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Trajectory Tracking Controller Design for A Tricycle Robot Using Piecewise Multi-Linear Models

Tadanari Taniguchi and Michio Sugeno

***Abstract*—This paper deals a tracking trajectory controller design of a tricycle robot as a non-holonomic system with a piecewise multi-linear (PML) model. The approximated model is fully parametric. Input-output (I/O) dynamic feedback linearization is applied to stabilize PML control system. We also apply a method for a tracking control based on PML models to the tricycle robot. Although the controller is simpler than the conventional I/O feedback linearization controller, the control performance based on PML model is the same as the conventional one. Examples are shown to confirm the feasibility of our proposals by computer simulations.**

***Index Terms*—piecewise model, tracking trajectory conrol, dynamic feedback linearization.**

I. Introduction

WE propose the tracking trajectory control of a tricycle robot using dynamic feedback linearization based on piecewise multi-linear (PML) models. Wheeled mobile robots are completely controllable. However they cannot be stabilized to a desired position using time invariant continuous feedback control [1]. The wheeled mobile robot control systems have a non-holonomic constraint. Non-holonomic systems are much more difficult to control than holonomic ones. Many methods have been studied for the tracking control of wheeled robots. The backstepping control methods are proposed in (e.g. [2], [3]). The sliding mode control methods are proposed in (e.g., [4], [5]), and also the dynamic feedback linearization methods are in (e.g., [6], [7], [8]). For non-holonomic robots, it is never possible to achieve exact linearization via static state feedback [9]. It is shown that the dynamic feedback linearization is an efficient design tool to solve the trajectory tracking and the setpoint regulation problem in [6], [7].

In this paper, we consider PML model as a piecewise approximation model of the tricycle robot dynamics. The model is built on hyper cubes partitioned in state space and is found to be bilinear (bi-affine) [10], so the model has simple nonlinearity. The model has the following features: 1) The PML model is derived from fuzzy if-then rules with singleton consequents. 2) It has a general approximation capability for nonlinear systems. 3) It is a piecewise nonlinear model and second simplest after the piecewise linear (PL) model. 4) It is continuous and fully parametric. The stabilizing conditions are represented by bilinear matrix inequalities (BMIs) [11], therefore, it takes long computing time to obtain a stabilizing controller. To overcome these difficulties, we have derived stabilizing conditions [12], [13], [14] based on feedback linearization, where [12] and [14] apply input-output linearization and [13] applies full-state linearization.

We propose a dynamic feedback linearization for PML control system and apply the tracking control [15] to a tricycle robot system. The control system has the following features: 1) Only partial knowledge of vertices in piecewise regions is necessary, not overall knowledge of an objective plant. 2) These control systems are applicable to a wider class of nonlinear systems than conventional I/O linearization. 3) Although the controller is simpler than the conventional I/O feedback linearization controller, the tracking performance based on PML model is the same as the conventional one. Wheeled robot dynamics has some trigonometric functions. The trigonometric functions are smooth functions and of class C^∞. The PML models are not of class of C^∞. In the tricycle robot control, we have to calculate the third derivatives of the output. Therefore the derivative PML models lose some dynamics. Thus we propose the derivative PML models of the trigonometric functions.

This paper is organized as follows. Section II introduces the canonical form of PML models. Section III presents a dynamic feedback linearization of the car-like robot. Section IV proposes a tracking controller design using dynamic feedback linearization based on PML model of the tricycle robot. Section V shows examples demonstrating the feasibility of the proposed methods. Finally, section VI summarizes conclusions.

II. Canonical Forms of Piecewise Bilinear Models

A. Open-Loop Systems

In this section, we introduce PML models suggested in [10]. We deal with the two-dimensional case without loss of generality. Define vector $d(\sigma,\tau)$ and rectangle $R_{\sigma\tau}$ in two-dimensional space as $d(\sigma,\tau) \equiv (d_1(\sigma), d_2(\tau))^T$,

$$R_{\sigma\tau} \equiv [d_1(\sigma), d_1(\sigma+1)] \times [d_2(\tau), d_2(\tau+1)]. \quad (1)$$

σ and τ are integers: $-\infty < \sigma, \tau < \infty$ where $d_1(\sigma) < d_1(\sigma+1), d_2(\tau) < d_2(\tau+1)$ and $d(0,0) \equiv (d_1(0), d_2(0))^T$. Superscript ^{T} denotes a *transpose* operation.

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For $x \in R_{\sigma\tau}$, the PML system is expressed as

$$\begin{cases} \dot{x} = \sum_{i=\sigma}^{\sigma+1}\sum_{j=\tau}^{\tau+1} \omega_1^i(x_1)\omega_2^j(x_2)f_o(i,j), \\ x = \sum_{i=\sigma}^{\sigma+1}\sum_{j=\tau}^{\tau+1} \omega_1^i(x_1)\omega_2^j(x_2)d(i,j), \end{cases} \quad (2)$$

where $f_o(i,j)$ is the vertex of nonlinear system $\dot{x} = f_o(x)$,

$$\begin{cases} \omega_1^\sigma(x_1) = (d_1(\sigma+1) - x_1)/(d_1(\sigma+1) - d_1(\sigma)), \\ \omega_1^{\sigma+1}(x_1) = (x_1 - d_1(\sigma))/(d_1(\sigma+1) - d_1(\sigma)), \\ \omega_2^\tau(x_2) = (d_2(\tau+1) - x_2)/(d_2(\tau+1) - d_2(\tau)), \\ \omega_2^{\tau+1}(x_2) = (x_2 - d_2(\tau))/(d_2(\tau+1) - d_2(\tau)), \end{cases} \quad (3)$$

and $\omega_1^i(x_1), \omega_2^j(x_2) \in [0,\ 1]$. In the above, we assume $f(0,0) = 0$ and $d(0,0) = 0$ to guarantee $\dot{x} = 0$ for $x = 0$.

