

Maximizing Procedures Covering Conduct of Examination in a Potential World Class University using Operational Research Technique

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Abstract—One of the good qualities of a world class university is their high standard examination procedures. This paper attempts to optimize the examination procedures in a potential world class university by minimizing the major variables: time, cost and human resources involved in the examination conduct. Covenant University, the best private university in Nigeria, is used as a case study. The examination procedures covering conduct of examination at Covenant University, were modelled as a linear programming with time, cost and human resources as our decision variables. The constraint, including the non-negativity constraint, were carefully formed. The standard form of the model is solved using Simplex method with the aid of a computer software - LIP Solver, which was used to evaluate the feasible solutions from the initial tableau. It was observed that some of the decision variables such as time to correct examination questions and address examination misconducts must be deemphasized, while decision variables such as time to admit students into the examination hall and time to verify students' identity in order to admit them into the examination hall should be given priority in order to optimize the resources related to the examination conduct at Covenant University.

Index Terms—Linear Programming Model, Optimization, Procedures Covering Examination Conduct

I. INTRODUCTION

Optimization is usually used to select the best element based on a set of criteria from a selection of available alternatives. [4,7] It involves finding an alternative with the most cost effective or highest achievable performance under given constraints, by maximizing desired factors and minimizing undesired ones [3,5]. In comparison, maximization means trying to attain the highest or maximum result or outcome without regard to cost or expense.[5,7] Practice of optimization is restricted by the lack of full information, and the lack of time to evaluate what information is available [4]. In computer simulation (modeling) of business problems, optimization is achieved usually by using linear programming techniques of operations research[3]. Linear programming model is a

planning technique which uses mathematical model in maximizing or minimizing appropriate measure to optimize the value of some objective after identifying some constraints [3,7]. Linear programming is a mathematical discipline, developed from the invention of the simplex method by G.B. Dantzig [1], in 1947. Historically, development in linear programming is driven by its applications in economics and management. A method he developed to solve the U.S. air force planning problem. The term linear programming was recommended by Dantzig by T.C. Koopmans in 1951 in place of what it was previously known as "programming in a linear structure". The aim of linear programming is to either minimize or maximize certain variables from a system putting into consideration some constraints, in order to generate the best possible output. [7] The goal of any Linear Programming algorithms is to find the optimum solution of a given problem. The problem is formulated by an objective function, which needs to be minimized or maximized under a set of limits and constraints. The basic requirements of an optimization problem, including linear programming, are the following: An objective function, which is made up of some scalar quantities that need to be minimized or maximized to get the most suitable output. Secondly, a predictive model used to describe the behavior of the system. In an optimization problem they are called constraints and are sets of equations and inequalities. These constraints cover a feasible region that defines limits of performance for the system. Thirdly, Variables that are given in the predictive model must be adjusted to satisfy the constraints [5]. In this paper, the objective function is represented by the examination procedures and their contribution to a successful examination conduct at Covenant University [9]. The optimization problem therefore is that of minimization, with the aim of minimizing time, cost and manpower require for conduct of examination. Minimizing the objective function, therefore, leads to minimizing money involved in conducting examination in the university. Time is money, cost involves money and manpower involves money too

II FORMULATION OF PROBLEMS

A. Decision Variables

Let X_1 represent the number of question papers produced.

Let X_2 represent the invigilators required for each examination.

