial\phi}{\partial n} = 0$ at $y = h + \varepsilon c(x)$, then these condition can be change to first order equation as

$$\frac{\partial\phi}{\partial y} - \varepsilon\left\{c(x)\frac{\partial\phi}{\partial x}\right\} = 0, \qquad y = h \quad (6)$$

III. Multiple Scale Expansion Method

Using multiple scale expansion, we expand

$$\begin{aligned} \phi &= \phi_0 + \varepsilon\phi_1 + O(\varepsilon^2) \\ R &= R_0 + \varepsilon R_1 + O(\varepsilon^2) \\ T &= 1 + \varepsilon T_1 + O(\varepsilon^2) \end{aligned} \quad (7)$$

Substitute equation (7) into (2), (3), (5), and (6) so we get the boundary value problem for orde O(1) and orde O(ε). Here is boundary value problem for order O(1):

$$\frac{\partial^2\phi_0}{\partial x^2} + \frac{\partial^2\phi_0}{\partial y^2} = 0, -\infty < x < \infty, \quad 0 \le y \le h + \varepsilon c(x)$$

$$\frac{\partial\phi_0}{\partial y} + K\phi_0 = 0, \quad y = 0 \quad (9)$$

$$\frac{\partial\phi_1}{\partial y} = h \quad (10)$$

$$\phi_0(x,y) \sim \begin{cases} e^{ik_0x}\cosh k_0(h-y), & x \to -\infty \\ e^{ik_0x}\cosh\ k_0(h-y), & x \to +\infty \end{cases} \quad (11)$$

and for order O(ε).

$$\frac{\partial^2\phi_1}{\partial x^2} + \frac{\partial^2\phi_1}{\partial y^2} = 0 \quad -\infty < x < \infty, \quad 0 \le y \le h + \varepsilon c(x) \quad (12)$$

$$\frac{\partial\phi_1}{\partial y} + K\phi_1 = 0, \qquad y = 0 \quad (13)$$

$$\frac{\partial\phi_1}{\partial n} = ik_0\frac{d}{dx}\{c(x)e^{ik_0x}\} \equiv p(x), \qquad y = h \quad (14)$$

$$\phi_1(x,y) \sim \begin{cases} R_1e^{-ik_0x}\cosh k_0(h-y), & x \to -\infty \\ T_1e^{ik_0x}\cosh\ k_0(h-y), & x \to +\infty \end{cases} \quad (15)$$

Assume that the characteristic of potential velocity at order O(ε) and order O(1) are the same, that is periodically monochromatic wave and expressed as a complex function. Because of that, we use Fourier transform to solve boundary value problem (12) – (15) which is infinite domain. After these transformations, we get the solution of this boundary problem as follows

$$\phi_1(\xi,y) = \int_{-\infty}^{\infty}\frac{\xi\cosh\xi y - K\sinh\xi y}{\xi[\xi\sinh\xi h - K\cosh\xi h]}\Lambda(\xi)e^{-i\xi x}d\xi \quad (16)$$

Let see that equation (16) has a singular solution when $\xi = 0$. Because of that, we use residue theorem of calculus to get reflected and transmitted wave coefficient as

$$\mathrm{R}_1 = \frac{-2ik_0^2}{2k_0h + \sinh\ 2k_0h}\int_{-\infty}^{\infty}c(x)e^{2ik_0x}dx \quad (17)$$

$$\mathrm{T}_1 = \frac{-2ik_0^2}{2k_0h + \sinh\ 2k_0h}\int_{-\infty}^{\infty}c(x)dx \quad (18)$$