A key point in the system is that state variable x is also expressed by a convex combination of $d(i,j)$ for $\omega_1^i(x_1)$ and $\omega_2^j(x_2)$, just as in the case of $\dot{x}$. As seen in equation (3), x is located inside $R_{\sigma\tau}$ which is a rectangle: a hypercube in general. That is, the expression of x is polytopic with four vertices $d(i,j)$. The model of $\dot{x} = f(x)$ is built on a rectangle including x in state space, it is also polytopic with four vertices $f(i,j)$. We call this form of the canonical model (2) parametric expression.

B. Closed-Loop Systems

We consider a two-dimensional nonlinear control system.

$$\begin{cases} \dot{x} = f_o(x) + g_o(x)u(x), \\ y = h_o(x). \end{cases} \quad (4)$$

The PML model (5) is constructed from a nonlinear system (4).

$$\begin{cases} \dot{x} = f(x) + g(x)u(x), \\ y = h(x), \end{cases} \quad (5)$$

where

$$\begin{cases} f(x) = \sum_{i=\sigma}^{\sigma+1}\sum_{j=\tau}^{\tau+1} \omega_1^i(x_1)\omega_2^j(x_2)f_o(i,j), \\ g(x) = \sum_{i=\sigma}^{\sigma+1}\sum_{j=\tau}^{\tau+1} \omega_1^i(x_1)\omega_2^j(x_2)g_o(i,j), \\ h(x) = \sum_{i=\sigma}^{\sigma+1}\sum_{j=\tau}^{\tau+1} \omega_1^i(x_1)\omega_2^j(x_2)h_o(i,j), \\ x = \sum_{i=\sigma}^{\sigma+1}\sum_{j=\tau}^{\tau+1} \omega_1^i(x_1)\omega_2^j(x_2)d(i,j), \end{cases} \quad (6)$$

and $f_o(i,j)$, $g_o(i,j)$, $h_o(i,j)$ and $d(i,j)$ are vertices of the nonlinear system (4). The modeling procedure in region $R_{\sigma\tau}$ is as follows:

1) Assign vertices $d(i,j)$ for $x_1 = d_1(\sigma)$, $d_1(\sigma+1)$, $x_2 = d_2(\tau)$, $d_2(\tau+1)$ of state vector x, then partition state space into piecewise regions (see Fig. 1).
2) Compute vertices $f_o(i,j)$, $g_o(i,j)$ and $h_o(i,j)$ in equation (6) by substituting values of $x_1 = d_1(\sigma)$, $d_1(\sigma+1)$ and $x_2 = d_2(\tau)$, $d_2(\tau+1)$ into original nonlinear functions $f_o(x)$, $g_o(x)$ and $h_o(x)$ in the system (4). Fig. 1 shows the expression of $f(x)$ and $x \in R_{\sigma\tau}$.

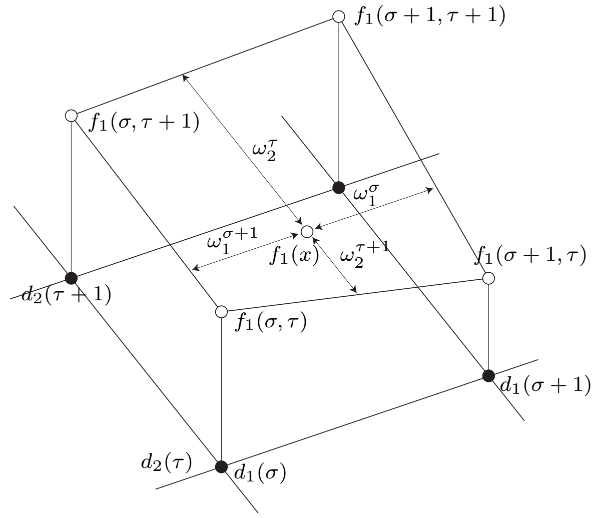

Fig. 1. Piecewise region ($f_1(x) = \sum_{i=\sigma}^{\sigma+1}\sum_{j=\tau}^{\tau+1}\omega_1^i\omega_2^j f_1(i,j)$, $x \in R_{\sigma\tau}$)

The overall PML model is obtained automatically when all vertices are assigned. Note that $f(x)$, $g(x)$ and $h(x)$ in the PML model coincide with those in the original system at vertices of all regions.

