Proceedings of the International MultiConference of Engineers and Computer Scientists 2017 Vol I,
IMECS 2017, March 15 - 17, 2017, Hong Kong

Discrete-Time PID×(*n*-2) Stage PD Cascade Controllers with First Order Hold and Delayed First Order Hold Discretizations

Channarong Chiengtee, Pittaya Pannil, Prapart Ukakimaparn, and Thanit Trisuwannawat

***Abstract*---This paper proposes a technique to design the (*n*-2) stage PD (Proportional-Derivative) controller cascaded with the PID (Proportional-Integral-Derivative) controller in accordance with *n*th order plants. The Continuous-Time (CT) design is firstly reviewed to show the advantages of the Kitti's method. The proposed technique is based on the Kitti's method in combination with the use of First Order Hold (FOH) to discretize the CT plant and Delayed First Order Hold (DFOH) to discretize the CT controller for obtaining the proper Discrete-Time (DT) controller structure. The simulation results confirm that the proposed design technique can be applied to the DT framework with better specifications than it was expected.**

***Index Terms*— Continuous-Time / Discrete-Time PID×(*n*-2) PD controllers, First Order Hold, Delayed First Order Hold**

I. Introduction

It is known that most industrial plants are type 0 and consist of three to five first order lags or dead time plus one first order lag [1]. However, the PID controller is properly applied to a typical second order plant only. In order to control a third order system to obtain the given specifications, an analytic PIDA (Proportional-Integral-Derivative-Acceleration) controller design technique is then proposed [2]. For a third or higher *n*th order plant, a design method based on root locus technique for the PID×(*n*-2) stage PD cascade controllers in CT framework has been presented [3]. This design technique is aimed to satisfy the desired specifications without trial and error. Then, the forward controller is employed to decrease the overshoot, and the controlled system structure becomes two degree of freedom (2-DOF) control system as shown in Fig. 1.

Manuscript received December 19, 2016; revised January 9, 2017.

C. Chiengtee is with Department of Instrumentation and Control Engineering, Faculty of Engineering, King Mongkut's Institute of Technology Ladkrabang, Bangkok, Thailand, 10520 (e-mail: atom_12@hotmail.com).

P. Pannil is with Department of Instrumentation and Control Engineering, Faculty of Engineering, King Mongkut's Institute of Technology Ladkrabang, Bangkok, Thailand, 10520, (phone : 662-329-8348; fax: 662-329-8349; e-mail: pittay.pa@kmitl.ac.th).

P. Ukakimaparn is with Department of Instrumentation and Control Engineering, Faculty of Engineering, King Mongkut's Institute of Technology Ladkrabang, Bangkok, Thailand, 10520 (e-mail: kuprapar@gmail.com).

T. Trisuwannawat is with Department of Instrumentation and Control Engineering, Faculty of Engineering, King Mongkut's Institute of Technology Ladkrabang, Bangkok, Thailand, 10520 (e-mail: ktthanit@gmail.com).

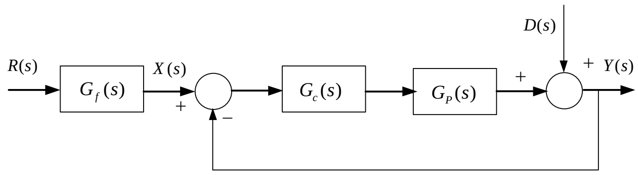

Fig. 1. Structure of the 2-DOF control system.

For DT framework, three generations of these PID×(*n*-2) stage PD cascade controllers have been proposed recently. The first design for the DT PID×(*n*-2) stage PD cascade controllers is using Zero Order Hold (ZOH) discretization method [4], while the second one is using "Tustin" or bilinear discretization method to design the controllers in *z*-plane [5]. The third concept to design DT controllers is also using "Tustin", but it is required to transform the CT designed controller from *s*-plane to *z*-plane [6]. In order to be an alternative method for DT controller designs, this paper presents an effective design technique using FOH and DFOH discretizations as well as using Kitti's method. The MATLAB simulation results for verifying the controller performances are also included.