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Let X_3 represent possession of one of the authorized materials.
Let X_4 represent the time to assign duties to invigilators.
Let X_5 represent the time required for students to be served with the question papers.
Let X_6 represent the required time to admit students into the examination hall.
Let X_7 represent the require time to inspect students for possession of laptops and other multi-media.
Let X_8 represent the duration of stopping the examination after expiration of allowed time..
Let X_9 represent the number of extra answer booklet produced.
Let X_{10} represent the time required to verify students' identity.
Let X_{11} represent number of examination officers/supervisors
Let X_{12} represent duration of time invigilators will stay in the hall after an examination.
Let X_{13} represent the require time to cross check the attendance.
Let X_{14} represent the duration of time required to control disturbances.
Let X_{15} represent the required time to correct examination questions.
Let X_{16} represent the number of answer booklets produced.
Let X_{17} represent the required time to address cases of misconduct.
Let X_{18} represent the required time to handle health issues
Let X_{19} represent the time the invigilators collect the answer booklets at the end of examination.
Let X_{20} represent the time to collect examination materials and prepare the hall for the next examination.

B. Contributions (C_j)

The following are the contributions, obtained by using the weighted score concept:

$C_1=25, C_2=22, C_3=16, C_4=9, C_5=20, C_6=22, C_7=23, C_8=24, C_9=22, C_{10}=22, C_{11}=24, C_{12}=21, C_{13}=18, C_{14}=15, C_{15}=12, C_{16}=12, C_{17}=12, C_{18}=12, C_{19}=16, C_{20}=22,$

C. Resources per unit of a Decision Variable (a_{ij})

Let a_{11} represent the cost of producing one question paper
Let a_{12} represent the cost of producing question paper for one examination
Let a_{21} represent amount paid to an invigilator per course (nonacademic)
Let a_{22} represent amount paid to an invigilator for all exam (nonacademic)
Let a_{31} represent the time to check one student for authorized materials
Let a_{32} represent the time to check one student for authorized materials throughout the examination
Let a_{41} represent the time to assign duty to one invigilator
Let a_{42} represent the time to assign duty to one invigilator throughout the examination
Let a_{51} represent the time to place a question paper to be on a student's desk.
Let a_{52} represent the time to place a question paper to be on a student's desk throughout the examination

Let a_{61} represent the required time to admit one student into the examination hall.
Let a_{62} represent the required time to admit one student into the examination hall throughout the examination
Let a_{71} represent the require time to inspect one student for possession of laptops and other multi-media.
Let a_{72} represent the require time inspect one student for possession of laptops and other multi-media throughout the examination.
Let a_{81} represent the duration of one course examination.
Let a_{91} represent the cost of answer booklet produced for one examination.
Let a_{92} represent the cost of answer booklet produced throughout the examination.
Let a_{101} represent the time required to verify one students' identity.
Let a_{102} represent the time required to verify one students' identity throughout the examination.
Let a_{111} represent cost of refreshment for one examination officer.
Let a_{122} represent duration of time invigilators will stay in the hall for one course.
Let a_{131} represent required time to verify one student name on the attendance sheet
Let a_{132} represent required time to verify one student name on the attendance sheet throughout the exam
Let a_{141} represent the required time to report disturbances for each examination
Let a_{142} represent the required time to control disturbances throughout the examination
Let a_{151} represent the required time to correct one examination question.
Let a_{152} represent the required time to correct one examination question throughout the examination.
Let a_{161} represent the cost of one supplementary paper produced
Let a_{162} represent the cost of supplementary paper produced for one course.
Let a_{171} represent the required time for one student to fill a misconduct form
Let a_{172} represent the required time for one student to fill a misconduct form throughout the examination.
Let a_{181} represent the required time to handle health issues for each examination
Let a_{182} represent the required time to handle health issues throughout the examination.
Let a_{191} represent time an invigilator announces the end of exam for one course
Let a_{192} represent time an invigilator announces the end of exam throughout the examination
Let a_{201} represent the time required to prepare the hall for the next paper
Let a_{202} represent the time required to prepare the hall for next paper throughout the examination

D. Available Resources (b_i)

Let b_1 represent the cost of producing one question paper
Let $b_{1.0}$ represent the cost of producing question paper for one examination
Let b_2 represent amount paid to an invigilator per course (nonacademic)
Let $b_{2.0}$ represent amount paid to an invigilator for all examination (nonacademic)