III. Dynamic Feedback Linearization of Tricycle Robot

We consider a tricycle robot model.

$$\begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \\ \dot{\psi} \end{pmatrix} = \begin{pmatrix} \cos\theta \\ \sin\theta \\ \frac{1}{L}\tan\psi \\ 0 \end{pmatrix} u_1 + \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} u_2, \quad (7)$$

where x and y are the position coordinates of the center of the rear wheel axis, θ is the angle between the center line of the car and the x axis, ψ is the steering angle with respect to the car. The control inputs are represented as

$$u_1 = v_s \cos\psi$$
$$u_2 = \dot{\psi},$$

where v_s is the driving speed. Fig 2 shows the kinematic model of tricycle robot. The steering angle ψ is constrained by

$$\|\psi\| \le M,\ 0 < M < \pi/2.$$

The constraint [8] is represented as

$$\psi = M \tanh w,$$

where w is an auxiliary variable. Thus we get

$$\dot{\psi} = M\text{sech}^2 w \mu_2 = u_2,$$
$$\dot{w} = \mu_2$$

We substitute the equations of ψ and w into the tricycle robot model. The model is obtained as

$$\begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \\ \dot{w} \end{pmatrix} = \begin{pmatrix} \cos\theta \\ \sin\theta \\ \frac{1}{L}\tan(M\tanh w) \\ 0 \end{pmatrix} u_1 + \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \mu_2 \quad (8)$$

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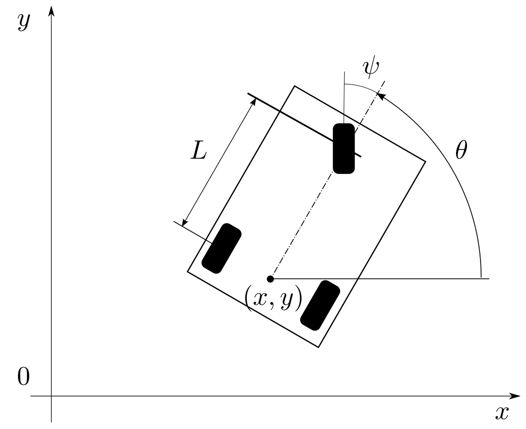

Fig. 2. Kinematic model of tricycle robot

In this case, we consider $\eta = (x, y)^T$ as the output, the time derivative of η is calculated as

$$\dot{\eta} = \begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} = \begin{pmatrix} \cos\theta & 0 \\ \sin\theta & 0 \end{pmatrix} \begin{pmatrix} u_1 \\ \mu_2 \end{pmatrix}.$$

The linearized system of (8) at any points (x, y, θ, w) is clearly not controllable and the only u_1 affects $\dot{\eta}$. To proceed, we need to add some integrators of the input u_1. Using dynamic compensators as

$$\dot{u}_1 = \nu_1, \quad \dot{\nu}_1 = \mu_1,$$

the tricycle robot model (8) can be dynamic feedback linearizable. The extended model is obtained as

$$\begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \\ \dot{w} \\ \dot{u}_1 \\ \dot{\nu}_1 \end{pmatrix} = \begin{pmatrix} u_1 \cos\theta \\ u_1 \sin\theta \\ u_1 \frac{1}{L} \tan(M \tanh w) \\ 0 \\ \nu_1 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \mu_1 + \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} \mu_2 \quad (9)$$

The time derivative of $\dot{\eta}$ is calculated as

$$\ddot{\eta} = \begin{pmatrix} L_f^2 h_1 \\ L_f^2 h_2 \end{pmatrix} = \begin{pmatrix} \nu_1 \cos\theta - u_1^2 \frac{1}{L} \tan(M \tanh w) \sin\theta \\ \nu_1 \sin\theta + u_1^2 \frac{1}{L} \tan(M \tanh w) \cos\theta \end{pmatrix},$$

where $(h_1, h_2) = (x, y)$. Since the controller (μ_1, μ_2) doesn't appear in the equation $\dot{\eta}$, we continue to calculate the time derivative of $\ddot{\eta}$. Then we get

$$\begin{aligned} \eta^{(3)} =& L_f^3 h + L_g L_f^2 h \mu \\ =& \begin{pmatrix} L_f^3 h_1 \\ L_f^3 h_2 \end{pmatrix} + \begin{pmatrix} L_{g_1} L_f^2 h_1 & L_{g_2} L_f^2 h_1 \\ L_{g_1} L_f^2 h_2 & L_{g_2} L_f^2 h_2 \end{pmatrix} \begin{pmatrix} \mu_1 \\ \mu_2 \end{pmatrix}. \quad (10) \end{aligned}$$

Equation (10) shows clearly that the system is input-output linearizable because state feedback control

$$\mu = -(L_g L_f^2 h)^{-1} L_f^3 h + (L_g L_f^2 h)^{-1} v$$

reduces the input-output map to $y^{(3)} = v$.