IV. Bragg Resonance Condition

Consider that the bottom undulation is a sinusoidal bed, and then c(x) can be written as

$$c(x) = \begin{cases} a\sin(lx), & L1 \le x \le L2 \\ 0, & \text{otherwise} \end{cases} \quad (19)$$

where a is amplitude of sinusoidal bed, l is wave number of sinusoidal beds, and choose

$$L_1 = \frac{-n\pi}{l} \qquad \text{and} \qquad L_2 = \frac{m\pi}{l}$$

So we get

$$R1 = \frac{2k_0a}{2k_0h + \sinh 2k_0h}\,\frac{(-1)^m\left(\frac{2k_0}{l}\right)}{\left(\frac{2k_0}{l}\right)^2}\sin\left(\frac{2k_0m\pi}{l}\right) \quad (20)$$

C.C. Mei, 2004 found that Bragg resonance will occurs when $l = 2k_0$, so we get

$$R1 = \frac{2k_0a}{2k_0h + \sinh 2k_0h}(-1)^m\sin m\pi$$

V. Simulation

There is no relevant data in Indonesia to this research. Because of that, in this paper, analytic simulation using the data of Martha, Bora, & Chakrabarti, 2009 shown in Table 1. We use these data because these data is relevant and more useful for this research. Our researches are the same topic, but use different method for solving the governing equation and numerical method. At the end, we compare the result of this research with them.

Table 1. Data of analytic simulation

Variable	Value
Amplitude of sinusoidal beds/ flat depth (a/h)	0.1
Wave number of sinusoidal beds ✕ flat depth $(l\,xh)$	1
Wave number of sinusoidal beds (l)	1,3,5

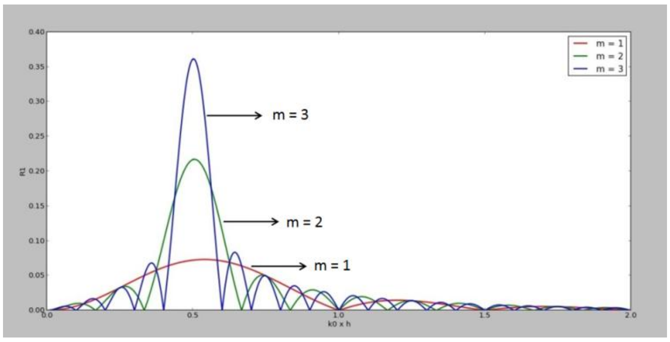


Figure 1. The reflected coefficient (Analytic Solution)

Figure 1 shows the reflected coefficient as a function of incident wave number multiply by flat bottom (k_0h). The reflected coefficient plotting for some cases wave numbers of sinusoidal beds. The reflected coefficient when k_0 = 1, 3, and 5 represented by red, green, and blue line, respectively.

Table 2.The maximum reflection coefficient for different wave number sinusoidal beds

Wave number of sinusoidal beds (l)	Flat depth (h)	k_0 x h	The maximum reflected coefficient (R1 maximum)	Wave number of incident wave (k_0) when R1 Maximum
1	1	0.5	0.073166	0.5
3	1/3	0.5	0.217668	1.5
5	1/5	0.5	0.361306	2.5

After see Figure 1 and table 2, we know that for same value of a/h, the larger wave number of sinusoidal beds leads larger amplitude of reflected wave. Besides that, it shows that the maximum reflected coefficient occur when the wave number of sinusoidal beds is twice the wave number of incident wave.

Table 3.The maximum reflection coefficient for different amplitude of sinusoidal beds

Amplitude of sinusoidal beds	*R*1 Maximum
0.03	0.217416
0.066	0.434832
0.099	0.652248