II. Methodology

Fig. 2 shows the steps for design of digital control systems [7], which are 2 major steps; plant modeling and controller design.

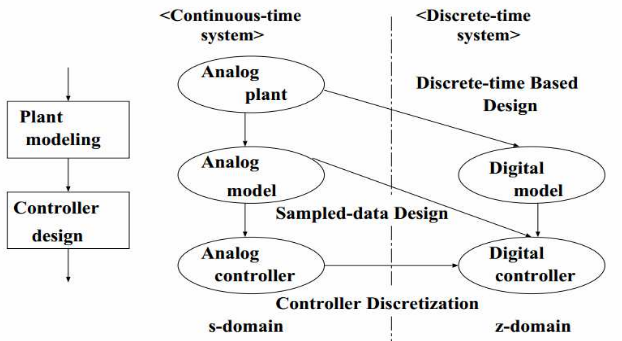

Fig. 2. Steps of the digital control system design.

A. Problem Statement

From a block diagram of Fig. 3, we need to find the PID×(*n*-2) stage PD cascade controllers $K(s)$ or $K(z)$ for the plant $G(s)$, so that the given desired specifications could be acceptably achieved.

ISBN: 978-988-14047-3-2
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

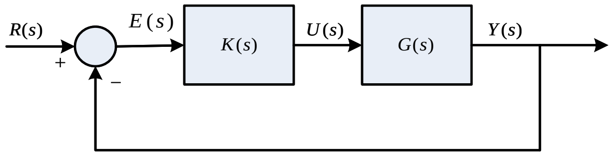

Fig. 3. Block diagram of typical control system.

B. Continuous–Time Framework

Let the *n*th order plant *G*(*s*) be controlled by the cascade controllers *K*(*s*), their transfer function is assumed to be

$$\begin{cases} G(s) = \dfrac{K_n}{s^N (T_1 s+1)(T_2 s+1)\cdots(T_p s+1)}, \\ \quad\;\; = \dfrac{1}{(s+1)(s+3)(s+6)}; \quad n=3,\ N=0. \end{cases} \tag{1}$$

The transfer function of the PID controller can be stated as

$$K_{PID}(s) = K_p + \frac{K_i}{s} K_d s = K_{pid}\frac{(s+z_1)(s+z_2)}{s}, \tag{2}$$

where K_p is a proportional gain, K_i is an integral gain, and K_d is a derivative gain. Hence, the PD controller transfer function is

$$K_{PD}(s) = K_p + K_d s = K_{pd}(s+z_{pd}). \tag{3}$$

The open-loop transfer function for the PID×(*n*-2) stage PD cascade controllers *K*(*s*) and the plant *G*(*s*) can be given by

$$\begin{cases} KG(s) = \dfrac{\overbrace{K_{pid}(s+z_1)(s+z_2)}^{\text{PID Controller}} \times \overbrace{K_{pd}(s+z_{pd})\cdots K_n}^{(n-2)\text{ PD}}}{s\cdot \underbrace{s^N (s+p_1)(s+p_2)\cdots(s+p_p)}_{n\text{th order Plant}}}, \\ \quad\;\;\; = K\dfrac{(s+3.1)(s+6.1)(s+z_{pd})}{s\cdot(s+1)(s+3)(s+6)}. \end{cases} \tag{4}$$

By using Kitti's method, z_1 = 3.1 and z_2 = 6.1 are firstly assigned, then find only z_{pd} and *K* from the root locus angle and magnitude conditions as follows.

$$\begin{cases} \measuredangle KG(s) = \pm(2k+1)\pi, \quad k = 0,1,2,\cdots, \\ |KG(s)| = 1. \end{cases} \tag{5}$$

The desired specifications to be designed are usually specified in terms of transient and steady state response characteristics of the control system to a unit-step input, exhibited by a pair of complex-conjugate dominant closed-loop poles $s_{d\pm} = -\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2}$ as follows:

$$\begin{cases} \text{Percent Overshoot } (P.O.) = e^{\left(\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}\right)} \times 100\% = 5\%, \\ \text{Settling Time } (t_s) = \dfrac{-\ln\left(0.02\sqrt{1-\zeta^2}\right)}{\zeta\omega_n} = \underset{(\pm 2\%)}{2\,\text{secs.}} \end{cases} \tag{6}$$