The matrix $L_g L_f^2 h$ multiplying the modified input (μ_1, μ_2) is non-singular if $u_1 \neq 0$. Since the modified input is obtained as (μ_1, μ_2), the integrator with respect to the input v is added to the original input (u_1, u_2). Finally, the stabilizing controller of the tricycle robot system (7) is presented as a dynamic feedback controller:

$$\begin{cases} \dot{u}_1 = \nu_1, \ \dot{\nu}_1 = \mu_1, \\ u_2 = M \mathrm{sech}^2 w \mu_2 \end{cases} \quad (11)$$

IV. PML Modeling and Tracking Controller Design of the Tricycle Robot Model

A. *PML Model of the Tricycle Robot Model*

We construct PML model of the tricycle robot system (9). The state spaces of θ and w in the tricycle robot model (9) are divided by the 13 vertices $x_3 \in \{-\pi, -5\pi/6, \ldots, \pi\}$ and the 13 vertices $x_4 \in \{-3.0, -2.5, \ldots, 3.0\}$. The state variable is $x = (x_1, x_2, x_3, x_4, x_5, x_6)^T = (x, y, \theta, w, u_1, v_1)^T$.

$$\dot{x} = \begin{pmatrix} \sum_{i_3=\sigma_3}^{\sigma_3+1} w_3^{i_3}(x_3) f_1(d_3(i_3)) x_5 \\ \sum_{i_3=\sigma_3}^{\sigma_3+1} w_3^{i_3}(x_3) f_2(d_3(i_3)) x_5 \\ \sum_{i_4=\sigma_4}^{\sigma_4+1} w_4^{i_4}(x_4) f_3(d_4(i_4)) x_5 \\ 0 \\ x_6 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \mu_1 + \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} \mu_2. \quad (12)$$

We can construct PML models with respect to $f_1(x)$, $f_2(x)$ and $f_3(x)$. The PML model structures are independent of the vertex positions x_5 and x_6 since x_5 and x_6 are the linear terms. This paper constructs the PML models with respect to the nonlinear terms of x_3 and x_4.

Note that trigonometric functions of the tricycle robot (9) are smooth functions and are of class C^∞. The PML models are not of class C^∞. In the tricycle robot control, we have to calculate the third derivatives of the output y. Therefore the derivative PML models lose some dynamics. In this paper we propose the derivative PML models of the trigonometric functions.

B. *Tracking Controller Design Using Dynamic Feedback Linearization Based on PML Model*

We define the output as $\eta = (x_1, x_2)^T$ in the same manner as the previous section, the time derivative of η is calculated as

$$\dot{\eta} = \begin{pmatrix} L_{f_p} h_1 \\ L_{f_p} h_2 \end{pmatrix} = \begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \sum_{i_3=\sigma_3}^{\sigma_3+1} w_3^{i_3}(x_3) \begin{pmatrix} f_1(d_3(i_3)) x_5 \\ f_2(d_3(i_3)) x_5 \end{pmatrix}$$

where the vertices are $f_1(d_3(i_3)) = \cos d_3(i_3)$ and $f_2(d_3(i_3)) = \sin d_3(i_3)$. The time derivative of η doesn't contain the control inputs (μ_1, μ_2). We calculate the time derivative of $\dot{\eta}$. We get

$$\begin{aligned} \ddot{\eta}_1 =& L_{f_p}^2 h_1 = \sum_{i_3=\sigma_3}^{\sigma_3+1} w_3^{i_3}(x_3) f_1(d_3(i_3)) x_6 \\ &+ \sum_{i_3=\sigma_3}^{\sigma_3+1} w_3^{i_3}(x_3) f_1'(d_3(i_3)) \sum_{i_4=\sigma_4}^{\sigma_4+1} w_4^{i_4}(x_4) f_3(d_4(i_4)) x_5^2, \\ \ddot{\eta}_2 =& L_{f_p}^2 h_2 = \sum_{i_3=\sigma_3}^{\sigma_3+1} w_3^{i_3}(x_3) f_2(d_3(i_3)) x_6 \\ &+ \sum_{i_3=\sigma_3}^{\sigma_3+1} w_3^{i_3}(x_3) f_2'(d_3(i_3)) \sum_{i_4=\sigma_4}^{\sigma_4+1} w_4^{i_4}(x_4) f_3(d_4(i_4)) x_5^2, \end{aligned}$$

where $f_3(d_4(i_4)) = \tan(M \tanh d_4(i_4))/L$. We continue to calculate the time derivative of $\ddot{\eta}$. We get

$$\begin{aligned}
\eta_1^{(3)} =& L_{f_p}^3 h_1 + L_{g_1} L_{f_p}^2 h_1 \mu_1 + L_{g_2} L_{f_p}^2 h_1 \mu_2 \\
=& x_5^3 \sum_{i_3=\sigma_3}^{\sigma_3+1} w_3^{i_3}(x_3) f_1^{''}(d_3(i_3)) \left(\sum_{i_4=\sigma_4}^{\sigma_4+1} w_4^{i_4}(x_4) f_3(d_4(i_4)) \right)^2 \\
&+ 3x_5 x_6 \sum_{i_3=\sigma_3}^{\sigma_3+1} w_3^{i_3}(x_3) f_1^{'}(d_3(i_3)) \sum_{i_4=\sigma_4}^{\sigma_4+1} w_4^{i_4}(x_4) f_3(d_4(i_4)) \\
&+ \sum_{i_3=\sigma_3}^{\sigma_3+1} w_3^{i_3}(x_3) f_1(d_3(i_3)) \mu_1 \\
&+ x_5^2 \sum_{i_3=\sigma_3}^{\sigma_3+1} w_3^{i_3}(x_3) f_1'(d_3(i_3)) \sum_{i_4=\sigma_4}^{\sigma_4+1} w_4^{i_4}(x_4) f_3'(d_4(i_4)) \mu_2,
\end{aligned}$$