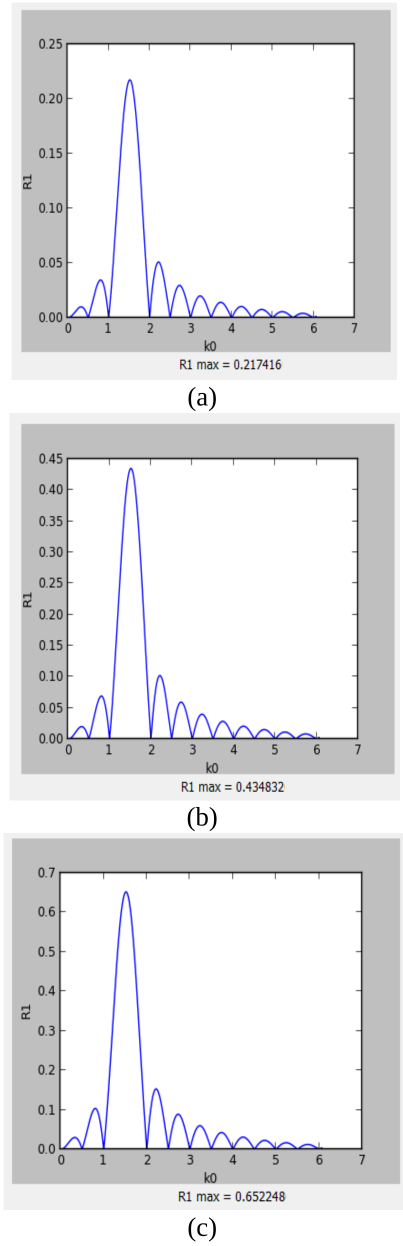
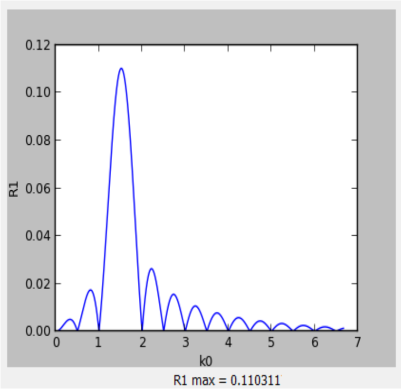


Figure 2. Reflection Coefficient for Different Amplitude: (a) 0.03, (b) 0.066, (c) 0.099

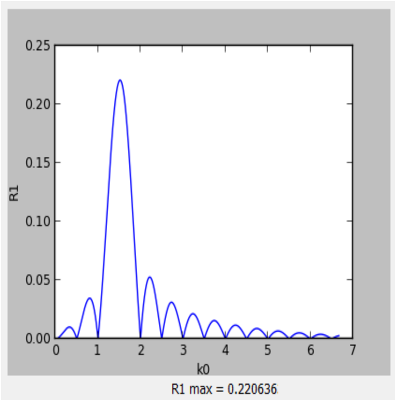
Table 4.The maximum reflection coefficient for different length of sinusoidal beds

Length of sinusoidal beds	*R*1 Maksimum
3.14	0.110311
6.28	0.220636
9.42	0.330487

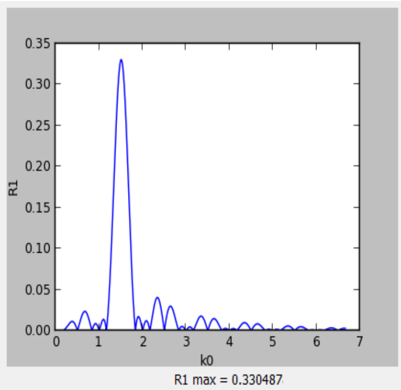
ISBN: 978-988-14047-6-3
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)



(a)



(b)



(c)

Figure 3. Reflection Coefficient for Different Length: (a) 3.14, (b) 6.28, (c) 9.42

From Table 3 and 4, Figure 2 and 3, we know that the larger amplitude and length of sinusoidal beds lead the larger reflection coefficient.

VI. Conclusion

Sinusoidal beds give the effect for surface wave propagation. The solution of wave equation over sinusoidal-beds based on potential velocity obtains by perturbation method and Fourier transform. Sinusoidal beds may lead to Bragg resonance. Bragg resonance occurs when wavelength of incident wave is twice of the wavelength of the periodic bottom disturbance. The larger wave number, amplitude, and length of sinusoidal beds lead larger amplitude of reflected wave and leads smaller amplitude of transmitted wave. It means that the potential velocity of incoming waves decreasing by sinusoidal beds. The maximum reflected coefficient occurs when there is Bragg condition.

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