From the given desired specification in term of the Percent Overshoot (*P.O.*), the damping ratio is

$$\zeta = \sqrt{\left[\ln\left(\frac{P.O.}{100}\right)\right]^2 \Big/ \left\{\pi^2 + \left[\ln\left(\frac{P.O.}{100}\right)\right]^2\right\}} = 0.69. \tag{7}$$

From the given Settling Time {t_s(±2%)}, the undamped natural frequency is

$$\omega_n = \frac{-\ln\left(0.02\sqrt{1-\zeta^2}\right)}{\zeta t_s} = 3.069 \text{ rad./sec}. \tag{8}$$

Hence, one of the dominant closed-loop poles is located at

$$s_d = -2.118 + j2.221. \tag{9}$$

The open-loop transfer function without z_{pd} at s_d is

$$\begin{cases} KGwozpd(s_d) = \dfrac{\overbrace{(s_d+3.1)}^{66.139^\circ}\overbrace{(s_d+6.1)}^{29.147^\circ}}{\underbrace{s_d}_{133.639^\circ}\ \underbrace{(s_d+1)}_{116.715^\circ}\ \underbrace{(s_d+3)}_{68.333^\circ}\ \underbrace{(s_d+6)}_{29.771^\circ}}, \\ \quad = -0.069 + j0.13 = 0.411\measuredangle 106.829^\circ. \end{cases} \tag{10}$$

The angle from the zero z_{pd} to s_d is

$$\arg[z_{pd}] = \pi - \arg\left(KGwozpd(s_d)\right) = 73.171^\circ. \tag{11}$$

The location of the zero z_{pd} can find from

$$z_{pd} = |\mathrm{Re}(s_d)| + \frac{|\mathrm{Im}(s_d)|}{\tan\left(\arg[z_{pd}]\right)} = 2.789. \tag{12}$$

Now, it is implied that

$$\begin{cases} \measuredangle KG(s_d) = \measuredangle\left(\dfrac{\overbrace{(s_d+3.1)(s_d+6.1)}^{95.286^\circ}\overbrace{(s_d+2.789)}^{73.171^\circ}}{\underbrace{s_d(s_d+1)(s_d+3)(s_d+6)}_{348.457^\circ}}\right) = -180^\circ. \end{cases} \tag{13}$$

The open-loop gain *K* can be found from the magnitude condition of the root locus technique as follows:

$$K = \frac{\overbrace{|s_d|}^{3.069}\ \overbrace{|s_d+1|}^{2.486}\ \overbrace{|s_d+3|}^{2.39}\ \overbrace{|s_d+6|}^{4.473}}{\underbrace{|s_d+3.1|}_{2.428}\ \underbrace{|s_d+6.1|}_{4.56}\ \underbrace{|s_d+2.789|}_{2.32}} = 3.174. \tag{14}$$

To decrease the overshoot caused by adding the zero (s+z_{pd}) to the open-loop transfer function *KG*(*s*), the forward controller can be stated as

$$K_f(s) = \frac{z_{pd}}{s+z_{pd}}. \tag{15}$$

ISBN: 978-988-14047-3-2
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

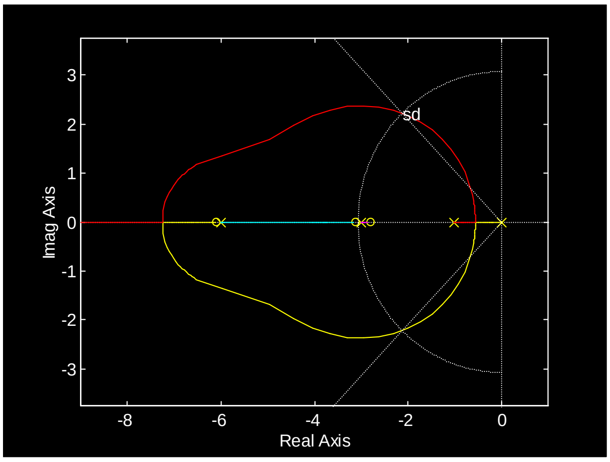

Fig. 4. Plots of root loci in s-Plane.