$$\begin{aligned}
\eta_2^{(3)} =& L_{f_p}^3 h_2 + L_{g_1} L_{f_p}^2 h_2 \mu_1 + L_{g_2} L_{f_p}^2 h_2 \mu_2 \\
=& x_5^3 \sum_{i_3=\sigma_3}^{\sigma_3+1} w_3^{i_3}(x_3) f_2^{''}(d_3(i_3)) \left(\sum_{i_4=\sigma_4}^{\sigma_4+1} w_4^{i_4}(x_4) f_3(d_4(i_4)) \right)^2 \\
&+ 3x_5 x_6 \sum_{i_3=\sigma_3}^{\sigma_3+1} w_3^{i_3}(x_3) f_2^{'}(d_3(i_3)) \sum_{i_4=\sigma_4}^{\sigma_4+1} w_4^{i_4}(x_4) f_3(d_4(i_4)) \\
&+ \sum_{i_3=\sigma_3}^{\sigma_3+1} w_3^{i_3}(x_3) f_2(d_3(i_3)) \mu_1 \\
&+ x_5^2 \sum_{i_3=\sigma_3}^{\sigma_3+1} w_3^{i_3}(x_3) f_2'(d_3(i_3)) \sum_{i_4=\sigma_4}^{\sigma_4+1} w_4^{i_4}(x_4) f_3'(d_4(i_4)) \mu_2.
\end{aligned}$$

The vertices $f_1^{''}(d_3(i_3))$, $f_2^{''}(d_3(i_3))$ and The controller of (12) is designed as

$$\begin{aligned}
(\mu_1, \mu_2)^T =& -(L_g L_{f_p}^2 h)^{-1} L_{f_p}^3 h + (L_g L_{f_p}^2 h)^{-1} v \\
=& - \begin{pmatrix} L_{g_1} L_{f_p}^2 h_1 & L_{g_2} L_{f_p}^2 h_1 \\ L_{g_1} L_{f_p}^2 h_2 & L_{g_2} L_{f_p}^2 h_2 \end{pmatrix}^{-1} \begin{pmatrix} L_{f_p}^3 h_1 \\ L_{f_p}^3 h_2 \end{pmatrix} \\
&+ \begin{pmatrix} L_{g_1} L_{f_p}^2 h_1 & L_{g_2} L_{f_p}^2 h_1 \\ L_{g_1} L_{f_p}^2 h_2 & L_{g_2} L_{f_p}^2 h_2 \end{pmatrix}^{-1} v
\end{aligned}$$

where v is the linear controller of the linear system (13).

$$\begin{cases} \dot{z} = Az + Bu, \\ y = Cz, \end{cases} \tag{13}$$

where $z = (h_1,\ L_{f_p} h_1,\ L_{f_p}^2 h_1,\ h_2,\ L_{f_p} h_2,\ L_{f_p}^2 h_2)^T \in \Re^6$,

$$A = \begin{pmatrix} 0&1&0&0&0&0 \\ 0&0&1&0&0&0 \\ 0&0&0&0&0&0 \\ 0&0&0&0&1&0 \\ 0&0&0&0&0&1 \\ 0&0&0&0&0&0 \end{pmatrix}, B = \begin{pmatrix} 0&0 \\ 0&0 \\ 1&0 \\ 0&0 \\ 0&0 \\ 0&1 \end{pmatrix}, C = \begin{pmatrix} 1&0 \\ 0&0 \\ 0&0 \\ 0&1 \\ 0&0 \\ 0&0 \end{pmatrix}^T.$$

If $x_5 \neq 0$, there exists a controller $(\mu_1, \mu_2)^T$ of the tricycle robot model (12) since $\det(L_g L_{f_p}^2 h) \neq 0$.

In this case, the state space of the tricycle robot model is divided into 13×13 vertices. Therefore the system has 12×12 local PML models. Note that all the linearized systems of these PML models are the same as the linear system (13).

In the same manner of (11), the dynamic feedback linearizing controller of the PML system is designed as

$$\begin{cases} \ddot{u}_1 = \mu_1, \\ u_2 = M \mathrm{sech}^2 x_4 \mu_2, \\ \begin{pmatrix} \mu_1 \\ \mu_2 \end{pmatrix} = L_{f_p}^3 h + L_g L_f^2 h v. \end{cases} \tag{14}$$

The stabilizing linear controller $v = -Fz$ of the linearized system (13) is designed so that the transfer function $C(sI - A)^{-1}B$ is Hurwitz.

Note that the dynamic controller (14) based on PML model is simpler than the conventional one (11). Since the nonlinear terms of controller (14) contain not the original nonlinear terms (e.g., $\sin x_3$, $\cos x_3$, $\tan(M \tanh x_4)$) but the piecewise approximation models.