Let b3 represent the time to check one student for authorized materials
 Let b3.0 represent the time to check one student for authorized materials throughout
 Let b4 represent the time to assign duty to one invigilator
 Let b4.0 represent the time to assign duty to one invigilator throughout
 Let b5 represent the time to place a question paper to be on a student's desk.
 Let b5.0 represent the time to place a question paper to be on a student's desk
 Let b6 represent the available time to admit one student into the examination hall.
 Let b6.0 represent the available time to admit one student into the examination hall
 Let b7 represent the available time inspect one student for possession of laptops and
 Let b7.0 represent the available time inspect one student for possession of laptops and
 Let b8 represent the duration of one course examination.
 Let b9 represent the cost of answer booklet produced for one examination
 Let b9.0 represent the cost of answer booklet produced throughout the examination.
 Let b10 represent the time available to verify one students' identity.
 Let b10.0 represent the time available to verify one students' identity throughout
 Let b11 represent cost of refreshment for one examination officer.
 Let b12 represent duration of time invigilators will stay in the hall for one course.
 Let b13 represent available time to verify one student name on the attendance sheet
 Let b13.0 represent available time to verify one student name on the attendance sheet
 Let b14 represent the available time to report disturbances for each examination
 Let b14.0 represent the available time to control disturbances throughout
 Let b15 represent the available time to correct one examination question.
 Let b15.0 represent the available time to correct one examination question throughout
 Let b16 represent the cost of one supplementary paper produced
 Let b16.0 represent the cost of supplementary paper produced for one course.
 Let b17 represent the available time for one student to fill a misconduct form
 Let b17.0 represent the available for one student time to fill a misconduct form
 Let b18 represent the available time to handle health issues for each examination
 Let b18.0 represent the available time to handle health issues throughout
 Let b19 represent time an invigilator announces the end of exam for one course
 Let b19.0 represent time an invigilator announces the end of exam throughout
 Let b20 represent the time available to prepare the hall for the next paper

Let b20.0 represent the time available to prepare the hall for next paper throughout

E. Examination procedures resources and utilization

For this study, we used a typical unit, Management information system (MIS), in the university as a case study. Also the average number of students, gotten by taking the average of the total number of students for all the levels was used for computation. From the data gathered and subsequent computations, the following were some of the values obtained:

b1=650, b1.0= 25000, b2=15000, b2.0=30000, b3=300, b3.0=3600, b4=300, b4.0=3600, b5=300, b5.0=3600, b6=300, b6.0=3600, b7=300, b7.0=3600, b8=345600, b9=2800, b9.0=122700, b10=300, b10.0=3600, b11= 2500, b12=345600, b13=1800, b13.0=21600, b14=120, b14.0= b15=1800, b15.0=21600, b16=400, b16.0=17600, b17=420seconds, b17.0= 5040 seconds, b18=180seconds, b18.0=2160seconds

Also, a₁ = 6 naira, a₉ = 28, a₁₆ = 3.46, a₂ = 15000, a₁₁ = 500, a₃ = 30, a₆ = 5, a₇ = 15, a₁₀ = 10, a₈ = 5000, a₄ = 60, a₅ = 5, a₁₃ = 18, a₁₅ = 300, a₁₂ = 10800, a₁₄ = 1800, a₁₇ = 1800, a₁₈ = 1800, a₁₉ = 900, a₂₀ = 1800

F. Budget for production of examination papers

The budget for production of examination papers is as follows:

Question paper = 144000 naira
 Answer booklet = 250000 naira
 Supplementary sheet = 27680
 Total budget for one course = 421,680
 Total budget for production of paper = 4,500,000