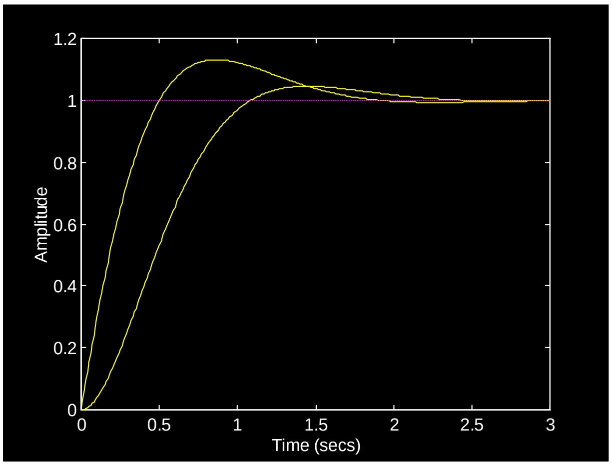

Fig. 5. Unit step responses.

The overall system is then approximated as if it is a standard second order system as follows:

$$\begin{cases} \dfrac{Y(s)}{R(s)} = \left(\dfrac{z_{pd}}{s+z_{pd}}\right)\left(\dfrac{K(s+z_{pd})}{s(s+1)+K(s+z_{pd})}\right), \\ = \dfrac{8.854}{s^2+2\cdot\underbrace{0.701}_{\zeta}\cdot\underbrace{2.976}_{\omega_n}s+\underbrace{8.854}_{\omega_n^2}}. \end{cases} \quad (16)$$

Fig. 4 shows the plots of root loci in *s*-Plane. The unit step responses with and without the forward controller are shown in Fig. 5, respectively.

C. Discrete-Time Framework

To design the DT controller, the CT plant (or CT system) and CT controller can be discretized by FOH [9] and by DFOH [10], respectively. Then we design the DT controller in the same way as the CT framework.

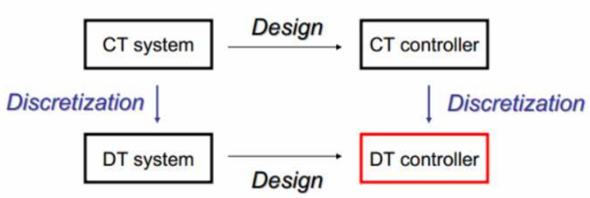

Fig. 6. Discretization.

The DT transfer function of the CT plant $G(s)$ with the sampling time T (sec/samples) is discretized by FOH is as follows:

$$G(z) = \frac{(z-1)^2}{Tz}\mathscr{Z}\left\{\frac{1}{s^2}G(s)\right\}. \quad (17)$$

Hence, from (1) yields

$$G(z) = \frac{(z-1)^2}{Tz}\mathscr{Z}\Bigg\{\overbrace{\frac{1}{s^2\cdot(s+1)(s+3)(s+6)}}^{F(s)}\Bigg\}, \quad (18)$$

where

$$\begin{cases} F(s) \equiv \dfrac{a}{s^2}+\dfrac{b}{s}+\dfrac{c}{(s+1)}+\dfrac{d}{(s+3)}+\dfrac{e}{(s+6)}, \\ F(s) = \left(\dfrac{\left(\frac{1}{18}\right)}{s^2}+\dfrac{\left(\frac{-27}{324}\right)}{s}+\dfrac{\left(\frac{1}{10}\right)}{(s+1)}+\dfrac{\left(\frac{1}{-54}\right)}{(s+3)}+\dfrac{\left(\frac{1}{540}\right)}{(s+6)}\right). \end{cases} \quad (19)$$