C. Tracking Control for PML System

We apply a tracking control [15] to the tricycle robot model (7). Consider the following reference signal model

$$\begin{cases} \dot{x}_r = f_r, \\ \eta_r = h_r. \end{cases}$$

The controller is designed to make the error signal $e = (e_1, e_2)^T = \eta - \eta_r \to 0$ as $t \to \infty$. The time derivative of e is obtained as

$$\dot{e} = \dot{\eta} - \dot{\eta}_r = \begin{pmatrix} L_{f_p} h_{p1} \\ L_{f_p} h_{p2} \end{pmatrix} - \begin{pmatrix} L_{f_r} h_{r1} \\ L_{f_r} h_{r2} \end{pmatrix}.$$

Furthermore the time derivative of $\dot{e}$ is calculated as

$$\ddot{e} = \ddot{\eta} - \ddot{\eta}_r = \begin{pmatrix} L_{f_p}^2 h_{p1} \\ L_{f_p}^2 h_{p2} \end{pmatrix} - \begin{pmatrix} L_{f_r}^2 h_{r1} \\ L_{f_r}^2 h_{r2} \end{pmatrix}$$

Since the controller μ doesn't appear in the equation $\ddot{e}$, we calculate the time derivative of $\ddot{e}$.

$$\begin{aligned}
e^{(3)} =& \eta^{(3)} - \eta_r^{(3)} \\
=& \begin{pmatrix} L_{f_p}^3 h_{p1} \\ L_{f_p}^3 h_{p2} \end{pmatrix} + L_g L_{f_p}^2 h \begin{pmatrix} \mu_1 \\ \mu_2 \end{pmatrix} - \begin{pmatrix} L_{f_r}^3 h_{r1} \\ L_{f_r}^3 h_{r2} \end{pmatrix}
\end{aligned}$$

The tracking controller is designed as

$$\begin{cases} \ddot{u}_1 = \mu_1, \\ u_2 = M \mathrm{sech}^2 x_4 \mu_2, \\ \begin{pmatrix} \mu_1 \\ \mu_2 \end{pmatrix} = L_{f_p}^3 h - L_{f_r}^3 h_r + L_g L_{f_p}^2 h v. \end{cases} \tag{15}$$

The linearized system (13) and controller $v = -Fz$ are obtained in the same manners as the previous subsection. The coordinate transformation vector is $z = (e_1,\ \dot{e}_1,\ \ddot{e}_1, e_2,\ \dot{e}_2,\ \ddot{e}_2)^T$.

Note that the dynamic controller (15) based on PML model is simpler than the conventional one on the same reason of the previous subsection.

V. Simulation results

We apply the previous tracking control method to the tricycle robot model (7). Although the controller is simpler than the conventional I/O feedback linearization controller, the tracking performance based on PML model is the same as the conventional one. In addition, the controller is capable to use a nonlinear system with chaotic behavior as the reference model. In the following simulations, the tricycle length L is 1.0 [m] and the angle constrain M is $\pi/3$ [rad.].

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A. Ellipse-shaped reference trajectory

Consider an ellipse model as the reference trajectory.

$$\begin{pmatrix} x_{r_1} \\ x_{r_2} \end{pmatrix} = \begin{pmatrix} R_1 \cos\theta + x_{r_1}(0) \\ R_2 \sin\theta + x_{r_2}(0) \end{pmatrix},$$

where R_1 and R_2 are the semiminor axes and $(x_{r_1}(0), x_{r_2}(0))$ is the center of the ellipse. Fig. 3 shows the simulation result. The dotted line is the reference signal and the solid line is the tricycle tracking trajectory. The semiminor parameters R_1 and R_2 are 10 and 25. The initial positions are set at $(x(0), y(0)) = (5, 0)$ and $(x_r(0), y_r(0)) = (10, 0)$. Fig. 4 shows the control inputs u_1 and ν_1 of the tricycle. Fig. 5 shows the error signals of the tricycle position (x, y).

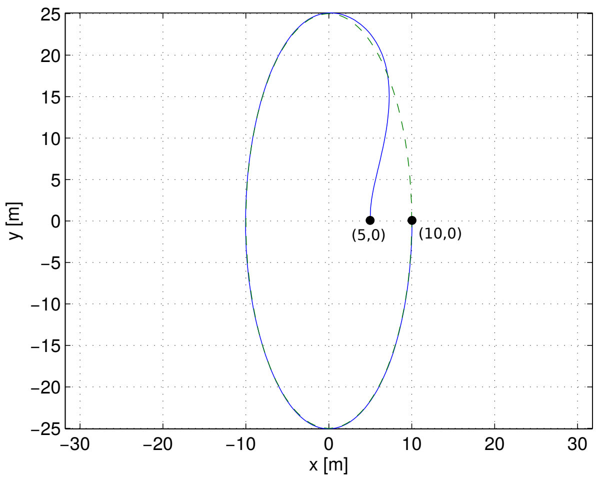

Fig. 3. Ellipse-shaped reference signal and the tricycle tracking trajectory

B. Trajectory tracking control using ellipse-shaped reference models

Arbitrary tracking trajectory control can be realized using the ellipse-shaped tracking trajectory method. The controller design procedure is as follows:

