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The Issues of Multipurpose Control Laws Construction

Maria A. Smirnova, Mikhail N. Smirnov, Tatyana E. Smirnova, Nikolay V. Smirnov

Abstract—**In the paper it is proposed a new approach to the formation of multipurpose control laws, based on the fixation of the uniform for all modes of the control law part with the ability to connect additional elements adaptively customized to separate mode. Two multipurpose structures of control laws are considered and compared.**

Index Terms—**multipurpose, control law, stability.**

I. Introduction

NOWADAYS much prominence in various scientific publications is given to automatic control laws [1-10], to their choice and construction. This fact is explained by the expansion of the requirements for automatic control systems and by the growing capabilities of the devices implementing control laws. Therefore there is a need to use multipurpose control laws allowing to consider the presence of the complex conditions, requirements and constraints, which should certainly be performed in all operation modes of the controlled object.

Modern system of automatic motion control, as a rule, are operate in different modes, determined by the specific definition of command signals and external disturbances acting on the controlled object. For each of the modes the complex of requirements, conditions and restrictions that must be strictly fulfilled during the motion is formed. It should be noted that these systems together are often contradictory in nature due to significant differences in the dynamic characteristics of motion modes.

A natural way to ensure all the required dynamic properties is to achieve a compromise on the quality of the control processes in different modes. This compromise is easy to provide with some common multipurpose control law for all modes, but the loss of the quality for the separate modes are obvious.

In connection with the above circumstances, it is proposed a different approach to the formation of multipurpose control laws, based on the fixation of the uniform for all modes of the control law part with the ability to connect additional elements adaptively customized to separate mode. As the mathematical basis for such a configuration is adopted the optimization approach, which allows to present design problems as the problems on searching the extremum.

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M. A. Smirnova is with Saint-Petersburg State University, 7/9 Universitetskaya nab., St. Petersburg, 199034, Russia (e-mail: mariya.smirnova@spbu.ru).

M. N. Smirnov is with Saint-Petersburg State University, 7/9 Universitetskaya nab., St. Petersburg, 199034, Russia (e-mail: mikhail.smirnov@spbu.ru).

T. E. Smirnova is with Saint-Petersburg State University, 7/9 Universitetskaya nab., St. Petersburg, 199034, Russia (e-mail: smi-tana@mail.ru).

N. V. Smirnov is with Saint-Petersburg State University, 7/9 Universitetskaya nab., St. Petersburg, 199034, Russia (e-mail: nvs_v@mail.ru).

II. Structures of Control Laws with Multipurpose Feedback

To represent a multipurpose structure, let us consider the mathematical model of the dynamics of the object in the form of a system of linear stationary equations

$$\begin{aligned} \dot{x} &= Ax + B\delta + d(t), \\ y &= Cx. \end{aligned} \tag{1}$$

Here $x \in E^n$ is a state vector, $\delta \in E^m$ is a control vector, $y \in E^k$ is a vector of measured variables, A, B, C are constant matrices of corresponding dimensions.

In practical control problems the mathematical model (1) is usually supplemented with separate actuators dynamics equations

$$\dot{\delta} = F_\delta(t, \delta, u), \tag{2}$$

where $u \in E^m$ is a vector of control signals. Functions F_δ are often reflect the existence of significant non-linearities (dead zone and saturation areas), but in the presence of linear zones.

We assume that control is carried out within the linear sections of the respective functions, and drives can be represented in its simplest form by the equation

$$\dot{\delta} = u. \tag{3}$$

In many cases, automatic control laws are formed as a linear feedback

$$u = W(p)\,(y - y^*(t)) + W_0(p)\delta, \ \ p = d/dt, \tag{4}$$

where $y^*(t)$ is the given command signal. Transfer matrices W, W_0 of the control law (4) are to be found in the process of the solution of the synthesis problem.

As noted above, the essence of the multipurpose control is as follows: the synthesized controller (4) must provide the desired quality of the dynamic process in all motion modes of the object. The use of a multi-purpose ideology in the synthesis of control laws allows to achieve the desired result.