Then,

$$\begin{cases} \mathscr{Z}\{F(s)\} = \left\{\left(\dfrac{1}{18}\right)\left[\dfrac{Tz}{(z-1)^2}\right]+\left(\dfrac{-27}{324}\right)\left[\dfrac{z}{(z-1)}\right] \right. \\ \quad +\left(\dfrac{1}{10}\right)\left[\dfrac{z}{(z-e^{-T})}\right]+\left(\dfrac{1}{-54}\right)\left[\dfrac{z}{(z-e^{-3T})}\right] \\ \quad \left. +\left(\dfrac{1}{540}\right)\left[\dfrac{z}{(z-e^{-6T})}\right]\right\}. \end{cases} \quad (20)$$

Finally, we have

$$\begin{cases} G(z) = \left\{\left(\dfrac{1}{18}\right)+\left(\dfrac{-27}{324}\right)\left[\dfrac{z-1}{T}\right]+\left(\dfrac{1}{10}\right)\left[\dfrac{(z-1)^2}{T(z-e^{-T})}\right] \right. \\ \quad \left. +\left(\dfrac{1}{-54}\right)\left[\dfrac{(z-1)^2}{T(z-e^{-3T})}\right]+\left(\dfrac{1}{540}\right)\left[\dfrac{(z-1)^2}{T(z-e^{-6T})}\right]\right\}. \end{cases} \quad (21)$$

For $T = 1/50$ sec/samples, we obtain

$$\begin{cases} G(z) = \dfrac{\beta_3 z^3+\beta_2 z^2+\beta_1 z+\beta_0}{(z-e^{-T})(z-e^{-3T})(z-e^{-6T})}, \\ \beta_3 = 3.203\times10^{-7}, \quad e^{-T} = 0.98, \\ \beta_2 = 3.386\times10^{-6}, \quad ,e^{-3T} = 0.942, \\ \beta_1 = 3.254\times10^{-6}, \quad e^{-6T} = 0.887. \\ \beta_0 = 2.841\times10^{-7}, \end{cases} \quad (22)$$

Then,

$$G(z) = 10^{-5}\frac{(z+9.5139)(z+0.9608)(z+0.970)}{(z-0.9802)(z-0.9418)(z-0.8869)}. \quad (23)$$

In this work, to obtain the structure of DT PID×(*n*-2) Stage PD cascade controllers for a third order plant, the

DFOH [10] is applied. Based on the DFOH, the desired DT transfer function can be stated as

$$K(z)=\left(1-2z^{-1}+z^{-2}\right)\mathscr{Z}\left\{\mathscr{L}^{-1}\left(\frac{K(s)}{Ts^{2}}\right)\right\}. \quad (24)$$

Here,

$$\begin{aligned}K(s)&=\left(K_p+\frac{K_i}{s}+K_d s\right)\left(K_p+K_d s\right),\\&=\left(b_3 s^3+b_2 s^2+b_1 s+b_0\right)/s.\end{aligned} \quad (25)$$

Then,

$$\begin{cases}\dfrac{K(s)}{Ts^2}=\dfrac{1}{Ts^2}\left(\dfrac{b_3 s^3+b_2 s^2+b_1 s+b_0}{s}\right),\\ \mathscr{L}^{-1}\left(\dfrac{K(s)}{Ts^2}\right)=\left\{\dfrac{b_3}{T}+\dfrac{b_2}{T}\left(\dfrac{z}{z-1}\right)\cdots\right.\\ \qquad\left.+\dfrac{b_1}{T}\left(\dfrac{Tz}{(z-1)^2}\right)+\dfrac{b_0}{T}\left(\dfrac{T^2 z(z+1)}{2(z-1)^3}\right)\right\},\\ K(z)=\left(\dfrac{(z-1)^2}{z^2}\right)\left\{\dfrac{b_3}{T}+\dfrac{b_2}{T}\left(\dfrac{z}{z-1}\right)\cdots\right.\\ \qquad\left.+\dfrac{b_1}{T}\left(\dfrac{Tz}{(z-1)^2}\right)+\dfrac{b_0}{T}\left(\dfrac{T^2 z(z+1)}{2(z-1)^3}\right)\right\}.\end{cases} \quad (26)$$