G. Objective Function

$$\text{Minimize } Z = \sum_{j=1}^{20} C_j X_j$$

$$\text{Minimize } Z = C_1 X_1 + C_2 X_2 + C_3 X_3 + C_4 X_4 + C_5 X_5 + C_6 X_6 + C_7 X_7 + C_8 X_8 + C_9 X_9 + C_{10} X_{10} + C_{11} X_{11} + C_{12} X_{12} + C_{13} X_{13} + C_{14} X_{14} + C_{15} X_{15} + C_{16} X_{16} + C_{17} X_{17} + C_{18} X_{18} + C_{19} X_{19} + C_{20} X_{20}$$

Substituting values of the contributions into the above objective function we have:

$$\text{Minimize } Z = 25X_1 + 22X_2 + 16X_3 + 9X_4 + 20X_5 + 22X_6 + 23X_7 + 24X_8 + 22X_9 + 22X_{10} + 24X_{11} + 21X_{12} + 18X_{13} + 15X_{14} + 12X_{15} + 12X_{16} + 12X_{17} + 12X_{18} + 16X_{19} + 22X_{20}$$

H. Constraints

$$\begin{aligned} a_1 X_1 + a_9 X_9 + a_{16} X_{16} &\leq b_1 + b_9 + b_{16} \\ a_{12} X_1 + a_{92} X_9 + a_{162} X_{16} &\leq b_{12} + b_{92} + b_{162} \\ a_2 X_2 + a_{11} X_{11} &\leq b_2 + b_{11} \\ a_8 X_8 + a_{12} X_{12} &\leq b_8 + b_{12} \\ a_3 X_3 + a_4 X_4 + a_5 X_5 + a_6 X_6 + a_7 X_7 + a_{10} X_{10} &\leq b_3 + b_4 + b_5 + b_6 + b_7 + b_{10} \\ a_{32} X_3 + a_{42} X_4 + a_{52} X_5 + a_{62} X_6 + a_7 X_7 + a_{102} X_{10} &\leq b_{32} + b_{42} + b_{52} + b_{62} + b_{72} + b_{102} \end{aligned}$$

$$\begin{aligned}
 a_{13}X_{13} + a_{15}X_{15} &\leq b_{13} + b_{15} \\
 a_{132}X_{13} + a_{152}X_{15} &\leq b_{132} + b_{152} \\
 a_{14}X_{14} + a_{17}X_{17} + a_{18}X_{18} &\leq b_{14} + b_{17} + b_{18} \\
 a_{142}X_{14} + a_{172}X_{17} + a_{182}X_{18} &\leq b_{142} + b_{172} + b_{182} \\
 a_{19}X_{19} + a_{20}X_{20} &\leq b_{19} + b_{20} \\
 a_{192}X_{19} + a_{202}X_{20} &\leq b_{192} + b_{202} \\
 a_1X_1 &\leq b_1 \\
 a_9X_9 &\leq b_9 \\
 a_{16}X_{16} &\leq b_{16} \\
 a_2X_2 &\leq b_2 \\
 a_{11}X_{11} &\leq b_{11} \\
 a_{2.1}x_2 + a_{11}x_{11} &\leq b_2 + b_{11} \\
 a_{22}X_2 &\leq b_{22} \\
 a_3X_3 &\leq b_3 \\
 a_4X_4 &\leq b_4 \\
 a_5X_5 &\leq b_5 \\
 a_6X_6 &\leq b_6 \\
 a_7X_7 &\leq b_7 \\
 a_{10}X_{10} &\leq b_{10} \\
 a_{13}X_{13} &\leq b_{13} \\
 a_{15}X_{15} &\leq b_{15} \\
 a_{132}X_{13} &\leq b_{132} \\
 a_{152}X_{15} &\leq b_{152}
 \end{aligned}$$