1) Assign passing points $(p_x(i), p_y(i))$, $i = 1, \ldots, n$. We consider the passing points: $(0, 0)$, $(10, 20)$, $(26, 30)$, $(18, 50)$ and $(2, 70)$
2) Construct some ellipses trajectories to connect the passing points smoothly.
   From $(0, 0)$ to $(10, 20)$, the trajectory 1:
   $$\begin{pmatrix} x_{r_1} \\ x_{r_2} \end{pmatrix} = \begin{pmatrix} 10\cos\theta + 10 \\ 20\sin\theta \end{pmatrix}, \tag{16}$$
   where $\pi/2 \le \theta \le \pi$.
   From $(10, 20)$ to $(26, 30)$, the trajectory 2:
   $$\begin{pmatrix} x_{r_1} \\ x_{r_2} \end{pmatrix} = \begin{pmatrix} 16\cos\theta + 10 \\ 10\sin\theta + 30 \end{pmatrix}, \tag{17}$$
   where $-\pi/2 \le \theta \le 0$.
   From $(26, 30)$ to $(18, 50)$, the trajectory 3:
   $$\begin{pmatrix} x_{r_1} \\ x_{r_2} \end{pmatrix} = \begin{pmatrix} 8\cos\theta + 18 \\ 20\sin\theta + 30 \end{pmatrix}, \tag{18}$$
   where $0 \le \theta \le \pi/2$.
   From $(18, 50)$ to $(2, 70)$, the trajectory 4:
   $$\begin{pmatrix} x_{r_1} \\ x_{r_2} \end{pmatrix} = \begin{pmatrix} 16\cos\theta + 18 \\ 10\sin\theta + 60 \end{pmatrix}, \tag{19}$$
   where $-\pi/2 \le \theta \le -\pi/2$.
3) Design the controllers (15) for the ellipse tracking trajectories (16)-(19).

We show a tracking trajectory control example for the tricycle robot system. Fig. 6 shows the reference signals (16)-(19) and the tricycle tracking trajectory. The dotted line is the reference signal and the solid line is the tricycle tracking trajectory. Fig. 7 shows the control inputs u_1 and ν_1 of the tricycle. Fig. 8 shows the error signals with respect to the tricycle position (x, y).

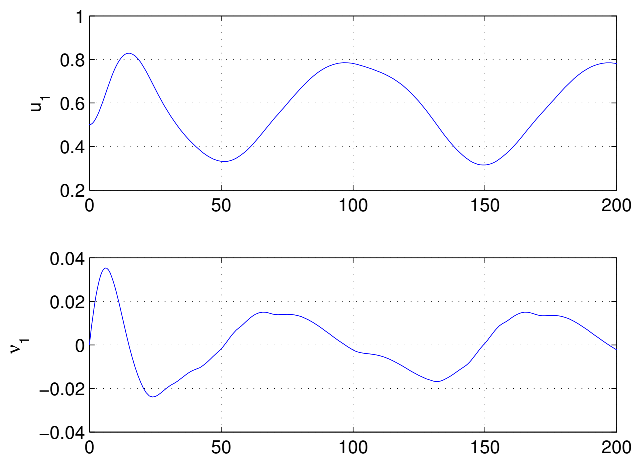

Fig. 4. Control inputs u_1 and ν_1 of the tricycle

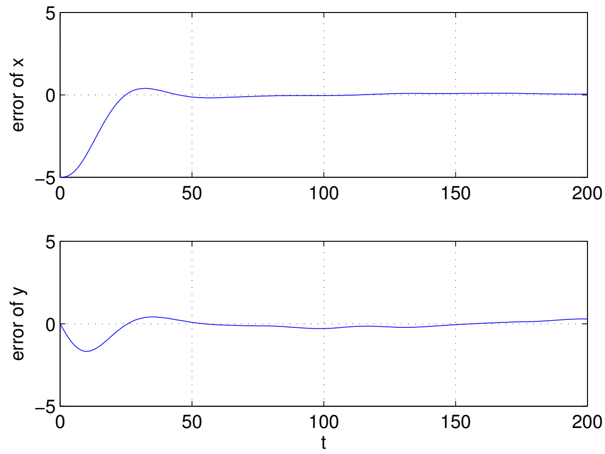

Fig. 5. Error signals of the tricycle position (x, y)