Finally, we have

$$\begin{cases}K(z)=\dfrac{\beta_3 z^3+\beta_2 z^2+\beta_1 z+\beta_0}{z^2(z-1)},\\ \begin{bmatrix}\beta_3\\ \beta_2\\ \beta_1\\ \beta_0\end{bmatrix}=\dfrac{1}{2T}\begin{bmatrix}2&2&0&0\\-6&-4&2T&T^2\\6&2&-2T&T^2\\-2&0&0&0\end{bmatrix}\begin{bmatrix}b_3\\ b_2\\ b_1\\ b_0\end{bmatrix},\\ K(z)\equiv K\dfrac{(z-z_a)(z-z_b)(z-z_c)}{z^2(z-1)}.\end{cases} \quad (27)$$

From (22) and (26), the open-loop transfer function used to design the DT PID×(n-2) stage PD cascade controllers can be written as

$$\begin{cases}K(z)G(z)=K\dfrac{(z-z_a)(z-z_b)(z-z_c)}{z^2(z-1)}\cdots\\ \qquad\times 10^{-5}\dfrac{(z+9.5139)(z+0.9608)(z+0.970)}{(z-0.9802)(z-0.9418)(z-0.8869)}.\end{cases} \quad (28)$$

By using Kitti's Method to design the cascade controllers $K(z)$, let $z_a=0.9518$ and $z_b=0.8969$. Then, the open-loop transfer function without $(z-z_c)$ is

$$\begin{cases}KGwoz_c(z)=K\dfrac{(z-0.9518)(z-0.8969)}{z^2(z-1)}\cdots\\ \qquad\times 10^{-5}\dfrac{(z+9.5139)(z+0.9608)(z+0.970)}{(z-0.9802)(z-0.9418)(z-0.8869)}.\end{cases} \quad (29)$$

The desired specifications for design of the controller $K(z)$ are given in (6). Then the dominant closed-loop pole in z-Plane is

$$\begin{cases}z_d=e^{T\cdot s_d}=0.958+j0.043,\\ T=1/50\ \text{sec/ sample}.\end{cases} \quad (30)$$

Then, the necessary angle of the open-loop transfer function without the zero (z–z_c) at the dominant closed-loop pole z_d is

$$\begin{cases}KGwoz_c(z_d)=-0.036+j0.057,\\ \arg\left[KGwoz_c(z_d)\right]=122.438^{\circ}.\end{cases} \quad (31)$$

From the angle condition of the root locus method, the angle from z_c to z_d can be written as

$$\arg\left[z_c\right]=\pi-\arg\left[KGwoz_c(z_d)\right]=57.562^{\circ}. \quad (32)$$

Since, the angle of the zero (z–z_c) is less than 90°, then z_c is located at the left hand side of z_d as follows.

$$z_c=\left|\mathrm{Re}(z_d)\right|-\frac{\left|\mathrm{Im}(z_d)\right|}{\tan\left(\arg\left[z_c\right]\right)}=0.931. \quad (33)$$

Another required parameter is the controller gain K, which can be found from the magnitude condition of the root locus method as follows.

$$K=1/\left|K(z_d)G(z_d)\right|=292.683. \quad (34)$$

Finally, the controller transfer function can be stated by

$$\begin{cases}K(z)=K\dfrac{(z-z_a)(z-z_b)(z-z_c)}{z^2(z-1)},\\ \quad=292.683\times\dfrac{(z-0.9518)(z-0.8969)(z-0.931)}{z^2(z-1)}.\end{cases} \quad (35)$$

The root loci for the DT controller with the sampling time $T=1/50$ sec/ sample are shown in Fig. 7. While, the unit step response are shown in Fig. 8, respectively.