To select matrices W, W_0 we will use optimization approach, the essence of which is to solve the problem

$$J = J(W, W_0) \to \inf_{(W,W_0)\in\Omega}, \tag{5}$$

where J is the functional characterizing the quality of the dynamics. It is defined on the motions of the closed-loop system (1), (3), (4), $\Omega^* \in \Omega$ is the feasible set of the desired matrices W, W_0 from the domain Ω of Hurwitz characteristic polynomial of the closed-loop system.

It should be noted that the restriction of the domain Ω is determined by several factors, which can be divided into two groups. The first is an objective, reflecting the need to consider the complex of the requirements for a closed-loop system, along with the requirement of its stability. The second group primarily reflects the subjective views of the systems' designer and, first of all, his views on the structure of the control law, which is appropriate to provide the desired efficiency and dynamic quality at solving specific problems.

In particular, it is proposed to form the multipurpose control laws with a special structure that contains adjustable components used as needed. Choosing of the customizable elements is performed by the specific techniques in the framework of the optimization approach that allows us to achieve the desired quality of the dynamic processes.

Let us form the proposed multipurpose control laws with the mathematical model

$$\begin{aligned} \dot{z} &= Az + B\delta + G(y - Cz), \\ u &= K_x z + K_0\delta + \zeta, \\ \zeta &= F(p)(y - Cz), \;\; p = d/dt. \end{aligned} \quad (6)$$

In the system (6) the first equation is an asymptotic observer, the second determines the control signal, ensuring the stability and astaticism on the vector of controlled variables y for a closed-loop system (1), (3), (6). The third equation describes the dynamic corrector, preserving stability and astaticism of the closed-loop system and at the same time providing the desired quality of the dynamic process. In the equations (6) $z \in E^n$ is the state vector of the observer, $\zeta \in E^m$ is the output of the corrector.

The synthesis problem of the multipurpose feedback (6) is to find the optimal matrices $G, K, K_x, F(s)$ for the desired dynamics requirements for a closed-loop system for the respective motion modes.

Alternative approach of multipurpose control law for moving objects is proposed in T. Fossen [1]. For the introduction of an appropriate structure, let us consider a non-linear model of the moving object with three degrees of freedom

$$\begin{aligned} M\dot{\nu} &= -D\nu + \tau + d(t), \\ \dot{\eta} &= R(\eta)\nu. \end{aligned} \quad (7)$$

Here vector $\nu = (u\ v\ r)'$ represents the speeds in the body-fixed frame, the vector $\eta = (x\ y\ \psi)'$ determines the object's position $(x\ y)$ and rotation angle ψ in the earth-fixed frame. Vector $\tau \in E^3$ determines the control action, and the vector $d \in R^3$ is the external impact on the object. Matrices M and D with constant components are positive definite, and $M = M'$.

The non-linearity of this system is determined by the orthogonal rotation matrix

$$R(\eta) = R(\psi) = \begin{pmatrix} \cos\psi & -\sin\psi & 0 \\ \sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

Mathematical model (7) is commonly used in the problem of dynamic positioning, the essence of which is to transfer the movable object from an arbitrary initial position $\eta(0)$ to the target position η_d.

In general case the control law for the object is constructed in the form

$$\begin{aligned} \dot{z} &= f(z, \tau, \eta), \\ \tau &= g(z, \eta, \eta_d). \end{aligned} \quad (8)$$

where $z \in E^k$ is the state vector of the controller, that provides the desired dynamics of the closed-loop system (7), (8).

Control law with a multipurpose structure proposed in [1], has the form

$$\begin{aligned} \dot{z}_b &= T_b z_b + \Gamma_b \tilde{y}, \\ \dot{z}_w &= \Omega z_w + K_0 \tilde{y}, \\ M\dot{z}_\nu &= -Dz_\nu + R(\eta)z_b + \tau + R'(\eta)K_1\tilde{y}, \quad (9) \\ \dot{z}_\eta &= R(\eta)z_\nu + K_2\tilde{y}, \\ \tilde{y} &= \eta - z_\eta - \Gamma_w z_w, \\ \tau &= -K_d\nu - R'(\eta)K_p(z_\eta - \eta_d) - R'(\eta)z_b. \quad (10) \end{aligned}$$