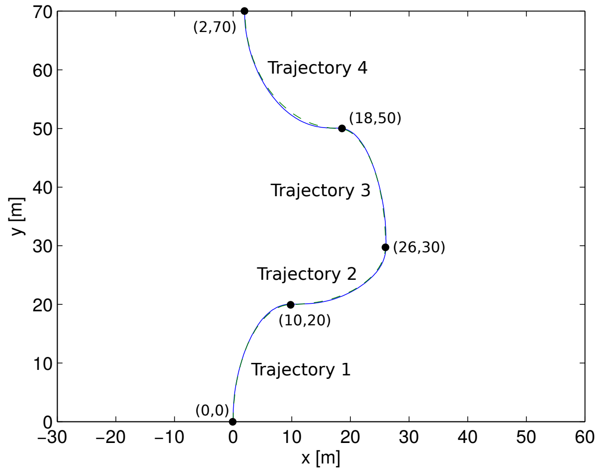

Fig. 6. Reference signals (16)-(19) and the tricycle tracking trajectory

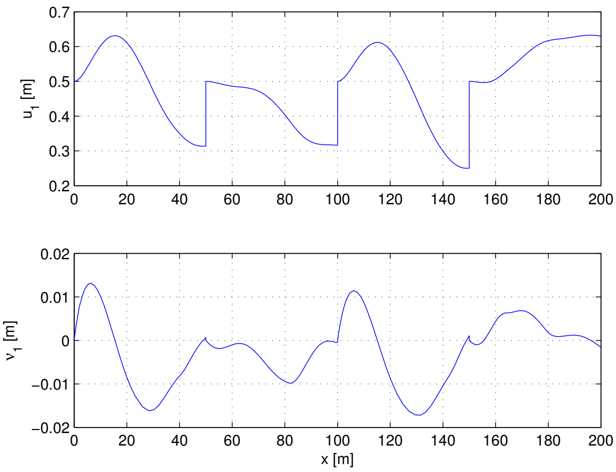

Fig. 7. Control inputs u_1 and ν_1 of the tricycle

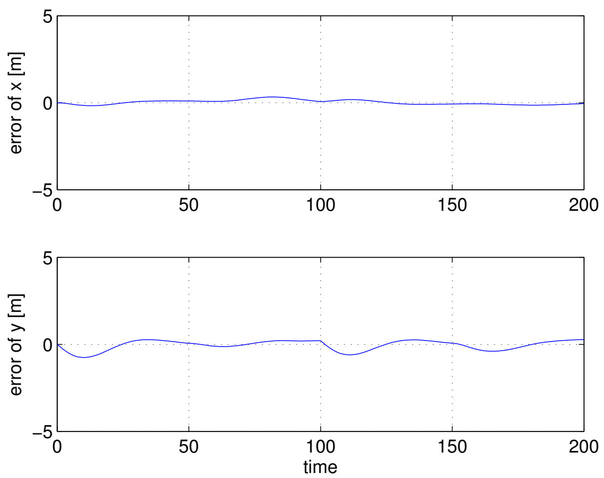

Fig. 8. Error signals of the tricycle position (x, y)

VI. Conclusions

We have proposed a trajectory tracking controller design of a tricycle robot as a non-holonomic system with PML models. The approximated model is fully parametric. I/O dynamic feedback linearization is applied to stabilize PML control system. PML modeling with feedback linearization is a very powerful tool for analyzing and synthesizing nonlinear control systems. We also have applied a method for tracking controller to the tricycle robot. Although the controller is simpler than the conventional I/O feedback linearization controller, the tracking performance based on PML model is the same as the conventional one. Examples have been shown to confirm the feasibility of our proposals by computer simulations.

References

[1] B. d'Andréa-Novel, G. Bastin, and G. Campion, "Modeling and control of non holonomic wheeled mobile robots," in *the 1991 IEEE International Conference on Robotics and Automation*, 1991, pp. 1130–1135.

[2] R. Fierro and F. L. Lewis, "Control of a nonholonomic mobile robot: backstepping kinematics into dynamics," in *the 34th Conference on Decision and Control*, 1995, pp. 3805–3810.

[3] T.-C. Lee, K.-T. Song, C.-H. Lee, and C.-C. Teng, "Tracking control of unicycle-modeled mobile robots using a saturation feedback controller," *IEEE Transactions on Control Systems Technology*, vol. 9, no. 2, pp. 305–318, 2001.

[4] J. Guldner and V. I. Utkin, "Stabilization of non-holonomic mobile robots using lyapunov functions for navigation and sliding mode control," in *the 33rd Conference on Decision and Control*, 1994, pp. 2967–2972.

[5] J. Yang and J. Kim, "Sliding mode control for trajectory tracking of nonholonomic wheeled mobile robots," *IEEE Transactions on Robotics and Automation*, pp. 578–587, 1999.

[6] B. d'Andréa-Novel, G. Bastin, and G. Campion, "Dynamic feedback linearization of nonholonomic wheeled mobile robot," in *the 1992 IEEE International Conference on Robotics and Automation*, 1992, pp. 2527–2531.

[7] G. Oriolo, A. D. Luca, and M. Vendittelli, "WMR control via dynamic feedback linearization: Design, implementation, and experimental validation," *IEEE Transaction on Control System Technology*, vol. 10, no. 6, pp. 835–852, 2002.

[8] E. Yang, D. Gu, T. Mita, and H. Hu, "Nonlinear tracking control of a car-like mobile robot via dynamic feedback linearization," in *University of Bath, UK*, no. ID-218, 2004.

[9] A. D. Luca, G. Oriolo, and M. Vendittelli, "Stabilization of the unicycle via dynamic feedback linearization," in *the 6th IFAC Symposium on Robot Control*, 2000.

[10] M. Sugeno, "On stability of fuzzy systems expressed by fuzzy rules with singleton consequents," *IEEE Trans. Fuzzy Syst.*, vol. 7, no. 2, pp. 201–224, 1999.

[11] K.-C. Goh, M. G. Safonov, and G. P. Papavassilopoulos, "A global optimization approach for the BMI problem," in *Proc. the 33rd IEEE CDC*, 1994, pp. 2009–2014.

[12] T. Taniguchi and M. Sugeno, "Piecewise bilinear system control based on full-state feedback linearization," in *SCIS & ISIS 2010*, 2010, pp. 1591–1596.

[13] ——, "Stabilization of nonlinear systems with piecewise bilinear models derived from fuzzy if-then rules with singletons," in *FUZZ-IEEE 2010*, 2010, pp. 2926–2931.

[14] ——, "Design of LUT-controllers for nonlinear systems with PB models based on I/O linearization," in *FUZZ-IEEE 2012*, 2012, pp. 997–1022.

[15] T. Taniguchi, L. Eciolaza, and M. Sugeno, "Look-Up-Table controller design for nonlinear servo systems with piecewise bilinear models," in *FUZZ-IEEE 2013*, 2013.