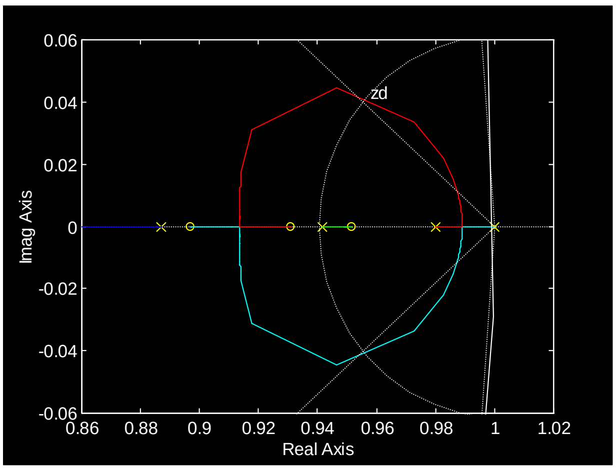

Fig. 7 Root Loci in z-Plane.

ISBN: 978-988-14047-3-2
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

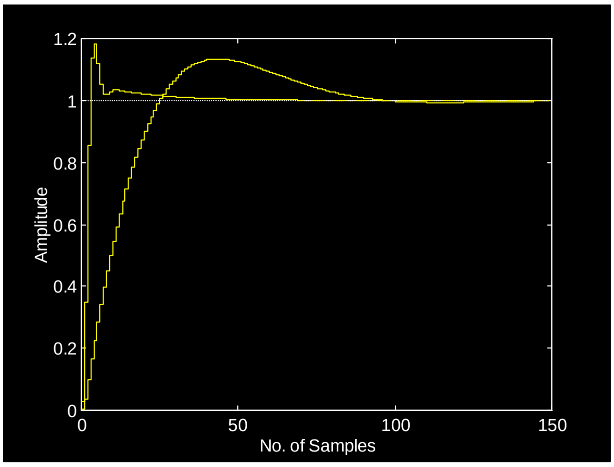

Fig. 8 Unit Step Responses.

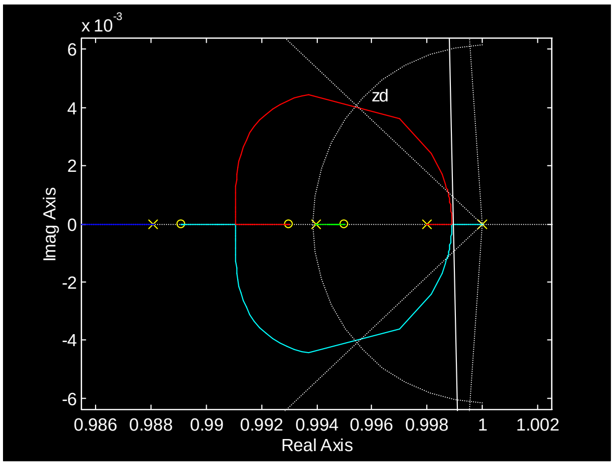

Fig. 9 Root Loci in z-Plane.

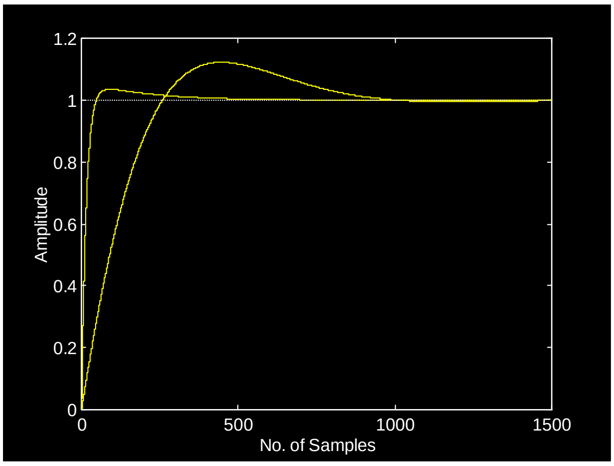

Fig. 10 Unit Step Responses.