Equations (9) are the equations of nonlinear asymptotic observer, where $z_\nu \in E^3$ and $z_\eta \in E^3$ are estimations of vectors ν and η respectively, $z_b \in E^3$ is the estimation of the vector of slowly changing (or constant) components of external influences, $z_w \in E^3$ is the estimation of the vector components of the external actions, defined by the rough sea. Here matrices K_1, K_2, Γ_b, Γ_w are subject to the choice in the process of adjustment of the observer to ensure the asymptotic stability of the zero equilibrium position on the estimation error. The matrices K_d and K_p that determine the control signal must ensure the global asymptotic stability of the target position η_d for the object (7) with the controller

$$\tau = -K_d\nu - R'(\eta)K_p(\eta - \eta_d). \quad (11)$$

In contrast to (9), (10), in [9–13] for dynamic positioning problem it is proposed to form the control law with a multipurpose structure

$$\begin{aligned} M\dot{z}_\nu &= -Dz_\nu + \tau + R'(\eta)K_1(\eta - z_\eta), \quad (12) \\ \dot{z}_\eta &= R(\eta)z_\nu + K_2(\eta - z_\eta), \\ \zeta &= F(p)(\eta - z_\eta), \;\; p = d/dt, \quad (13) \\ \tau &= -K_d\nu - R'(\eta)K_p(z_\eta - \eta_d) + \zeta. \quad (14) \end{aligned}$$

Note that the structure (12)–(14) is much simpler than the structure of the control law (9), (10) because of an asymptotic observer does not include the elements for estimation of external influences.

To implement the structure of the control law (12)–(14), it is necessary to determine numerically the matrices K_1, K_2, K_d, K_p. On the basis of (7), (12) we form the differential equations satisfied by observation errors $\varepsilon_\nu(t) = \nu(t) - z_\nu(t)$, $\varepsilon_\eta(t) = \eta(t) - z_\eta(t)$:

$$\begin{aligned} M\dot{\varepsilon}_\nu &= -D\varepsilon_\nu - R'(\eta)K_1\varepsilon_\eta + d(t), \\ \dot{\varepsilon}_\eta &= R(\eta)\varepsilon_\nu - K_2\varepsilon_\eta. \end{aligned} \quad (15)$$

Note that in the absence of external influence, i.e. under the condition $d(t) \equiv 0$ the system (15) has the desired zero equilibrium position. As shown in [9], a sufficient condition for global exponential stability is a diagonal structure and positive definiteness of matrices K_1, K_2.

As for the choice of matrices K_d and K_p, then, as to the structure (9), (10), the asymptotic stability of the desired equilibrium position is reached in the case if it holds for the object (7) with the controller

$$\tau = -K_d\nu - R'(\eta)K_p(\eta - \eta_d). \quad (16)$$

Both the structure of (9), (10) and the structure (12)–(14) are multipurpose in the sense that they provide not only the balance of the desired position, which is globally asymptotically stable, i.e., the equality

$$\lim_{t\to\infty}[\eta(t) - \eta_d(t)] = 0. \tag{17}$$

but the performance of additional requirements to quality of dynamic processes. In particular, both the structures allow to provide the astaticism of the closed-loop system, which is important under the action of constant disturbances, and filtering properties, which are required to work under the action of oscillating nature of perturbations.

III. Conclusion

Two multipurpose structures of control laws providing not only the balance of the desired position, but the performance of additional requirements to quality of dynamic processes are compared. Advantages and disadvantages of each structure is considered. Multipurpose control law for dynamic positioning problem is proposed. Perspective direction of research is related to the multiprogram stabilization approach [14–20].

References

[1] T. I. Fossen, *Guidance and Control of Ocean Vehicles*. John Wiley & Sons, New York, 1999, 480 p.

[2] E. I. Veremey, "Synthesis of multiobjective control laws for ship motion," *Gyroscopy and Navigation*, vol. 1, no. 2, pp. 119–125, 2010.

[3] E. I. Veremey, "Dynamical Correction of Positioning Control Laws," *Control Applications in Marine Systems*, vol. 9, Part 1, pp. 31–36, 2013.

[4] N. V. Smirnov, T. Ye. Smirnova, M. A. Smirnova, M. N. Smirnov, "Multiprogram digital control," *Lecture Notes in Engineering and Computer Science*, vol. 1, pp. 268–271, 2014.