Substituting the values for the a_{ij} 's and b_i 's we have,

$$\begin{aligned}
 6.08X_1 + 28X_9 + 3.6X_{16} &\leq 4000 \\
 620.16X_1 + 2856X_9 + 367.2X_{16} &\leq 165300 \\
 750X_2 + 500X_{11} &\leq 17500 \\
 10800X_8 + 10800X_{12} &\leq 345600 \text{ seconds or } 5760 \text{ minutes} \\
 2.94X_3 + 2.94X_4 + 2.94X_5 + 2.94X_6 + 2.94X_7 + 2.94X_{10} &\leq 1800 \\
 35.28X_3 + 35.28X_4 + 35.28X_5 + 35.28X_6 + 35.28X_7 + 35.28X_{10} &\leq 21600 \\
 17.65X_{13} + 300X_{15} &\leq 1800 \\
 211.8X_{13} + 3600X_{15} &\leq 21600 \\
 120X_{14} + 420X_{17} + 180X_{18} &\leq 10800 \\
 1440X_{14} + 5040X_{17} + 2160X_{18} &\leq 345600 \\
 180X_{19} + 1800X_{20} &\leq 10800 \\
 2160X_{19} + 21600X_{20} &\leq 345600 \\
 6.08 X_1 &\leq 625 \\
 28X_9 &\leq 2900 \\
 3.6X_{16} &\leq 370 \\
 750X_2 &\leq 3000 \\
 500X_{11} &\leq 14500 \\
 15000 x_2 + 500x_{11} &\leq 74500 \\
 15000X_2 &\leq 60000 \\
 2.94X_3 &\leq 352.8 \\
 2.94X_4 &\leq 352.8 \\
 2.9X_5 &\leq 352.8 \\
 2.94X_6 &\leq 352.8 \\
 2.94X_7 &\leq 352.8 \\
 2.94X_{10} &\leq 352.8 \\
 17.65X_{13} &\leq 1800 \\
 300X_{15} &\leq 1800 \\
 211.8X_{13} &\leq 21600 \\
 3600X_{15} &\leq 21600
 \end{aligned}$$

II. PROBLEM SOLUTION

A. Standardized model

$$\text{Minimize } Z = 25X_1 + 22X_2 + 16X_3 + 9X_4 + 20X_5 + 22X_6 + 23X_7 + 24X_8 + 22X_9 + 22X_{10} + 24X_{11} + 21X_{12} + 18X_{13} + 15X_{14} + 12X_{15} + 12X_{16} + 12X_{17} + 12X_{18} + 16X_{19} + 22X_{20}$$

Subject to

$$\begin{aligned}
 6.08X_1 + 28X_9 + 3.6X_{16} + S_1 &= 4000 \\
 620.16X_1 + 2856X_9 + 367.2X_{16} + S_2 &= 165300 \\
 750X_2 + 500X_{11} + S_3 &= 17500 \\
 10800X_8 + 10800X_{12} + S_4 &= 345600 \\
 2.94X_3 + 2.94X_4 + 2.94X_5 + 2.94X_6 + 2.94X_7 + 2.94X_{10} + S_5 &= 1800 \\
 35.28X_3 + 35.28X_4 + 35.28X_5 + 35.28X_6 + 35.28X_7 + 35.28X_{10} + S_6 &= 21600 \\
 17.65X_{13} + 300X_{15} + S_7 &= 1800 \\
 211.8X_{13} + 3600X_{15} + S_8 &= 21600 \\
 120X_{14} + 420X_{17} + 180X_{18} + S_9 &= 10800 \\
 1440X_{14} + 5040X_{17} + 2160X_{18} + S_{10} &= 345600 \\
 180X_{19} + 1800X_{20} + S_{11} &= 10800 \\
 2160X_{19} + 21600X_{20} + S_{12} &= 345600 \\
 6.08 X_1 + S_{13} &= 625 \\
 28X_9 + S_{14} &= 2900 \\
 3.6X_{16} + S_{15} &= 370 \\
 750X_2 + S_{16} &= 3000 \\
 500X_{11} + S_{17} &= 14500 \\
 15000 x_2 + 500x_{11} + S_{18} &= 74500 \\
 15000X_2 + S_{19} &= 60000 \\
 2.94X_3 + S_{20} &= 352.8 \\
 2.94X_4 + S_{21} &= 352.8 \\
 2.9X_5 + S_{22} &= 352.8 \\
 2.94X_6 + S_{23} &= 352.8 \\
 2.94X_7 + S_{24} &= 352.8 \\
 2.94X_{10} + S_{25} &= 352.8 \\
 17.65X_{13} + S_{26} &= 1800 \\
 300X_{15} + S_{27} &= 1800 \\
 211.8X_{13} + S_{28} &= 21600 \\
 3600X_{15} + S_{29} &= 21600 \\
 x_i &\geq 0 \text{ (where } i = 1, 2, 3, \dots, 20) \\
 \text{where } S_i \text{ (} i = 1, 2, 3, \dots, 29) &\text{ are the slack variables.}
 \end{aligned}$$