For the sampling time T = 1/500 sec/samples, the corresponding root loci and unit step responses are shown in Fig. 9 and Fig. 10, respectively. Where, the plant transfer function, the controller transfer function and the dominant closed loop pole are as follows:

$$\begin{cases} G(z) = \dfrac{10^{-8}(z+9.8595)(z+0.9960)(z+0.1006)}{(z-0.9980)(z-0.9940)(z-0.9881)}, \\ K(z) = \dfrac{27660(z-0.9950)(z-0.9891)(z-0.993)}{z^2(z-1)}, \\ z_d = 0.996 + j4.423\times10^{-3}. \end{cases} \quad (36)$$

III. Conclusion

The design of PID×(n-2) stage PD cascade controllers in CT framework has been described to point out the aim of Kitti's method, which provides that all desired specifications to be designed can be achieved without trial and error steps in the design process. However, the original design based on Kitti's method uses the forward controller for decreasing undesired overshoot. Nowadays, the forward controller is rarely or never used, because there is alternate way to decrease the maximum percent overshoot by increasing the controller gain to be greater than the designed value, so that the plots of root loci are always toward the real axis along circular shape. If the sampling time is enough, all desired specifications are easily obtained.

References

[1] David W. Pessen, “A New Look at PID Controller Tuning”, *Transactions of the ASME, Journal of Dynamic Systems, Measurement, and Control*, Vol. 116, pp. 553-557, Sept. 1994.

[2] Seul Jung and Richard C. Dorf, “Analytic PIDA Controller Design Technique for a Third Order System,” *Proceedings of the 35th Conference on Decision and Control*, pp. 2513-2518, Kobe, Japan, December, 1996.

[3] Thanit Trisuwannawat, Kitti Tirasesth, Jongkol Ngamwiwit and Michihiko Iida, “PID×(n-2) stage PD cascade controller for SISO systems,” *SICE'98 Proceedings of the 37th SICE Annual Conference, International session Papers*, pp. 965-968, 1998.

[4] Pittaya Pannil, Suksiri Kanchanasomranvong, Prapart Ukakimaparn, Thanit Trisuwannawat, and Kitti Tirasesth, “Discrete PID×(n-2) stage Cascade Controller for SISO Systems,” *SICE Annual Con- ference*, pp. 1784-1787, The University Electro-Communications, Japan, August 20-22, 2008.

[5] Krit Smerpitak, Prapart Ukakimaparn, Thanit Trisuwannawat, and Prera Lavanprakai, “Bilinear Discrete PID×(n-2) stage PD Cascade Controller for SISO Systems,” *2012 12th International Conference on Control, Automation and Systems*, pp. 1591-1596, in ICC (International Convention Center), Jeju Island, Korea, Oct. 17-21, 2012.

[6] Suralak Charoonsote, Prapart Ukakimaparn, Pittaya Pannil, and Thanit Trisuwannawat, “Discrete PID×(n-2) Stage PD Cascade Controllers Proposed by Kitti,” *Proceedings of the 4th IIAE International Conference on Industrial Application Engineering*, pp. 258-262, B-Con Plaza, Beppu, Japan, March 26-30,2016.

[7] Hiroshi Fujimoto, "General Framework of Multirate Sampling Control and Applications to Motion Control Systems", *Ph.D. Dissertation, The University of Tokyo*, submitted in December 2000 and published in March 2001.

[8] Ryozo Nagamune, MECH468/550P: Modern Control Engineering / Foundations in Control Engineering, *Lecture Note # 6, Department of Mechanical Engineering, University of British of Columbia (UBC)*, Canada, 2008/2009.

[9] Jesse B. Hoagg, Seth L. Lacy, R. Scott Erwin, and Dennis S. Bernstein, “First-Order-Hold Sampling of Positive Real Systems And Subspace Identification of Positive Real Models,” *Proceeding of the 2004 American Control Conference*, pp. 861-866, Boston, Massachusetts, June 30 - July 2, 2004.

[10] Masayoshi Tomizuka, Tsu-Chin Tsao and Kok-Kia Chew, “Analysis and Synthesis of Discrete-Time Repetitive Controllers,” *Journal of Dynamic Systems, Measurement, and Control, Transaction of the ASME*, pp. 353-358, Vol.111, September, 1989.