[5] T. Ye. Smirnova, M. A. Smirnova, M. N. Smirnov, "Astaticism in the motion control systems of marine vessels," *Lecture Notes in Engineering and Computer Science*, vol. 1, pp. 258–261, Mar. 2014.

[6] N. V. Smirnov, M. A. Smirnova, M. N. Smirnov, "The method of accounting of bounded external disturbances for the synthesis of feedbacks with multi-purpose structure," *Lecture Notes in Engineering and Computer Science*, vol. 1, pp. 301–304, Mar. 2014.

[7] Maria S., Mikhail S., "Synthesis of Astatic Control Laws of Marine Vessel Motion," in *Proc. 18th Int. Conf. Methods and Models in Automation and Robotics*, 2013, pp. 678–681.

[8] Mikhail S., Maria S., "Dynamical Compensation of Bounded External Impacts for Yaw Stabilisation System," in *Proc. XXIV Int. Conf. Information, Communication and Automation Technologies*, 2013, pp. 1–3.

[9] Maria S., Mikhail S., "Modal Synthesis of Astatic Controllers for Yaw Stabilization System," in *Proc. XXIV Int. Conf. Information, Communication and Automation Technologies*, 2013, pp. 1–5.

[10] N. V. Smirnov, "Multiprogram control for dynamic systems: A point of view," in *ACM Int. Conf. Proc. Series, Joint International Conference on Human-Centered Computer Environments, HCCE 2012*, Mar. 2012, pp. 106–113.

[11] E. I. Veremey, M. N. Smirnov, M. A. Smirnova, "Synthesis of stabilizing control laws with uncertain disturbances for marine vessels," *2015 International Conference on "Stability and Control Processes" in Memory of V. I. Zubov, SCP 2015 – Proceedings*, pp. 1-3, Oct. 2015.

[12] N. K. Arzumanyan, M. N. Smirnov, M. A. Smirnova, "Synthesis and modeling of anti-lock braking system," *2015 International Conference on "Stability and Control Processes" in Memory of V. I. Zubov, SCP 2015 – Proceedings*, pp. 552-554, Oct. 2015.

[13] T. Ye. Smirnova, M. A. Smirnova, M. N. Smirnov, N. V. Smirnov "Astaticism in Tracking Control Systems," *Lecture Notes in Engineering and Computer Science*, vol. 1, pp. 200–204, Mar. 2016.

[14] T. Ye. Smirnova, M. A. Smirnova, M. N. Smirnov, N. V. Smirnov "Optimization of the Size of Minimal Invariant Ellipsoid with Providing the Desired Modal Properties," *Lecture Notes in Engineering and Computer Science*, vol. 1, pp. 238–241, Mar. 2016.

[15] N. V. Smirnov, T. E. Smirnova, "Multiprogram digital control for bilinear object," in *2014 Int. Conf. on Computer Technologies in Physical and Engineering Applications (ICCTPEA). Editor: E.I. Veremey*, Jun.–Jul. 2014, pp. 168–169.

[16] N. V. Smirnov, T. E. Smirnova, Ya. A. Shakhov, "Stabilization of a given set of equilibrium states of nonlinear systems," *Journal of Computer and Systems Sciences International*, vol. 51, no. 2, pp. 169–175, Apr. 2012.

[17] T. Ye. Smirnova, M. A. Smirnova, M. N. Smirnov, N. V. Smirnov "Control laws with integral action for marine vessels," *CACS 2015 - 2015 CACS International Automatic Control Conference*, vol. 1, pp. 414–417, Nov. 2015.

[18] T. Ye. Smirnova, M. A. Smirnova, M. N. Smirnov, N. V. Smirnov "Modernization of the approach for bounded external disturbances compensation," *Lecture Notes in Engineering and Computer Science*, vol. 1, pp. 418–421, Nov. 2015.

[19] M. A. Smirnova, M. N. Smirnov, "Multipurpose Control Laws in Trajectory Tracking Problem," *International Journal of Applied Engineering Research*, vol. 11(22), pp. 11104-11109, 2016.

[20] M. N. Smirnov, M. A. Smirnova, T. E. Smirnova, N. V. Smirnov, "Questions of digital Control Laws Synthesis for Marine Vessels in View of Uncertainties," *International Journal of Applied Engineering Research*, vol. 11(23), pp. 11566-11570, 2016.

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