IV. RESULTS AND DISCUSSION

After solving the above initial tableau with a computer software “LINDO”, the following values of the decision variables X1, ..., X20, were obtained:

VARIABLE	VALUE
X1	102.796051
X2	4.000000
X3	120.000000
X4	10.589725
X5	121.655174
X6	120.000000
X7	120.000000
X8	32.000000
X9	22.342438
X10	120.000000
X11	29.000000
X12	0.000000
X13	101.983002
X14	90.000000
X15	0.000000
X16	102.777779
X17	0.000000
X18	0.000000
X19	60.000000
X20	0.000000

From the above values we can deduce the following: Firstly, substituting these values into the objective function gave us 22480.87, which implies the optimal value. This suggests the minimum units of time, cost and manpower that should be involved in the examination conduct procedures in order to achieve the optimal result. Specifically, about 103 question papers should be produced, at most 4 invigilators should be used for each paper, and all the students should be in possession of the authorized materials for the examination. It should not take more than 10.6 seconds to assign duties to each invigilator, and not more than 122seconds should be spent to serve all the students with question papers. Students should be admitted into the examination hall within 120 seconds. Also 120 seconds should be used to check for unauthorized material such as laptops, tablets, phones and so on that might be in possession of the students. Every student should stop written within 32 seconds after expiration of an examination, and about 22 extra answer booklets should be produced. 120 seconds should be used to check students' identity. Not more than 29 examination officers/ supervisors should be in charge of an examination. To cross check the attendance should not take more than 102seconds. Maximum of 90 seconds should be used all through the examination to control disturbance during the examination. Just like question papers, 103 answer booklets should be produced for an examination. The invigilators should spend more than one minute to collect the answer booklets at the end of an examination. However, there might be no need to prepare the hall for next paper, as the same hall can be used for the next paper immediately. There should not be any special time set aside to handle health issues, as health

problems rarely occur during the examinations, and when they occur, they can handled without disturbing the flow of

an examination. Cases of misconduct can be handled without affecting the time allotted for an examination. Examination questions should be looked through by the course lecturer before the examination papers get to the examination hall. This will avoid time wasting in the examination hall. Invigilators are expected to leave the examination hall immediately after collecting the answer booklets from the students. It also be deduced from the results that in order to optimize the time for an examination, there should not be any special time assigned for the following activities; X₁₂, X₁₅, X₁₇, X₁₈, X₂₀, during the examination period. Such activities can either be carried out simultaneously with other specific activities or carried out some other time outside the examination period.

V. CONCLUSION

Interest was on making best use of the three major resources namely: Time, Cost and Manpower, needed for conducting examination in order to optimize the procedures covering the conduct of examination in Covenant University. The analysis is carried out using mathematical programming approach. The peculiar situation considered was modeled as a linear programming problem with the sole objective of minimizing the objective function. The objective function is made up of the resources mentioned above resulting in maximizing the procedures considered. Clearly, almost the exact amount of the resources, time, cost and manpower, needed should be used if the procedures covering the conduct of examination in Covenant University is to be optimized.

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